Location Information Driven Formation Control for Swarm Return-to-Base Application

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***Abstract*—On Mars there is no global positioning system available. In this paper we present an analysis of a relative localization system that acts as a moving swarm to estimate the location of its base. Our algorithm jointly processes two objectives. First to shape an optimized swarm structure to estimate the location of the base reliable and second to move the swarm together towards the base to return home. The estimate of the base location is used to return to it by controlled movements considering constraints such as the minimum distance between the swarm elements to avoid collisions. The performance comparison of our location information driven algorithm with goal approaching or flocking algorithm shows a robust behavior with a much higher efficiency.**

I. Introduction

In the past missions to Mars only single rover is used to explore the area and to perform different experiments. However, a single rover jeopardizes the whole mission with the risk of a single point of failure. A swarm brings in additional benefits such as simultaneous and fast exploration of larger areas through cooperation between the rovers. On Mars no global positioning system exists, such as the Global Navigation Satellite System on Earth. To navigate on Mars, a rover often relies on visual and inertial navigation systems, which strongly depends on environment, or external navigation infrastructure, including multiple anchors, e.g. mission base or fly-by orbiters, which is limited by its temporal/spatial availability.

In this paper we highlight the navigation capability of a swarm by exploiting relative position. The swarm expands its own reference system and collaboratively achieves navigation objectives, such as return-to-base, without any absolute position information. The rovers of the swarm, also known as the agents (AGs), estimate the distance to each other and to the mission base with received radio communication and navigation signal. Then the formation of swarm and the position of base relative to the swarm are jointly estimated, similarly to the simultaneous network localization and target tracking, for example, in [1]. The information quality is closely linked to the ranging model that is based on the radio signal [2]. The performance of the range estimate is distance dependent and even limited by a hear-ability distance. The base broadcasts its beacon signal much stronger to the swarm than the intra-swarm communications to cover a larger exploration area. Fundamentals of cooperative localization was introduced in [3] and extended in [4]. In [5] swarm self-localization with limited radio resource was discussed. Beacon direction estimation with uncertain swarm formation information is investigated in [6].

Moreover, the swarm can be controlled to form certain formations in order to enhance its capability. Flocking algorithm is proposed by Olfati-Saber in [7], where a regular triangular lattice formation is formed, which is preferable for collaborative observation and message exchange. In [8] the authors have the explicit aim to control agents to improve or maintain the localization quality through information-seeking. In [9] an alternating goal approaching and location information seeking formation control is proposed with the help from multiple anchors. We propose a location information-driven formation control, where the swarm intends to build an optimal formation for better estimating the relative position of the mission base in order to move to base efficiently. The swarm movement is constrained by a limited overall velocity and a minimum distance between the agents for collision avoidance. We exploit the position Cramér-Rao bound (CRB) to design such a multi-objective controller. As the consequence, the swarm spreads out to improve the geometrical constellation and keep connected within the hear-ability range. Meantime, the swarm moves towards the base with a certain minimal velocity. We show that the sensibility of the algorithm relies on the accurate relative position of the swarm and that there are substantial improvements compared to the well-known flocking algorithm.

The paper is structured as follows: In Section II we formulate the system model and the return-to-base problem. In Section III we describe how the rovers of the swarm control their move towards the base together with a collision avoidance distance. In Section IV we compare the presented algorithm against the goal approaching and flocking algorithm. Finally, we conclude with Section V.

II. System and Problem Formulation

A. System Formulation for Swarm Return-to-Base

We consider a return-to-base application with $M+1$ nodes, including M mobile AGs and a single stationary mission base. AGs form a swarm, trying to return to the base. Neither the swarm location nor the base location is known to the swarm. The swarm needs to seek for sufficient location information by exploiting inter-node radio signal in order to approach the base. We define the set of all nodes as $\mathbb{K}$, and the set of agents as $\mathbb{M}$. For a generic AG u, the state space at time step

k corresponds to the two-dimensional position, i.e., $\mathbf{p}_u^{(k)} = [x_u^{(k)}, y_u^{(k)}]^{\mathrm{T}}$. The mission base is fixed to a position $\mathbf{p}_b^{(k)} = [x_b, y_b]^{\mathrm{T}}$. The positions of agents are controlled as follows: agent u moves step-wise with a control command $\mathbf{u}_u^{(k)}$ and a transition noise $\boldsymbol{\epsilon}_u^{(k)}$:

$$\mathbf{p}_u^{(k)} = \mathbf{p}_u^{(k-1)} + \mathbf{u}_u^{(k)} + \boldsymbol{\epsilon}_u^{(k)}, \qquad u \in \mathbb{M}. \tag{1}$$

