A Novel Angular Parameters Estimator for Incoherently Distributed Sources

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***Abstract*—In this work, a new algorithm for angular parameters estimation of incoherently distributed sources is proposed. By using the general array manifold model, the nominal DOAs can be firstly separated from the original array manifold. Then a generalized shift invariance property inside the array manifold is identified, based on which the nominal DOAs can be estimated. The angular spreads are next estimated from the central moments of the angular distribution. Compared with the popular ESPRIT-ID algorithm, the proposed one could achieve higher accuracy, could handle more sources, and could be applied on a much more general array structure. Numerical simulations are provided to show the superior performance of the proposed algorithm over the existing works.**

I. Introduction

Classical direction-of-arrival (DOA) algorithms, e.g., MUSIC [1] and ESPRIT [2], are always developed for *point source model* that corresponds to the 1-ray scenario [3], i.e., the signal of the source arrives at the array through one direct path. However, in many applications like radar, sonar, speech processing and mobile communications, the *spatial distributed source model* [4] is more appropriate since the signal will reach the array through many rays reflected or scattered from the vicinity of the source. In spatial distributed source model, the angular parameters contain the nominal DOA (usually defined as the mean value of DOAs of multiple rays) and the angular spread (usually defined as the standard deviation of angular distribution around the nominal DOA). When the signal components from different rays are completely uncorrelated, the source is categorized into incoherently distributed (ID) source [4].

Since the nominal DOAs and angular spreads are coupled parameters, it is much more complex to estimate the angular parameter estimation in ID source case than in the point source case. Many existing works [3]–[8] for this topic suffer from the heavy computational burden due to multi-dimensional spectral search. An ESPRIT-based estimator for parameters estimation of multiple ID sources, called ESPRIT-ID, was proposed in [9]. This approach has closed-form solutions for decoupled estimations of the nominal DOA and angular spread. However, it restricts the array geometry to be two identical, closely-deployed arrays to ensure an approximate shift invariance property.

In this paper, a novel algorithm for estimating angular parameters of multiple ID sources is proposed with fewer restrictions for actual implementation. Applying general array manifold (GAM) model [10], we first construct a one-dimensional (1-D) spectrum function for estimating nominal DOAs based on a newly introduced *generalized shift invariance property*. Then, the angular spreads can be computed with closed-form from the central moments of the angular distribution. The proposed algorithm outperforms the conventional ESPRIT-ID algorithm, removes the restrictions on array geometry and can handle almost twice more sources. Compared to conventional multi-dimensional search approaches, e.g., [3], the proposed algorithm has comparable estimation performance and much lower computational complexity.

II. System Model

Assume that there are narrow-band signals $\{s_k(t)\}_{k=1}^{K}$ from K ID sources impinging on an arbitrary line array with M omnidirectional sensors[1]. The received signal at the array can be expressed as

$$\mathbf{r}(t) = \sum_{k=1}^{K} s_k(t) \sum_{l=1}^{L_k} \gamma_{k,l}(t)\mathbf{a}(\bar{\theta}_{k,l}(t)) + \mathbf{n}(t), \tag{1}$$

where $\bar{\theta}_{k,l}(t) \in (-90^\circ, 90^\circ)$ is the DOA of the lth ray from the kth signal; L_k is the number of rays inside the kth signal; $t = 1, 2, ..., T$ is the sampling time, $\mathbf{a}(\bar{\theta}_{k,l}(t))$ is the array manifold vector, and T is the total number of snapshots; $\gamma_{k,l}(t)$ is the complex-valued ray gain and $\mathbf{n}(t) \in \mathbb{C}^{M\times 1}$ is the complex-valued additive noise, which is spatially and temporally white with variance σ_n^2.

As the sources are incoherently distributed sources, the ray gains $\{\gamma_{k,l}(t)\}_{t=1}^{T}$ are temporally white, independent from ray to ray, and zero-mean, whose covariance is given by [6]

$$E\{\gamma_{k,l}(t)\gamma_{k',l'}^{*}(t')\} = \frac{\sigma_{\gamma_k}^2}{L_k}\delta(k-k')\delta(l-l')\delta(t-t'). \tag{2}$$

[1] Note the arbitrary line array still has to obey the half-wavelength rule to avoid phase ambiguity.

