Radar Target Recognition via 2-D Sparse Linear Prediction In Missing Data Case

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***Abstract*—Classical linear prediction methods based on least-square estimation yields radar images with high side lobes and many spurious scattering centers while singular value decomposition (SVD) truncation used to address these issues decreases the dynamic range of the image. So, radar images provided by these methods are not appropriate for classification purposes. In this work, sparsity constraints are induced on the prediction coefficients. The classification results demonstrate that the proposed sparse linear prediction methods give better accuracy rates compared to Multiple Signal Classification (MUSIC) method conventionally used for limited bandwidth-observation angle data. Classification performances of proposed methods are also investigated in case of the missing backscattered data. It is shown that the proposed methods are not affected from the missing data unlike the MUSIC method whose performance decreases with the increase in the percentage of the missing data.**

Keywords—radar imaging; sparse linear prediction ; BPDN; LASSO; BPDN with regularization term; missing data, classification.

I. Introduction

Nowadays, imaging radars play an important role for civil and military applications such as identification of targets. Since the performance of target identification is mainly related to the feature vectors obtained from inverse synthetic aperture radar (ISAR) image of target, the quality of the generated ISAR image has very important role in these applications. In general, the polar format algorithm (PFA) based on the 2-D Fourier transform is used for ISAR imaging [1]. However, this method fails to provide high resolved images for limited bandwidth – observation angle scenarios. One approach to overcome this issue is the use of spectral estimation methods such as MUSIC or linear prediction (autoregressive, AR) model.

MUSIC algorithm depends on the decomposition of the correlation matrix of the measurement data into signal and noise subspaces and provides radar image of the target using the noise eigenvalues [2]. Since the subspace decomposition relies on the distinction between noise and signal eigenvalues, the number of scattering centers must be determined which may represent some difficulties for complex targets. In linear prediction, collected data are modelled using 2-D forward and backward linear prediction and ISAR image is provided by the power spectrum of the resulting prediction coefficients. Since classical *l-2* norm minimization of the prediction error leads to many spurious scattering centers and high side lobes in generated ISAR image, singular value decomposition (SVD) truncation is used to suppress spurious scattering centers and side lobes [3]. Similar to MUSIC method, SVD truncation needs the number of scattering centers. A wrong choice severely affects the quality of the ISAR image: a higher value causes the elimination of weak scattering centers while a lower value causes the truncation to be inefficient. Moreover, all the methods mentioned above provide deteriorate results for the missing data case where the collected data is incomplete due to jamming or interference.

Recently, another truncation method based on the regularization of the AR parameters has been proposed [4]. In this method, basis pursuit denoising (BPDN) with penalty is used to produce the sparse linear prediction coefficients yielding radar images with decreased spurious scattering centers and side lobes. The major drawback of the method is the need to calculate the regularization parameter.

In this work, least absolute shrinkage and selection (LASSO) [5, 6] and basis pursuit denoising (BPDN) [7, 8] minimization methods are proposed to get sparse linear prediction coefficients. LASSO method yields the sparse coefficients by minimizing *l-2* norm of the residue, difference between the estimated and observed signal, when *l-1* norm of the coefficients are limited. In BPDN, sparse solution is obtained by minimizing the *l-1* norm of the coefficients while *l-2* norm of the residue is bounded with a predefined threshold value. ISAR images are obtained by drawing the power spectrum of the linear prediction with sparse linear prediction coefficients. In these images, side lobes and spurious scattering centers are suppressed. Then, the effects of the suggested methods on the classification performance are investigated. The method proposed by [9] is employed for the classification. Classification accuracy of methods are analysed when collected data are incomplete. In section 2, theory of linear prediction method and sparse linear prediction model and classification method are presented. In section 3, reconstructed ISAR images are shown and classification accuracy for full and missing data cases is presented. Conclusions are given in section 4.