The state $\mathbf{p}^{(k)}$ of all AGs is obtained by stacking the states of the individual agents into a vector, and can be expressed as

$$\mathbf{p}_{\mathbb{M}}^{(k)} = \mathbf{p}_{\mathbb{M}}^{(k-1)} + \mathbf{u}_{\mathbb{M}}^{(k)} + \boldsymbol{\epsilon}_{\mathbb{M}}^{(k)}. \tag{2}$$

where $\boldsymbol{\epsilon}_{\mathbb{M}}^{(k)}$ is the global transition noise. The global state vector of all nodes is denoted as: $\mathbf{p}^{(k)} = [(\mathbf{p}_{\mathbb{M}}^{(k)})^T, (\mathbf{p}_b^{(k)})^T]^T$. For simplicity, we will omit the superscript (k) excluding cases of ambiguity.

B. Swarm Localization

In order to return to the base, swarm localization is needed. Agent transmits radio signal for intra-swarm communication and localization. We assume the AGs equipped with stable clocks, which makes the location information observable from the received radio signal waveform. This signal is exploited for propagation time-based inter-agent distance estimate . Meantime, the base emits radio signal as beacon, in order to guide the swarm to return. The distance $d_{u,v} = \|\mathbf{p}_u - \mathbf{p}_v\|$ between two generic nodes u and v is estimated as $z_{u,v}$. The estimate noise is distance dependent with a variance $\sigma_{u,v}^2(d_{u,v})$. The distances of all node pairs and their estimates are denoted by vectors $\mathbf{d}$ and $\mathbf{z} \in \mathbb{R}^L$ respectively, where L is the number of measurement links. We model the swarm system as a fully connected network with independent links, i.e. $L = M(M+1)/2$. Similarly as in [9], links with distance larger than the node hear-ability are modeled with a large a-priori ranging variance. For conventional localization systems, sufficient number of anchors with known position, e.g. navigation satellites or base stations, form an absolute reference system. The task is to locate agents in this reference system. In contrast, in our system only the inter-node measurement is available, thus nodes can only be located with respect to a swarm-level reference system. Swarm localization aims to simultaneously estimate the swarm formation, generating a swarm-level reference from the formation and locating the base relative to the swarm. This relative location information is sufficient to generate control commands, which determine moving directions of AGs relative to the swarm reference. Choosing an appropriate swarm reference is important in swarm localization, as discussed in [10]. Three arbitrary entities in $\mathbf{p}_{\mathbb{M}}$ can define a swarm reference, since three is the minimal sufficient number of constraints to prevent the global rigid motion of the swarm estimate. Without loss of generality, we choose the first three entities, i.e. $\mathbf{r} = [x_1, y_1, x_2]^T$, to span the reference system. The interpretation of the choice is that the two reference AGs form a baseline. The remaining global state vector, defined as $\mathbf{s} = [\mathbf{s}_{\mathbb{M}}^T, \mathbf{p}_b^T]^T$, are to be estimated with respect to this baseline. The vector $\mathbf{s}_{\mathbb{M}}$ is a sub-vector from $\mathbf{p}_{\mathbb{M}}$, excluding $\mathbf{r}$. The global state vector can be simplified as $\mathbf{p} = [\mathbf{r}^T, \mathbf{s}^T]^T$. Relative localization is a well addressed topic, e.g. in [10] and [5]. In order to focus on formation controller design, in this paper we model the position estimate relative to the baseline from a generic estimator by the CRB described in III-C.