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Moreover, the mth element of the array manifold vector $\mathbf{a}(\bar{\theta}_{k,l}(t)) \in \mathbb{C}^{M\times 1}$ is [1]

$$\left[\mathbf{a}(\bar{\theta}_{k,l}(t))\right]_m = \exp\left\{j\frac{2\pi}{\lambda}x_m \sin(\bar{\theta}_{k,l}(t))\right\}, \qquad (3)$$

where λ is the signal wavelength, and x_m is the coordinate of the mth senor.

We may represent $\bar{\theta}_{k,l}(t)$ as

$$\bar{\theta}_{k,l}(t) = \theta_k + \varphi_{k,l}(t), \qquad (4)$$

where θ_k is the nominal DOA of the kth source; $\varphi_{k,l}(t)$ is the deviation from the nominal direction and is assumed to be a zero-mean random variable with probability density function $p_k(\zeta;\sigma_k)$. It is generally assumed that $p_k(\zeta;\sigma_k)$ is a symmetric function in ζ and is parameterized by the unknown standard deviations σ_k. Moreover, $p_k(\zeta;\sigma_k)$ is generally considered to be uniform or Gaussian distribution [9].

The following assumptions are adopted in this paper:1) The number of sources, K, is known as a prior and the number of array sensors M is larger than $2K$; 2) As [9]–[11], we assume the angular spreads $\{\sigma_k\}_{k=1}^K$ are small; 3) The numbers of incoming rays $\{L_k\}_{k=1}^K$ are large.

III. The Proposed Algorithm

A. GAM Modeling

With the aid of (4), the manifold $\mathbf{a}(\bar{\theta}_{k,l}(t))$ can be well approximated by the first-order Taylor series as

$$\mathbf{a}(\bar{\theta}_{k,l}(t)) \approx \mathbf{a}(\theta_k) + \mathbf{a}'(\theta_k)\varphi_{k,l}(t), \qquad (5)$$

where $\mathbf{a}'(\theta_k)$ is the partial derivative of $\mathbf{a}(\theta_k)$ with respect to θ_k. Then the received signal in (1) can be re-expressed as

$$\mathbf{r}(t) \approx \sum_{k=1}^{K}\Big(\mathbf{a}(\theta_k)\upsilon_{k,0}(t) + \mathbf{a}'(\theta_k)\upsilon_{k,1}(t)\Big) + \mathbf{n}(t), \qquad (6)$$

where

$$\upsilon_{k,0}(t) = s_k(t)\sum_{l=1}^{L_k}\gamma_{k,l}, \qquad (7)$$

$$\upsilon_{k,1}(t) = s_k(t)\sum_{l=1}^{L_k}\gamma_{k,l}\varphi_{k,l}(t). \qquad (8)$$

We can reformulate (1) into the GAM model as [9], [10]

$$\mathbf{r}(t) \approx \mathbf{B}(\boldsymbol{\theta})\mathbf{g}(t) + \mathbf{n}(t), \qquad (9)$$

where

$$\mathbf{B}(\boldsymbol{\theta}) = [\mathbf{A}(\theta_1), \mathbf{A}(\theta_2), \ldots, \mathbf{A}(\theta_K)] \in \mathbb{C}^{M\times 2K}, \qquad (10)$$

$$\mathbf{A}(\theta_k) = [\mathbf{a}(\theta_k), \mathbf{a}'(\theta_k)] \in \mathbb{C}^{M\times 2}, \qquad (11)$$

$$\mathbf{g}(t) = [\mathbf{g}_1^T, \mathbf{g}_2^T, \ldots, \mathbf{g}_K^T]^T \in \mathbb{C}^{2K\times 1}, \qquad (12)$$

$$\mathbf{g}_k = [\upsilon_{k,0}(t), \upsilon_{k,1}(t)]^T \in \mathbb{C}^{2\times 1}, \qquad (13)$$

$$\boldsymbol{\theta} = [\theta_1, \ldots, \theta_K]^T. \qquad (14)$$

Note that the generalized manifold matrix $\mathbf{B}(\boldsymbol{\theta})$ is only determined by the nominal DOAs and is independent of the specific distribution of angular deviation. Hence, unlike many existing works [3]–[6], [8], [11], the proposed algorithms is insensitive to the uncertainties of the angular distributions and can handle the case when multiple sources exhibit different angular distributions.