II. Radar Imaging with Sparse Linear Prediction

A. Problem Formulation

The ISAR image is obtained by processing the backscattered data that are collected at frequency-aspect domain. Backscattered signal from the target at different frequencies and different angles can be approximated as [1]:

$$E(f_m,\emptyset_n)=u(f_m,\emptyset_n)+\sum_{i=1}^{d}A_i e^{-j4\pi\frac{f_m}{c}(x_i\cos\emptyset_n+y_i\sin\emptyset_n)} \quad (1)$$

Frequency aspect domain data are transformed into spatial frequency domain data to get focused image. As a result of this transformation, data are uniformly spaced in spatial frequency domain. Transformation of $E^s(f,\emptyset)$ to spatial frequency domain is shown below:

$$E\big(f_x(l),f_y(n)\big)=\sum_{i=1}^{d}A_i e^{-j2\pi(f_x(l)x_i+f_y(n)y_i)} \quad (2)$$

$$f_x(l)=f_x(0)+l\Delta f_x \qquad l=0,1,\dots N-1$$
$$f_y(n)=f_y(0)+n\Delta f_y \qquad n=0,1,2,\dots.M-1$$

where $f_x(0)$ and $f_y(0)$ are the initial values of f_x and f_y.

Quarter plane models can be used for 2-D linear prediction of the backscattered data [3]. The first and third quadrants for linear prediction of backscatter field are given as:

$$\hat{E}(l,n)=-\sum_{i=0}^{p}\sum_{j=0}^{p}a_{ij}\,E(l-i,n-j) \quad (3)$$
$$i=j\neq 0 \qquad l,n=p,p+1,\dots,N-1$$

$$\hat{E}(l,n)=-\sum_{i=0}^{p}\tilde{a}_{ij}\,E(l+i,n+j) \quad (4)$$
$$i=j\neq 0 \quad l,n=0,1,\dots,N-p-1$$

Since a_{ij}=$\tilde{a}_{ij}$, (4) is written as:

$$\hat{E}^*(l,n)=-\sum_{i=0}^{p}a_{ij}\,E^*(l+i,n+j) \quad (5)$$
$$i=j\neq 0 \quad l,n=0,1,\dots,N-p-1$$

By combining the (3) and (5),

$$\boldsymbol{Ea}=-\boldsymbol{e} \quad (6)$$

After obtaining equation set, linear prediction coefficients $\boldsymbol{a}$ are calculated by the least square error solution. Least square error solution is given below [3]:

$$\boldsymbol{a}=-(\boldsymbol{E^H E})^{-1}\boldsymbol{E^H e} \quad (7)$$

Using the second quadrant and forth quadrant models consist of backward and forward predictions, equation set is written as:

$$\boldsymbol{Eb}=-\boldsymbol{e} \quad (8)$$

Power spectrum of linear prediction whose peaks give the radar image is calculated from [3]:

$$P(x,y)=\frac{1}{\left|1+\sum_{i=0}^{p}\sum_{j=0}^{p}a_{ij}\,z_1^{-i}z_2^{-j}\right|^2+\left|1+\sum_{i=0}^{p}\sum_{j=0}^{p}b_{ij}\,z_1^{-i}z_2^{-j}\right|^2} \quad (9)$$
$$i=j\neq 0$$

Where $z_1=e^{j4\pi/c\Delta f_x x}$ and $z_2=e^{j4\pi/c\Delta f_y y}$.

Obtained ISAR image has many spurious scattering centers and high side lobes when linear prediction coefficients are calculated by using least square error solution. SVD can suppress these spurious scattering centers and sidelobes by predicting the number of scattering centers [3]. However, prediction of number of scattering centers is not easy. MUSIC algorithm based on the decomposition of the signal and noise subspaces is another spectral estimation method. Detailed information and implementation of this method can be found in [2].

B. Sparse Linear Prediction

In this study, linear prediction coefficients are calculated by using sparsity approach and ISAR images are generated depending on the these sparse coefficients. Sparse linear prediction coefficients are obtained by solving:

$$min(\|\boldsymbol{a}\|_0) \quad \text{subject to } \boldsymbol{e}=\boldsymbol{Ea} \quad (10)$$

minimization problem. Since *l-0* norm minimization is NP hard problem, *l-0* norm is replaced by *l-1* norm.

In the literature, several approaches exist to obtain sparse solution. The most conventional ones among them are LASSO, BPND and BPDN with regularization term. In [4], BPDN with regularization term is investigated to get the sparse linear prediction coefficients. The minimization problem of this approach is given as:

$$\min\big(\ \|\boldsymbol{Ea}-\boldsymbol{e}\|_2^2+\lambda\|\boldsymbol{a}\|_1\big) \quad (11)$$

In (11), selection of λ plays an important role. Optimal value of λ is found form L-curve [10].