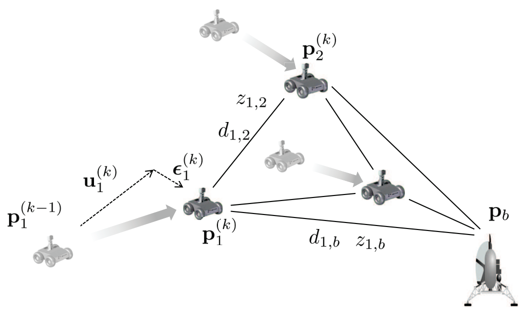

Figure 1. A swarm navigation system with three AGs and a single base

III. Swarm Formation Control

A. Navigation Objectives and Constraints

A swarm navigation application is normally a multi-objective optimization problem. The global control command for swarm $\mathbf{u}_{\mathbb{M}}$ is generated based on the objectives of the application and the position estimates at previous step to decide where to go next. For the swarm return-to-base application, the objectives include:

Problem $\mathcal{P}_1$ - location information seeking:

$$\underset{\mathbf{u}_{\mathbb{M}} \in \mathbb{U}_{\mathbb{M}}}{\text{minimize}} \|\hat{\mathbf{s}} - \mathbf{s}\|; \tag{3}$$

Problem $\mathcal{P}_2$ - collision avoidance:

$$d_{u,v} > d_{\text{min}} > 0, \quad \forall u \neq v \in \mathbb{K}; \tag{4}$$

Problem $\mathcal{P}_3$ - return-to-base:

$$\mathbf{u}_{\mathbb{M}}^T \cdot \mathbf{e}_{b,\mathbb{M}} > \kappa > 0, \tag{5}$$

$$\text{where } \mathbf{e}_{b,\mathbb{M}} = \frac{\mathbf{1}_{M\times 1} \otimes \mathbf{p}_b - \mathbf{p}_{\mathbb{M}}}{\|\mathbf{1}_{M\times 1} \otimes \mathbf{p}_b - \mathbf{p}_{\mathbb{M}}\|}. \tag{6}$$

The symbol $\otimes$ denotes the Kronecker product, which stacks the base position into a vector with the same size as $\mathbf{p}_{\mathbb{M}}$. The set $\mathbb{U}_{\mathbb{M}}$ is the applicable controller set. The problem $\mathcal{P}_1$ aims to seek location information jointly for swarm and base. It can be solved with a location information driven formation control. The problem $\mathcal{P}_2$ and $\mathcal{P}_3$ are considered as the primary and secondary constraints respectively to the optimization solution. The constraint $\mathcal{P}_2$ guarantees that any two nodes are distanced to avoid potential collisions. Whereas $\mathcal{P}_3$ guaranties the swarm arrive at the base after certain number of time steps, controlled by κ. A swarm system is illustrated in Fig. 1, where three AGs move from time step $k-1$ to time step k towards a base.

B. Flocking

Flocking is a heuristical algorithm proposed in [7] using collective virtual potentials to form a regular formation.

An agent is affected by attractive/replusive force to keep a fixed distance with neighboring agents in the hear-ability. The swarm objective direction is modeled as a virtual agent with only attractive force to lead the flock towards a goal. With these simple rules, complex swarm collective behavior can emerge, e.g. rendezvous, split, rejoin, squeezing, etc. depending on the environment. In free space, the swarm will be stabilized to a regular triangular lattice formation. With this formation, the swarm seamlessly covers the whole area and has bounded stretch, which is preferable primarily for collaborative observation and message exchange. We found that this formation is also beneficial for the return-to-base application. The controllable bounded stretch of the formation offers a large aperture for observing the direction of the base without losing precision for formation estimate, which partially solves the problem $\mathcal{P}_1$. The fixed desired inter-node distance inherently guaranties collision avoidance in $\mathcal{P}_2$. Last but not least, the flocking algorithm can be easily adapted to the return-to-base objective, i.e. $\mathcal{P}_3$, by setting the virtual agent moving towards the base. However, in flocking, local geometric information is assumed to be precisely known, and also the inter-node distance is pre-defined, which makes the algorithm less adaptive to an application with little location knowledge. In this work, we use a modified flocking algorithm as a benchmark for comparison.

