A Novel Block-shifted Pilot Design for Multipair Massive MIMO Relaying

Leyuan Pan, Yongyu Dai and Xiaodai Dong
Department of Electrical and Computer Engineering
University of Victoria
Victoria, BC V8P 5C2, Canada
Email: {leyuan,yongyu}@uvic.ca and xdong@ece.uvic.ca

Wei Xu
National Mobile Communications Research Laboratory
Southeast University
Nanjing, Jiangsu 210096, China
Email: wxu@seu.edu.cn

***Abstract*—We propose a design of block-shifted pilot scheme for the multipair massive multiple-input multiple-output (MIMO) relaying system. The proposed scheme balances the tradeoff between pilot transmission overhead and channel estimation accuracy in scenarios with limited length of coherence time interval, and thereby the system performance is dramatically improved. In the proposed scheme, pilots are properly designed so that they can be transmitted simultaneously with data to decrease channel estimation overheads. By exploiting the asymptotic orthogonality of massive MIMO channels, the source data can be exactly detected from the received signal, and then pilot-data interference can be effectively suppressed with assistance of the detected data in the destination channel estimation. In the block-shifted transmission pattern, the effective data transmission period is extended to improve system throughput. Both theoretical and numerical results confirm the superiority of the proposed scheme to conventional ones in limited coherence time interval scenarios.**

I. Introduction

Massive multiple-input multiple-output (MIMO) technology is growing to be one of the most promising solutions for the next-generation wireless communication to meet the urgent demands of both high-speed data transmissions and explosive growing numbers of user terminals, including not only mobile communication equipments such as cellphones, but also the Internet-of-Thing (IoT) devices [1]–[3]. However, despite of the advantages, massive MIMO is also facing great challenges. How to obtain precise channel state information (CSI) by consuming limited resources is most critical and fundamental.

In present massive MIMO studies, time-division duplex (TDD) is the most concerned implementation mode due to more effective approaches of obtaining CSI [4]. Thanks to the channel reciprocity, the TDD system exploits uplink pilot training to estimate channels, which saves large amounts of resources on channel estimation of a large-scale antenna array. However, as the growing of user numbers in a multiuser system, it requires a large amount of resources to transmit orthogonal pilot sequences, where the sequence number is bounded by the length of coherence time intervals. Hence the overhead of channel estimation increases correspondingly, which degrades the system performance. Some previous work studied this critical topic [5]–[7]. Nevertheless, all these researches are regarding point-to-point (P2P) communications. Few works conduct the pilot scheme design and optimization in massive MIMO relaying systems, especially multipair ones.

The relaying technique is an emerging cooperative technology capable of scaling up the system performance by orders of magnitude, extending the coverage and reducing power consumption [8]–[10]. Similar to the P2P system, the multiuser massive MIMO relaying system also suffers from the channel estimation overhead problem, which is even worse as both source and destination users need to transmit pilots within a coherence interval.Further, different from a P2P system, the throughput of a relaying system is determined by the weaker one of uplink and downlink connections. Thus, it is critical to co-consider interactions of both links while designing pilot scheme for a relaying system. Minorities of prior works conducted this consideration.

In order to reduce the overhead of channel estimation, in this paper, we propose a novel design of block-shifted pilot scheme in the multipair massive MIMO relaying system. By exploiting the asymptotic orthogonality of massive MIMO channels, we demonstrate that source data and pilots can be well separated from each other by applying the MRC processing. Therefore, the effective data transmission duration is extended and hence improving the overall system performance. Moreover, we derive the *closed-form expression* of the ergodic achievable rate of the considered relaying system and conduct the asymptotic analysis at both ultra-high and -low SNRs, and thus prove the superiority of the proposed scheme theoretically.

Organization: The rest of this paper is organized as follows. The system model is presented in Section II. Section III and Section IV conducts channel estimations and achievable rate derivations. Section V presents the numerical performance of the proposed scheme. Section VI concludes our works.

Notations: The superscript $(\cdot)^{\mathrm{H}}$ stands for the conjugate-transpose. $\mathbf{I}_M$ represents the identity matrix of size M. The operator $\|\cdot\|$, $\|\cdot\|_{\mathrm{F}}$ and $\mathrm{tr}(\cdot)$ denotes the Euclidean norm of a vector, the Frobenius norm and the trace of a matrix, respectively. For statistical vectors and matrices, we utilize $\mathbb{E}\{\cdot\}$ and $\mathbb{V}\mathrm{ar}\{\cdot\}$ to represent the expectation and variance, respectively. Finally, $\overset{\text{a.s.}}{=}$ denotes the almost sure convergence.