LASSO minimization problem is given as [5, 6]

$$\min\big(\|\boldsymbol{Ea}-\boldsymbol{e}\|_2^2\ \big) \quad subject\ to\ \|\boldsymbol{a}\|_1<\tau \quad (12)$$

l- 2 norm of residue is minimized by limiting the *l-1* norm coefficients. Sparse AR coefficients are determined by applying this minimization problem on (6) and (8). While the number of spurious scattering centers is too many for high τ values, it can be decreased by choosing lower values of τ. The optimum value of τ is obtained by using empirical method which is given in (13). In this method, the first value of τ is determined by *l-1* norms of linear prediction coefficients produced from (7).

$$a = -(E^H E)^{-1} E^H e \quad \rightarrow \quad \tau_0 = \|a\|_1 \tag{13}$$

The optimum value of τ is obtained by decreasing from τ_0 .

BPDN sparsity approach which minimize *l-1* norm coefficients by limiting the *l-2* norm of residue is given as [7]

$$\min(\ \|\boldsymbol{a}\|_1) \quad subject\ to\ \|\boldsymbol{Ea} - \boldsymbol{e}\|_2 < \in \tag{14}$$

Although the suppression of spurious scattering centers is not possible for small $\in$ values, it is successfully performed by increasing the $\in$ value. The optimum $\in$ value is determined empirically which is given in (15). The first value of $\in$ is calculated by using the *l-2* norm of the residue obtained from linear prediction coefficients resulting of the least square error.

$$a = -(E^H E)^{-1} E^H e \quad \rightarrow \ \in_0 = \ \|Ea - e\|_2 \tag{15}$$

Then, the $\in_0$ is increased and the spurious scattering centers are suppressed. All optimization problems are solved by using CVX optimization packet [11].

C. *Classification of ISAR Images*

For the classification of generated ISAR images, feature extraction method which is proposed in [9]. The proposed algorithm whose steps are given in Fig. 1 can be summarized as:

i. Polar ISAR image $I_p(r, \theta)$ is obtained from ISAR image space by using polar mapping.
ii. Size of the polar image is decreased by using principal component analysis (PCA)
iii. Additionally, the set of feature is obtained by using the projections of polar image onto r and θ axis.
iv. Classification is performed by using k-nearest neighbourhood (kNN) algorithm.

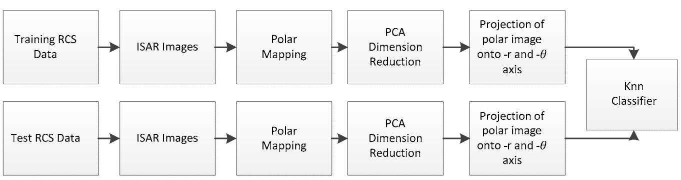

Figure 1: Block diagram of classification method [9]

III. Experimental Results

A. *ISAR IMAGES BASED SPARSE RESPRESENTATIONS*

Radar images are obtained by applying the proposed sparse linear prediction methods on the simulated Mig25 data with size, center frequency, bandwidth and aspect angle are 64x64, 9 GHz, 531 MHz and 3.67° , respectively [1]. SNR is set to 30 dB by adding White Gauss noise. In [12], ISAR images are obtained by employing the mentioned methods on the wideband and narrowband data. In this work, we handle the narrowband data case so size of data is decreased to 32x32 data whose center frequency is 9 GHz and bandwidth is 256 MHz to observe the performance of the methods on the narrowband data.