C. Location Information Driven Formation Optimization

To find a controller dedicated to the problem $\mathcal{P}_1$, we exploit the theory of Fisher information and the CRB. The CRB is a lower bound on the estimation error variance of any unbiased estimator, and is expressed as

$$\mathbf{E}\left[\|\hat{\mathbf{s}}-\mathbf{s}\|^2\right] \geq \mathrm{CRB}[\mathbf{s}] = \mathrm{tr}\left(\mathbf{F}_{\mathbf{s}}^{-1}\right), \tag{7}$$

where $\mathbf{F}_{\mathbf{s}} = \mathbf{H}_{\mathbf{s}}\mathbf{F}_{\mathbf{d}}\mathbf{H}_{\mathbf{s}}^T$ is the Fisher information matrix (FIM) of $\mathbf{s}$. The geometry matrix $\mathbf{H}_{\mathbf{s}} = \nabla_{\mathbf{s}}\mathbf{d}^T \in \mathbb{R}^{L\times(2M-1)}$ expresses the effect from the geometry relation among the nodes. The diagonal ranging FIM $\mathbf{F}_{\mathbf{d}}$ shows the impact from the distance estimate accuracy. Under a conservative Gaussian ranging assumption as in [9], the l-th diagonal entry in $\mathbf{F}_{\mathbf{d}}$, which corresponds to link (u,v), can be derived as in [11, eq. (3.31)]

$$\lambda_{u,v}^{-2} \triangleq (\mathbf{F}_{\mathbf{d}})_{l,l} = \sigma_{u,v}^{-2} + \frac{1}{2}\left(\frac{\partial\sigma_{u,v}^2}{\partial d_{u,v}}\right)^2 / \sigma_{u,v}^4. \tag{8}$$

We can reformulate the location information problem $\mathcal{P}_1$ as

$$\underset{\mathbf{u}_{\mathbb{M}}\in\mathbb{U}_{\mathbb{M}}}{\text{minimize}} \quad \mathrm{tr}\left(\mathbf{F}_{\mathbf{s}}^{-1}\right). \tag{9}$$

We will now proceed to solve the modified problem in (9), which is a highly non-convex problem. We propose a gradient approach to find the locally optimal solution similar to the scheme in [12] and [9]. The major difference to the previously mentioned works is that the swarm formation is optimized to jointly minimizing the formation estimate error and the base location error relative to the swarm-level reference system. The gradient $\mathbf{c}_{\mathbb{M}} \in \mathbb{R}^{2M}$ of the objective function is

$$\mathbf{c}_{\mathbb{M}} = \left[\mathbf{c}_1^{\mathrm{T}}, \ldots, \mathbf{c}_u^{\mathrm{T}}, \ldots, \mathbf{c}_M^{\mathrm{T}}\right]^{\mathrm{T}} = \nabla_{\mathbf{p}_{\mathbb{M}}}\mathrm{tr}\left(\mathbf{F}_{\mathbf{s}}^{-1}\right). \tag{10}$$

We further define $\mathbf{X}_{\mathbf{s}} = \mathbf{F}_{\mathbf{s}}^{-1}\mathbf{F}_{\mathbf{s}}^{-1}$,

$$\mathbf{X} = \begin{pmatrix} \mathbf{0}_{3\times(2M-1)} & \mathbf{0}_{3\times3} \\ \mathbf{X}_{\mathbf{s}} & \mathbf{0}_{(2M-1)\times3} \end{pmatrix}, \tag{11}$$

$\mathbf{X}_{u,v} \in \mathbb{R}^{2\times2}$ as the sub-matrix $\mathbf{X}_{2u-1:2u,2v-1:2v}$ and $\mathbf{Y}_{u,v} = \mathbf{X}_{u,u} + \mathbf{X}_{v,v} - \mathbf{X}_{u,v} - \mathbf{X}_{v,u}$. The gradient component of AG u, $\mathbf{c}_u \in \mathbb{R}^2$, is expressed as

$$\begin{aligned}\mathbf{c}_u = -\sum_{v\neq u\in\mathbb{K}} & 2\frac{\left(\mathbf{I}-\mathbf{e}_{u,v}\mathbf{e}_{u,v}^{\mathrm{T}}\right)\mathbf{Y}_{u,v}\mathbf{e}_{u,v}}{\lambda_{u,v}^2 d_{u,v}} \\ & + \frac{\partial\lambda_{u,v}^{-2}}{\partial d_{u,v}}\mathbf{e}_{u,v}\mathbf{e}_{u,v}^{\mathrm{T}}\mathbf{Y}_{u,v}\mathbf{e}_{u,v}.\end{aligned} \tag{12}$$

The vector $\mathbf{e}_{u,v} = (\mathbf{p}_u - \mathbf{p}_v)/d_{i,v}$ is directional vector pointing from node v to AG u.