Since the transmitted signals, ray gains, and the angular deviations are uncorrelated from each other, it can be proven that the central moments of the angular distribution, i.e., σ_k^2, is included in the variance of $\upsilon_{k,1}(t)$ and the covariance of $\mathbf{g}(t)$ can be derived as

$$\boldsymbol{\Lambda} = E\{\mathbf{g}(t)\mathbf{g}^H(t)\} = \mathrm{diag}\{\boldsymbol{\Lambda}_1, \boldsymbol{\Lambda}_2, \ldots, \boldsymbol{\Lambda}_K\}, \qquad (15)$$

where $\boldsymbol{\Lambda}_k = \rho_k \mathrm{diag}\{1, \sigma_k^2\}$, $\rho_k = E\{|s_k(t)|^2\}\sigma_{\gamma_k}^2$ is the received signal power of the kth source. We see that the estimate of $\boldsymbol{\Lambda}$ can be used for angular spread estimation.

Moreover, the covariance matrix $\mathbf{R}$ of the received signal can be approximated as

$$\mathbf{R} = E\{\mathbf{r}(t)\mathbf{r}^H(t)\} \approx \mathbf{B}(\boldsymbol{\theta})\boldsymbol{\Lambda}\mathbf{B}^H(\boldsymbol{\theta}) + \sigma_n^2\mathbf{I}_M. \qquad (16)$$

B. The Nominal DOA Estimation

Divide the entire array into two different subarrays with equivalent number of sensors. The two subarrays are allowed to have overlapping sensors, and hence the number of sensors in each subarray N could take the value from $\{2, \ldots, M-1\}$. Re-denote $\{x_{1,n}\}_{n=1}^N$ and $\{x_{2,n}\}_{n=1}^N$ as the locations of the sensors in each subarray, respectively. The received signals of both subarray can be expressed as

$$\mathbf{r}_1(t) = \mathbf{B}_1(\boldsymbol{\theta})\mathbf{g}(t) + \mathbf{n}(t), \qquad (17)$$

$$\mathbf{r}_2(t) = \mathbf{B}_2(\boldsymbol{\theta})\mathbf{g}(t) + \mathbf{n}(t), \qquad (18)$$

where

$$\mathbf{B}_1(\boldsymbol{\theta}) = [\mathbf{A}_1(\theta_1), \mathbf{A}_1(\theta_2), \ldots, \mathbf{A}_1(\theta_K)] \in \mathbb{C}^{N\times 2K}, \qquad (19)$$

$$\mathbf{B}_2(\boldsymbol{\theta}) = [\mathbf{A}_2(\theta_1), \mathbf{A}_2(\theta_2), \ldots, \mathbf{A}_2(\theta_K)] \in \mathbb{C}^{N\times 2K}, \qquad (20)$$

are the generalized manifold matrices of the two subarrays, respectively.

A key observation is that

$$\mathbf{A}_2(\theta_k) = \mathbf{A}_1(\theta_k) \odot \boldsymbol{\Phi}_k, \qquad (21)$$

where

$$\boldsymbol{\Phi}_k = \begin{bmatrix} e^{j\phi_{1k}} & \beta_1 e^{j\phi_{1k}} \\ \vdots & \vdots \\ e^{j\phi_{Nk}} & \beta_N e^{j\phi_{Nk}} \end{bmatrix} \in \mathbb{C}^{N\times 2}, \qquad (22)$$

and $\odot$ denotes the Schur-Hadamard product, $\phi_{nk} = \frac{2\pi}{\lambda}\Delta x_n \sin\theta_k$, $\Delta x_n = x_{2,n} - x_{1,n}$, $\beta_n = \frac{x_{2,n}}{x_{1,n}}$, $n = 1, \ldots, N$.[2] In fact, it is found that (21) exhibits a shift relationship between the generalized manifold of two subarrays, called *the generalized shift invariance property*.

Therefore, $\mathbf{B}_2(\boldsymbol{\theta})$ can be expressed as

$$\mathbf{B}_2(\boldsymbol{\theta}) = [\boldsymbol{\Phi}_1 \odot \mathbf{A}_1(\theta_1), \ldots, \boldsymbol{\Phi}_K \odot \mathbf{A}_1(\theta_K)]. \qquad (23)$$

[2]The sensors of the first subarray cannot be located at the original in order to make sure that the denominator in β_n is not zero.

Through eigen-decomposition of $\mathbf{R}$, we obtain the signal subspace $\mathbf{E}_s$, which is of $M \times 2K$ and is composed of the eigenvectors of $\mathbf{R}$ corresponding to the largest $2K$ eigenvalues. It is known from [9] that the signal subspace $\mathbf{E}_s$ spans the column space of the generalized manifold matrix $\mathbf{B}(\boldsymbol{\theta})$, which yields

$$\mathbf{E}_s = \mathbf{B}(\boldsymbol{\theta})\mathbf{T}, \tag{24}$$

where $\mathbf{T}$ is an invertible $2K \times 2K$ matrix.