II. System Model

A. Signal and Channel Model

We consider the half-duplex one-way relaying system where K pairs of single-antenna source and destination users are served by the relay station (RS) equipped with M ($M \gg K \gg 1$) antennas. In the following sections, we normalize the noise power as 1. Let ρ_{p}, ρ_{s} and ρ_{d} be the transmission power of pilot, source and forwarding data, respectively. The channel matrices from sources and destinations to the RS are denoted by $\mathbf{G}_{\mathrm{s}} \in \mathbb{C}^{M\times K}$ and $\mathbf{G}_{\mathrm{d}} \in \mathbb{C}^{M\times K}$, which are concisely named as source and destination channels, respectively, where the $k\mathrm{th}$ column of either matrix, $\mathbf{g}_{\mathrm{s}k}$ or $\mathbf{g}_{\mathrm{d}k}$, stands for the channel vector from the $k\mathrm{th}$ corresponding user to the RS. Further, we decompose both channel matrices as $\mathbf{G}_{\mathrm{s}} = \mathbf{H}_{\mathrm{s}}\mathbf{D}_{\mathrm{s}}^{1/2}$ and $\mathbf{G}_{\mathrm{d}} = \mathbf{H}_{\mathrm{d}}\mathbf{D}_{\mathrm{d}}^{1/2}$, where the large-scale fading matrices $\mathbf{D}_{\mathrm{s}}$ and $\mathbf{D}_{\mathrm{d}}$ are both diagonal with the $k\mathrm{th}$ entry as $\beta_{\mathrm{s}k}$ and $\beta_{\mathrm{d}k}$, respectively, and the small-scale fading matrices $\mathbf{H}_{\mathrm{s}}$ and $\mathbf{H}_{\mathrm{d}}$ are constructed by independent identically distributed (i.i.d.) $\mathcal{CN}(0,1)$ random variables (RVs). Finally, we assume that the system reuses the same frequency band for both uplink and downlink transmissions, and they obey the reciprocity, i.e., the channel matrices are consistent within a coherence time interval for both uplink and downlink communications.

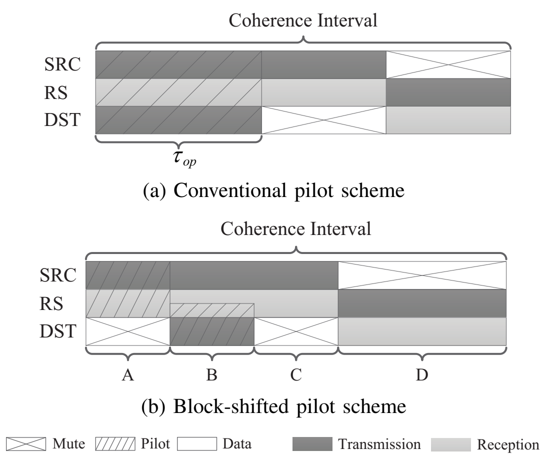


Fig. 1: Transmission diagrams of both conventional and block-shifted pilot schemes in half-duplex one-way relaying systems. (SRC: source users, RS: relay station, DST: destination users.)

B. Conventional Pilot Scheme

As shown by Fig. 1(a), TDD is selected as the working mode where the source and destination users firstly transmit training pilots to help RS estimate channel information, following which the sources send uplink data to RS, and then the data is forwarded to destination users.

In the MIMO system, the pilot scheme is designed by utilizing orthogonal pilot sequences to prevent pilot contamination, which requires the length of pilot sequences to be not smaller than the number of users, i.e., $\tau_{\mathrm{op}} \geq 2K$, where τ_{op} is the pilot length. Thus, the overhead of channel estimation is at least $2K/T_{\mathrm{c}}$ of each terminal, where T_{c} is the length of coherence time interval. In massive MIMO relaying systems, this overhead is extremely large because of large user numbers. Therefore, most of the resource is occupied by pilots, which makes the data transmission ineffective. To reduce such overhead and accordingly improve the data transmission efficiency, we propose a block-shifted pilot scheme in the next subsection.