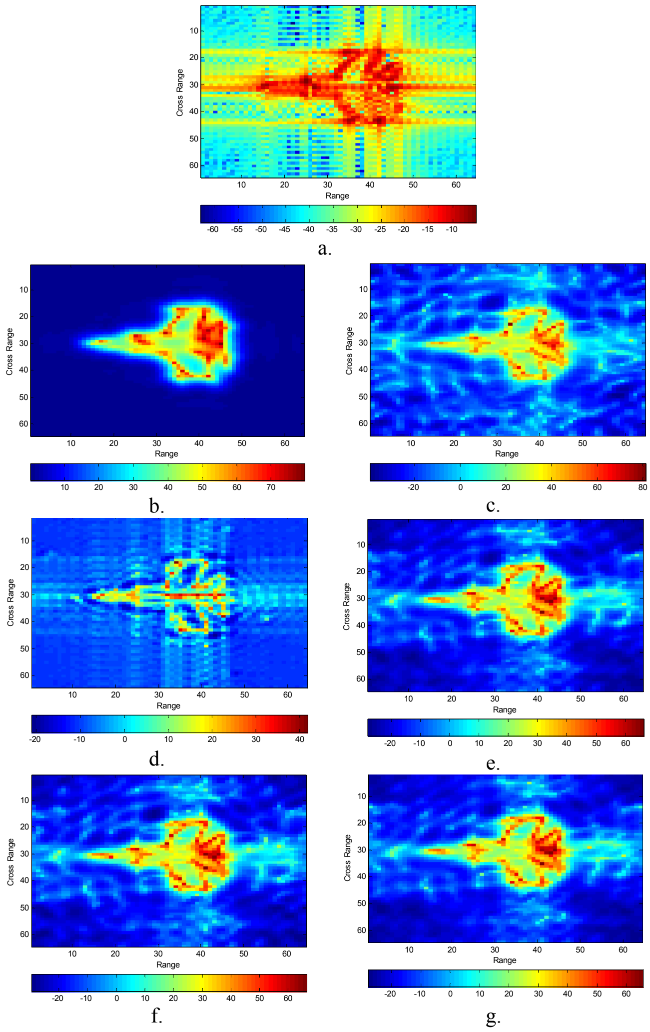

Figure 2: ISAR image of Mig25 narrowband data (SNR=30 dB) a.)PFA, b.) MUSIC , Linear prediction order :12, c.) LSE solution , d.) SVD , e.)LASSO ($\tau = \tau_i/4$), f.) BPDN ($\in = 1.10 \in_i$) , g.) BPDN with penalty ($\boldsymbol{\lambda}$ =1.5).

MUSIC algorithm losses the some scattering centers at the tail of the plane as seen from Fig. 2.b. If the AR based models are compared, one can easily see that the best results are obtained by using sparse linear prediction. When the least square solution is considered, less side lobes at the background occur by using the sparse linear predictions. When MUSIC and sparse linear prediction methods are compared, it is observed that scattering centers at the tail of plane are successfully resolved by the use of the sparse linear prediction method.

B. CLASSIFICATION OF RECONSTRUCTED ISAR IMAGES

For classification of ISAR image, the real data which are collected by Sandia National Laboratory for MSTAR (Moving Stationary Target Acquisition and Recognition) program [13]. The data are collected at 9.6 GHz center frequency and 591 MHz bandwidth. The backscattered data at different aspect angles belongs to T-72, BMP2 and BTR70. 232 different images for training and 196 different image for test are available for each target. So, training and test data sets contain 696 and 588 images, respectively. The size of wideband data is decreased to 32x32 to provide narrowband data whose center frequency is 9.6 GHz.

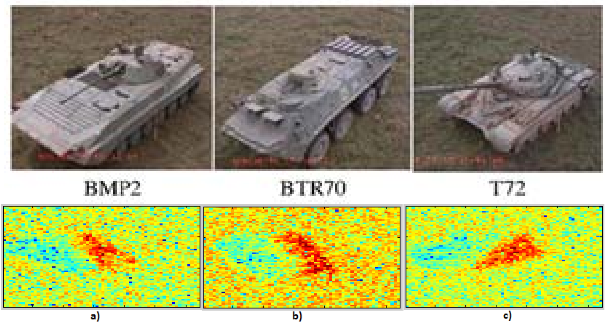

Figure 3: Real and ISAR images of MSTAR targets [14]

While radar system receives the backscattered signal from target, some burst may be missing in the collected signal. Fig. 4 shows the how missing backscattered data is generated from the received signal. These data are used to reconstruct the ISAR images and these images are classified. To get more reliable results, 5 Monte Carlo simulations are performed using different missing burst.

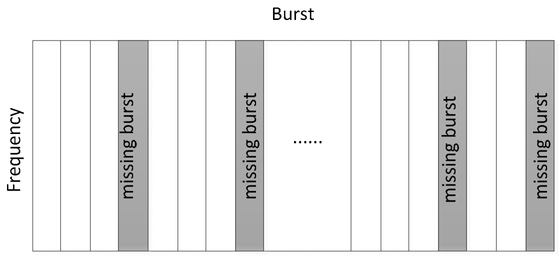

Figure 4: Received missing backscattered data

In Table 1, classification results are presented for original and 10 % missing narrowband data. For full narrowband data, it is seen that MUSIC algorithm's performance is better than the other well-known methods such as PFA, Least Square Error and SVD but not better than proposed sparse linear prediction methods. As seen from the table, classification performance accuracy is increased by employing sparse linear prediction methods. When we handle the classification results for 10% missing narrowband data, still linear prediction methods are the best choice to enhance the classification ratio. In addition to that, it is observed that classification result of least square error is higher than MUSIC's since linear prediction coefficients are calculated by using the linear combination of forward and backward samples.