We then extract two $N \times 2K$ sub-matrices $\mathbf{E}_1$ and $\mathbf{E}_2$ from $\mathbf{E}_s$ whose rows correspond to the partition of the two subarrays. Obviously, there are

$$\mathbf{E}_1 = \mathbf{B}_1(\boldsymbol{\theta})\mathbf{T}, \tag{25}$$

$$\mathbf{E}_2 = \mathbf{B}_2(\boldsymbol{\theta})\mathbf{T}. \tag{26}$$

Define a new matrix as a function of θ:

$$\boldsymbol{\Psi}(\theta) = \begin{bmatrix} e^{j\psi_1} & \beta_1 e^{j\psi_1} \\ \vdots & \vdots \\ e^{j\psi_N} & \beta_N e^{j\psi_N} \end{bmatrix}, \tag{27}$$

where $\psi_n = \frac{2\pi}{\lambda}\Delta x_n \sin\theta, n = 1, \ldots, N$. Further define another $N \times 2K$ matrix $\mathbf{C}(\theta)$ as

$$\mathbf{C}(\theta) = [\underbrace{\boldsymbol{\Psi}(\theta), \boldsymbol{\Psi}(\theta), \ldots, \boldsymbol{\Psi}(\theta)}_{K}] \in \mathbb{C}^{N\times 2K}. \tag{28}$$

Let us then formulate

$$\begin{aligned} \mathbf{D}(\theta) &= \mathbf{E}_2 - \mathbf{C}(\theta) \odot \mathbf{E}_1 \\ &= (\mathbf{B}_2 - \mathbf{C}(\theta) \odot \mathbf{B}_1)\mathbf{T} = \mathbf{Q}(\theta)\mathbf{T}, \end{aligned} \tag{29}$$

where

$$\mathbf{Q}(\theta) = [(\boldsymbol{\Phi}_1 - \boldsymbol{\Psi}(\theta)) \odot \mathbf{A}_1(\theta_1), \ldots, (\boldsymbol{\Phi}_K - \boldsymbol{\Psi}(\theta)) \odot \mathbf{A}_1(\theta_K)]. \tag{30}$$

It can be found that when $\theta = \theta_k$, all the elements of $(\boldsymbol{\Phi}_k - \boldsymbol{\Psi}(\theta))$ will become zero. Thus, if $2K \leq N$, then $\mathbf{D}(\theta)$ will become rank deficient, and the determinant of $\mathbf{D}^H(\theta)\mathbf{D}(\theta)$ will become zero. Hence, the nominal DOA estimates $\{\hat{\theta}_k\}_{k=1}^K$ can be obtained by finding the highest K peaks of the following function:

$$f(\theta) = \frac{1}{\det\{\mathbf{D}(\theta)^H \mathbf{D}(\theta)\}}. \tag{31}$$

In practice, the covariance matrix $\mathbf{R}$ can be estimated with finite samples via $\hat{\mathbf{R}} = \frac{1}{T}\sum_{t=1}^{T} \mathbf{r}(t)\mathbf{r}^H(t)$.

C. The Angular Spread Estimation

After getting the nominal DOA estimates $\{\hat{\theta}_k\}_{k=1}^K$, $\boldsymbol{\Lambda}$ can be obtained from

$$\hat{\boldsymbol{\Lambda}} = \mathbf{B}(\hat{\boldsymbol{\theta}})^{\dagger}(\hat{\mathbf{R}} - \hat{\sigma}_n^2 \mathbf{I}_M)(\mathbf{B}^H(\hat{\boldsymbol{\theta}}))^{\dagger}, \tag{32}$$

where $\mathbf{B}(\hat{\boldsymbol{\theta}})$ is the estimate of the generalized manifold by substituting the estimates of nominal DOAs into (10), and $\hat{\sigma}_n^2$ is the estimate of the variance of the noise which is the average of the $(M - 2K)$ smallest eigenvalues of $\hat{\mathbf{R}}$.