The steepest descent gradient controller solving $\mathcal{P}_1$ with a step size $\mu_{\mathbb{M}}$ can be expressed as

$$\mathbf{u}_{\mathbb{M}} = -\mu_{\mathbb{M}}\frac{\mathbf{c}_{\mathbb{M}}}{\|\mathbf{c}_{\mathbb{M}}\|}. \tag{13}$$

The gradient $\mathbf{c}_u$ from (12) is evaluated in the position estimates $\hat{\mathbf{p}}$.

D. Formation Optimization under Constraints

In order to find a solution for formation optimization fulfilling $\mathcal{P}_2$ and $\mathcal{P}_3$, a projection gradient method is applied. When a constraint is activated, the control command generated according to location information seeking $\mathbf{u}_{\mathbb{M}}$ is projected on to the tangent subspace of that constraint. For the collision avoidance, i.e. $\mathcal{P}_2$, the constraints in (4) can be re-described as:

$$g_{u,v} \triangleq \|d_{u,v}\| - d_{\text{min}} > 0, \forall u \neq v \in \mathbb{K}. \tag{14}$$

We define a constraint violation set $\mathbb{C}_2$ with a size of $\|\mathbb{C}_2\| = V$ and stack all the violation distances into a vector $\mathbf{d}_{\text{c}2}$. We stack all the $g_{u,v}(\mathbf{p}) \in \mathbb{C}_2$ into a vector $\mathbf{g}_{\text{c}2} = \mathbf{d}_{\text{c}2} - \mathbf{1}_{V\times1}\cdot d_{\text{min}}$ and its partial derivative with respect to $\mathbf{p}_{\mathbb{M}}$ into a matrix $\mathbf{N}_{\text{c}2}$. The projection matrix to the tangent subspace of these constraints can be expressed as

$$\mathbf{P}_{\text{c}2} = \mathbf{I} - \mathbf{N}_{\text{c}2}\left(\mathbf{N}_{\text{c}2}^T\mathbf{N}_{\text{c}2}\right)^{-1}\mathbf{N}_{\text{c}2}^T. \tag{15}$$

The control command after projection is

$$\mathbf{u}_{\mathbb{M}2} = \mathbf{P}_{\text{c}2}\cdot\mathbf{u}_{\mathbb{M}} - \mathbf{N}_{\text{c}2}\left(\mathbf{N}_{\text{c}2}^T\mathbf{N}_{\text{c}2}\right)^{-1}\mathbf{g}_{\text{c}2}. \tag{16}$$

Interestingly, $\mathbf{N}_{\text{c}2}$ is the geometry matrix of the distance of the violation links and the controllable state $\mathbf{p}_{\mathbb{M}}$, i.e.

$$\mathbf{N}_{\text{c}2} = \mathbf{H}_{\text{c}2} = \nabla_{\mathbf{p}_{\mathbb{M}}}\mathbf{d}_{\text{c}2}^T. \tag{17}$$

Similarly, the return-to-base objective $\mathcal{P}_3$ can be also written as a constraint:

$$g_{\text{c}3} = \mathbf{u}_{\mathbb{M}}^T\cdot\mathbf{e}_{b,\mathbb{M}} - \kappa > 0 \tag{18}$$

$$\mathbf{u}_{\mathbb{M}3} = \left(\mathbf{I} - \mathbf{e}_{b,\mathbb{M}}\cdot\mathbf{e}_{b,\mathbb{M}}^T\right)\mathbf{u}_{\mathbb{M}} - \mathbf{e}_{b,\mathbb{M}}\cdot g_{\text{c}3}. \tag{19}$$

The projection becomes active when the constraint is violated.

IV. Numerical Results

In the following we present numerical results of four different algorithms. These are the (1) goal approaching algorithm, where formation and base position is estimated, and swarm moves towards the estimated base position only trying to keep safety distance; (2), (3) the flocking algorithm described in III-B with desired distance of 10 m and 20 m; and (4) the location information driven algorithm proposed in III-C.