Table 1: Classification performance for narrowband and narrowband missing data

Method	Narrowband Result (%)	Narrowband Missing Result (%)
PFA	73.12 %	73.46 %
MUSIC (smoothing parameter=8)	73.8%	71.42 %
Least Square Error (AR order=8)	72.1 %	72.62 %
SVD (AR order=8)	70.74 %	72.44%
Sparse AR BPDN (ϵ=1.02)	76.7 %	**74.32 %**
Sparse ARBPDN with penalty (λ=4)	**77.3 %**	72.9 %
Sparse AR LASSO (ϵ=0.5)	76.1 %	73.2 %

In Table 2 and Table 3, confusion matrices of whole methods are presented for original and 10% missing narrowband data, respectively.

Table 2: Confusion matrix for narrowband data

Methods	Targets	BMP2	BTR70	T72
PFA	BPM2	124	28	30
	BTR70	18	142	2
	T72	54	26	164
MUSIC	BPM2	134	33	31
	BTR70	31	142	7
	T72	31	21	158
Least squaree	BPM2	130	36	29
	BTR70	21	133	6
	T72	45	27	161
SVD	BPM2	114	36	31
	BTR70	25	143	6
	T72	57	17	159
BPDN with penalty	BPM2	132	31	16
	BTR70	17	145	2
	T72	47	20	178
BPDN	BPM2	129	30	16
	BTR70	19	144	2
	T72	48	22	178
LASSO	BPM2	125	31	20
	BTR70	14	151	4
	T72	57	14	172

Table 3: Confusion matrix for narrowband missing data

Methods	Targets	BMP2	BTR70	T72
PFA	BPM2	126	27	31
	BTR70	27	147	6
	T72	43	22	159
MUSIC	BPM2	116	42	14
	BTR70	28	132	10
	T72	52	22	172
Least squaree	BPM2	124	37	24
	BTR70	25	140	9
	T72	47	19	163
SVD	BPM2	126	36	29
	BTR70	28	141	8
	T72	42	19	159
BPDN with penalty	BPM2	127	32	20
	BTR70	24	146	12
	T72	45	18	164
BPDN	BPM2	127	31	24
	BTR70	25	143	14
	T72	44	22	158
LASSO	BPM2	125	26	26
	BTR70	27	150	15
	T72	44	20	155

In Fig. 5, effect of the percentage of the missing backscattered data on the methods is presented. One may observe that linear prediction based methods are more robust to missing data compared to MUSIC method.

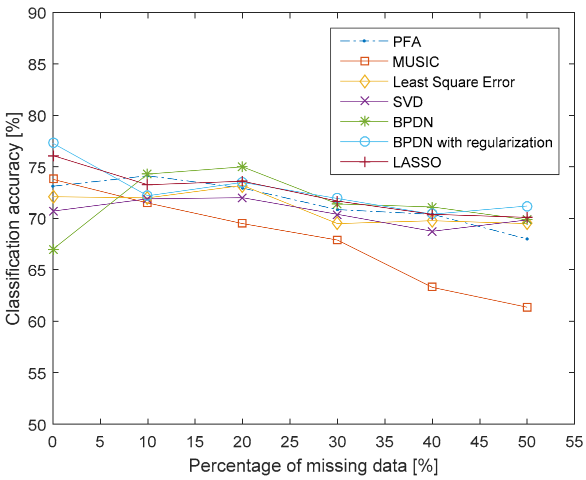

Figure 5: Classification result versus missing data rate

IV. CONCLUSION

In this work, several sparse linear prediction models are introduced to obtain radar images with less spurious scattering centers and decreased side lobes. Since the distinction of the target from the background becomes more obvious, these approaches increase the radar target classification rates, as expected. Moreover, linear prediction methods especially the proposed sparsity based ones are more robust to missing data unlike conventional MUSIC method whose performance is highly deteriorated with the increase of the percentage of missing data. They do not suffer from the decreased dynamic range as *l-2* norm minimization based least-square method. Thus sparse linear prediction based radar images can be considered as a good alternative to MUSIC for radar imaging and classification applications.

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