According to the definition of $\boldsymbol{\Lambda}$ in (15), the angular spread estimates are given by

$$\hat{\sigma}_k = \sqrt{\frac{[\hat{\boldsymbol{\Lambda}}]_{2k,2k}}{[\hat{\boldsymbol{\Lambda}}]_{2k-1,2k-1}}}, \quad k = 1, 2, ..., K. \tag{33}$$

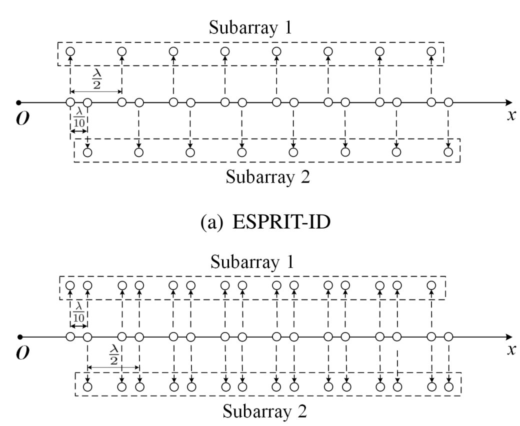
D. Comparison with ESPRIT-ID

In ESPRIT-ID, the value of N can only be $M/2$, where M is even, whereas in the proposed algorithm, the value of N can be $(M-1)$ at most. Hence, the most sources the proposed algorithm can estimate is $\lfloor (M-1)/2 \rfloor$, where $\lfloor \cdot \rfloor$ returns the maximum integer that is not bigger than the inside argument. Meanwhile, in ESPRIT-ID, the maximum number of detectable sources will be $\lfloor M/4 \rfloor$ only. It is obvious that the proposed algorithm can handle almost twice more sources than ESPRIT-ID.

ESPRIT-ID restricts the two subarrays to be identical and closely-placed in order to ensure the approximate shift-invariance property it exploits. However, the approximation brings severe performance degradation in nominal DOA estimation. Such degradation is avoided in the proposed algorithm since the generalized shift invariance property that contains no approximation is exploited, as in (21). The superiority of accuracy by the proposed algorithm will be proved in simulations.

IV. Simulations

In this section, we provide numerical results to demonstrate the performance of the proposed algorithm. The root mean square error (RMSE) is adopted as the figure of merit. Totally 500 Monte-Carlo runs are used for average. The variance of ray-gain is set as $\{\sigma_{\gamma_k}^2\}_{k=1}^K = 1$. The number of scattering paths is set as $\{L_k\}_{k=1}^K = 75$. The signal-to-noise ratio (SNR) is defined as $\text{SNR} = \rho_k/\sigma_n^2$. The number of snapshot is $T = 300$. For the proposed algorithm, the number of sensors included in each subarray is $N = M - 1$. The search region for nominal DOA estimation is $(-90°, 90°)$ and the search region for angular spread estimation is $(0°, 3°)$.



(a) ESPRIT-ID

(b) The proposed algorithm

Fig. 1. Subarray formation of the proposed algorithm and ESPRIT-ID.

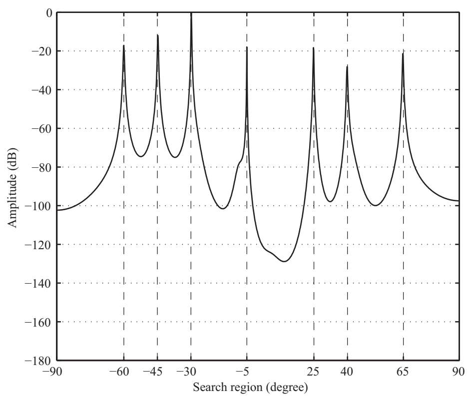


Fig. 2. Spectrum of the proposed algorithm for nominal DOA estimation in a seven-source case, SNR=15dB, $\theta_1 = -60°, \theta_2 = -45°, \theta_3 = -30°, \theta_4 = -5°, \theta_5 = 25°, \theta_6 = 40°, \theta_7 = 65°, \sigma = 1°$.

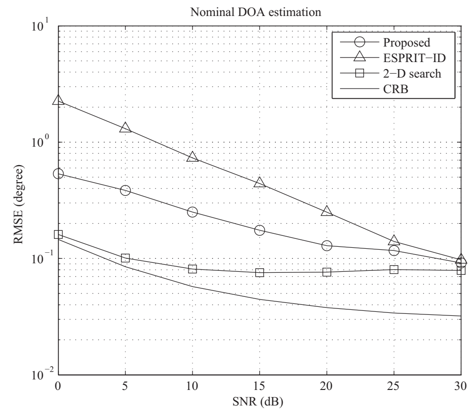


Fig. 3. RMSEs versus SNR for nominal DOA estimation, $\theta_1 = 30°$, $\theta_2 = 50°$, $\sigma_1 = 1°$, $\sigma_2 = 1.5°$.