A. Setup and Evaluation Metrics

Ranging Model: We conducted numerical simulations to assess the performance of the proposed formation controller. The distance estimate is acquired from a radio-based OFDM modulated ranging signal that is attenuated by a free-space pathloss model with additive white Gaussian noise at the receiver. The ranging model is similar to the one used in [9], where ranging variance is quadratically proportional to the distance until a low signal-to-noise ratio is reached, then the ranging variance rapidly rises to the a-priori level due to the misdetection. The diverging distance for a intra-swarm link is set to around 20 m, considering the low energy consumption requirement for a Mars rover. The mission base is assumed to transmit with much larger power and the ranging variance is always quadratically proportional to the distance in the operational area.

Simulation Scenario: A swarm of 24 rovers intends to return to a base 10 km away with an unknown position. The total speed of the swarm is set to 1 m per time step for all the four algorithms. For the location information driven control, a maximum 5-percent of the speed is allocated for the formation optimization, i.e. $\mu_{\mathbb{M}} = 0.05m/$step. The minimum safety distance between nodes is set to two meters. A snapshot of the four algorithms after 25000 time steps shows in Fig. 2- 5.

Performance Metrics: Besides for the snapshot, we consider the remaining distance versus time steps in Fig. 6, joint error of the formation and the base position estimate error in Fig. 7, angular error of base direction estimate in Fig. 8 and minimum inter-node distance in Fig. 9 as the metrics to evaluate the algorithms performance.

B. Discussion

From the snapshot and the result curves, we can observe that the goal approaching algorithm keeps all nodes close by and therefore, the geometry is not beneficial enough to determine the base. Both the error in joint position and angle-to-base is very significant and explains also why the swarm progresses towards the base only slowly. The minimum iter-node distance is mainly above the minimum safety distance.

The flocking algorithms ensure that the AGs build a regular structure in all dimensions. The angular error is low compared to the goal approaching algorithm, see in Fig. 8. Between two different desired distance, the flock with 10 m is not spread enough to acquire a precise direction estimate, whereas the flock with 20 m spread over a larger aperture to observe the base direction, also in Fig. 8. However, it suffers from the loss of formation estimation precision. Therefore, the difference

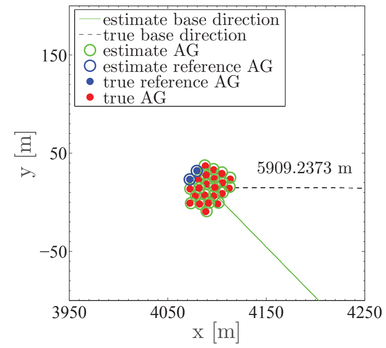

Figure 2. Flocking with a desired distance of 10 m after 25000 time steps

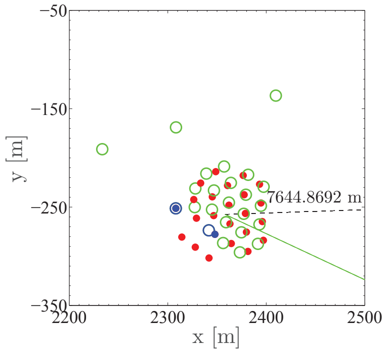

Figure 3. Flocking with a desired distance of 20 m after 25000 time steps

in joint position error between two flocking is insignificant. Additionally, the loss of formation estimation precision for flocking with 20 m leads to a less effective controlling command. Hence, a major portion of the total speed is spent on stabilizing and maintaining the flocking formation, which distracts the attempt of return-to-base in flocking with a 20 m desired distance, see in Fig. 6. The minimum inter-node distance mostly follows the desired distance, which on one hand, avoids collision, but on the other hand, is not adaptive to the scenario changes.

For our location information optimized swarm formation in Fig. 5 the swarm automatically spreads out evenly along the vertical direction towards the base, which significantly increases the effective aperture for observing the homing direction. Meantime, The AGs are not going too far , avoiding loss of connectivity from the swarm. From the snapshot, we can intuitively see that our proposed algorithm outperforms the others in the sense of smaller remaining distance and smaller angular error for the base direction. It is also proved in Fig. 6 and Fig. 8. From Fig. 7 we can see that our approach outperforms the others in the joint positioning error. The minimum inter-node distance is continuously changing, which indicates that the algorithm is online optimizing the formation in order to adapt different scenarios. The safety distance is also guarantied.