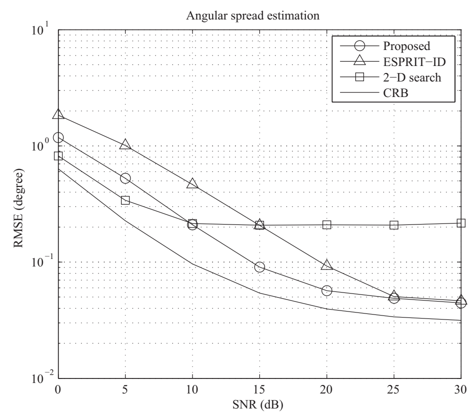


Fig. 4. RMSEs versus SNR for angular spread estimation, $\theta_1 = 30°$, $\theta_2 = 50°$, $\sigma_1 = 1°$, $\sigma_2 = 1.5°$.

In the first example, we consider large number of ID sources to exhibit the superiority of the proposed algorithm over ESPRIT-ID. For the 16-element array in Fig. 1, the subarray formation in ESPRIT-ID is limited into two non-overlap 8-sensor uniform linear arrays with half-wavelength spacing, and the distance between the two subarrays is $\Delta d = \lambda/10$, as shown in Fig. 1(a). On the other side, the proposed algorithm divides the array into two overlap 15-sensor subarray as shown in Fig. 1(b). The maximum number of ID sources that could be handled by ESPRIT-ID is 4 while that for the proposed algorithm is 7. Let us then set the number of the ID sources to be 7 with the corresponding nominal DOAs as $\theta_1 = -60°, \theta_2 = -45°, \theta_3 = -30°, \theta_4 = -5°, \theta_5 = 25°, \theta_6 = 40°, \theta_7 = 65°$, respectively. The SNR is taken as 15dB, and the distribution of all sources is assumed to be Gaussian with a same angular spread $\sigma = 1°$. Fig. 2 shows the spectrum (31) of the proposed algorithm for the nominal DOA estimation. Clearly, the proposed algorithm can provide valid nominal DOA estimation in this case while ESPRIT-ID completely fails.

In the second example, we compare the proposed algorithm with the classical ESPRIT-ID [9] and the 2-D spectral search algorithm [3] [3] in estimation accuracy. The Cramér-Rao bound (CRB) [11] is also plotted to make the comparison complete. Consider two ID sources with nominal DOAs $\theta_1 = 30°$, $\theta_2 = 50°$. The 16-element array in Fig. 1 is still exploited. The angular distribution is Gaussian with angular spreads $\sigma_1 = 1°$, $\sigma_2 = 1.5°$. Fig. 3 and Fig. 4 illustrate the RMSE performance versus SNR for the proposed algorithm, ESPRIT-ID algorithm [9] and the 2-D search algorithm [3], respectively. It is seen that for both estimation of nominal DOA and angular spread, the proposed algorithm outperforms ESPRIT-ID significantly due to the fact that it exploits as many sensors as possible in the overall array, whereas ESPRIT-ID can only exploit half sensors. Moreover, the generalized shift invariance property is exploited accurately in the proposed algorithm while ESPRIT-ID only approximates shift invariance property. Under high SNRs, the signal subspace $\mathbf{E}_s$ spans almost the same space as the generalized array manifold $\mathbf{B}(\theta)$. In this case, the number of sensors in each subarray imposes less impacts on estimation accuracy. Hence, the estimation performance of both the proposed algorithm and ESPRIT-ID for nominal DOA and angular spread reach unanimity under high SNRs. Meanwhile, when compared with 2-D search algorithm, the proposed algorithm performs worse for the nominal DOA estimation, but performs better in terms of angular spread estimation. Nevertheless, please keep in mind that the 2-D search algorithm suffers from very high computational complexity.

[3]The choice of this 2-D algorithm is based on the fact that it achieves good accuracy, outperforms some popular methods, such as DISPARE [5]. See [3] for detail.

V. Conclusions

In this paper, we developed a new angular parameters estimation algorithm for multiple ID sources. The nominal DOAs were first estimated through 1-D spectral peak search, based on a rank reduction criterion. Then the angular spreads were estimated with closed-form solutions. Compared to ESPRIT-ID algorithm, the proposed algorithm improves the estimation accuracy, removes the limitation of subarray formation as well as handles more ID sources. When compared to conventional multi-dimensional search approaches, the proposed algorithm has comparable estimation performance and much lower computational complexity.

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