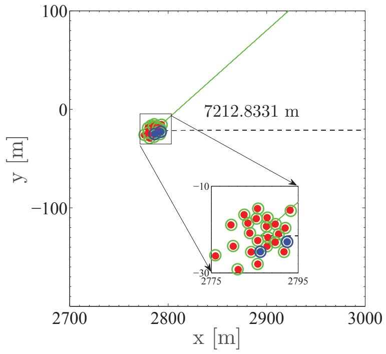

Figure 4. Goal approaching algorithm after 25000 time steps

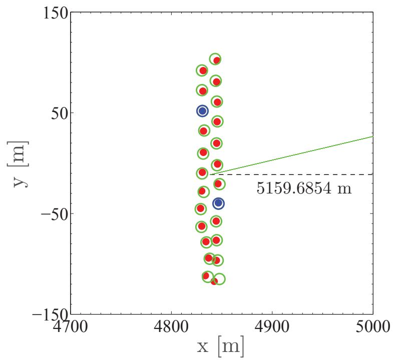

Figure 5. Location information driven algorithm after 25000 time steps

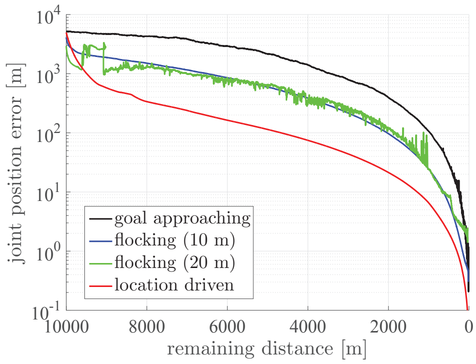

Figure 7. Joint location error of the swarm rovers and the base

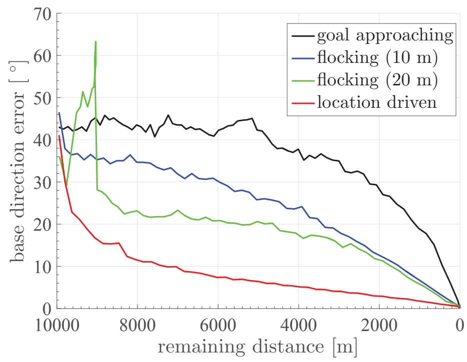

Figure 8. Angular error of base direction

V. Conclusion

In this paper we presented an algorithm that guides a swarm of rovers going home to a base based on a beacon signal from a single base. The proposed algorithm jointly considers an optimized formation of the rovers to estimate their position as well as the position of the base relative to the baseline. The performance analysis shows that our location information driven algorithm outperforms the classical flocking and goal approaching algorithm in accuracy of the individual rovers as well as the total amount of time.

Acknowledgment

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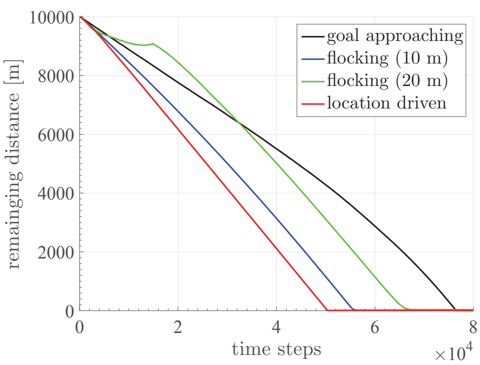

Figure 6. Remaining distance versus time steps

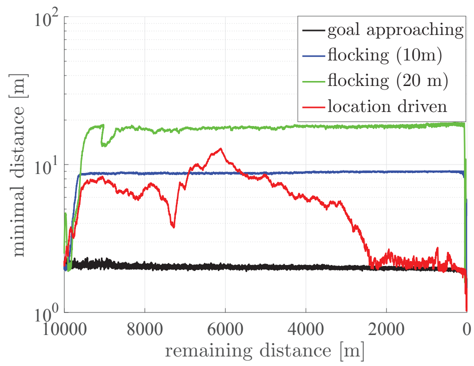

Figure 9. Minimum inter-node distance

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