C. Block-shifted Pilot Design

In this subsection, we propose a block-shifted pilot scheme for the half-duplex one-way relaying system. Thanks to the orthogonality of massive MIMO channels, the proposed pilot scheme improves the overall data rate performance by decreasing the pilot overhead. The block-shifted pilot transmission scheme is depicted in Fig. 1(b). To concisely describe signal transmissions, we separate a coherence interval into four phases denoted by A, B, C and D, respectively. During phases A and B, pilots are transmitted from users to the RS, while in C and D, the sources and RS conduct data transmissions, respectively. Within phase A, all source users send piece-wisely orthogonal pilot sequences while destination users keep mute. Thus, source channels are estimated at RS without contaminations. Subsequently, destination users start transmitting pilots in phase B, while sources can send uplink data to RS simultaneously. The RS observes both source data and destination pilots. With the source channel estimated in phase A, RS firstly detects source data. Then, RS cancels the data interference with assistance of the detected source data in destination channel estimation. Thereafter, sources keep sending uplink data in phase C and then RS forwards downlink data to destinations in phase D. In following coherence intervals, the relaying system repeats such communication procedures.

Remark 1. Note that the pilot overhead of the proposed scheme is K/T_{c} while that of the conventional one is $2K/T_{\mathrm{c}}$. It is obvious that $K/T_{\mathrm{c}} \ll 2K/T_{\mathrm{c}}$, especially for the massive MIMO scenario where a large K is comparable to T_{c}.

III. Channel Estimation

A. Source Channel Estimation

During phase A (see Fig. 1(b)), supposing that source users send the pilot matrix $\mathbf{\Phi} \in \mathbb{C}^{K\times K}$ to RS with power ρ_{p} per user, where the kth row of the matrix, $\boldsymbol{\phi}_k$, is the pilot sequence sent by the kth source user and $\mathbf{\Phi}\mathbf{\Phi}^{\mathrm{H}} = \mathbf{I}_K$ due to orthogonality, the received signal at RS can be expressed as

$$\mathbf{R}^{\mathrm{A}} = \sqrt{K\rho_{\mathrm{p}}}\mathbf{G}_{\mathrm{s}}\mathbf{\Phi} + \mathbf{N}^{\mathrm{A}} \tag{1}$$

where $\mathbf{N}^{\mathrm{A}}$ is the additive white Gaussian noise (AWGN) matrix constructed by $\mathcal{CN}(0,1)$ RVs. We employ the MMSE criteria to estimate source channels and obtain estimates as

$$\hat{\mathbf{G}}_{\mathrm{s}} = \mathbf{G}_{\mathrm{s}}\tilde{\mathbf{D}}^{\mathrm{A}} + \frac{1}{\sqrt{K\rho_{\mathrm{p}}}}\tilde{\mathbf{N}}^{\mathrm{A}}\tilde{\mathbf{D}}^{\mathrm{A}} \tag{2}$$

where $\tilde{\mathbf{D}}^{\mathrm{A}} \triangleq (\mathbf{D}_{\mathrm{s}}^{-1}/K\rho_{\mathrm{p}} + \mathbf{I}_K)^{-1}$ and $\tilde{\mathbf{N}}^{\mathrm{A}} \triangleq \mathbf{N}^{\mathrm{A}}\mathbf{\Phi}^{\mathrm{H}}$ is the effective noise constructed byssssss i.i.d. $\mathcal{CN}(0,1)$ RVs. Due to the property of MMSE estimation, we can decompose the channel matrix into two independent components as

$$\mathbf{G}_{\mathrm{s}} = \hat{\mathbf{G}}_{\mathrm{s}} + \mathcal{E}_{\mathrm{s}} \tag{3}$$

where $\mathcal{E}_{\mathrm{s}}$ is the error matrix. Denoting the kth ($k = 1, 2, \cdots, K$) column vector of $\hat{\mathbf{G}}_{\mathrm{s}}$ and $\mathcal{E}_{\mathrm{s}}$ by $\hat{\mathbf{g}}_{\mathrm{s}k}$ and $\boldsymbol{\varepsilon}_{\mathrm{s}k}$, respectively, we have $\hat{\mathbf{g}}_{\mathrm{s}k}$ is mutually independent of $\boldsymbol{\varepsilon}_{\mathrm{s}k}$ and $\hat{\mathbf{g}}_{\mathrm{s}k} \sim \mathcal{CN}(\mathbf{0}, \sigma_{\mathrm{s}k}^2 \mathbf{I}_M)$, $\boldsymbol{\varepsilon}_{\mathrm{s}k} \sim \mathcal{CN}(\mathbf{0}, \varepsilon_{\mathrm{s}k}^2 \mathbf{I}_M)$ with $\sigma_{\mathrm{s}k}^2 \triangleq K\rho_{\mathrm{p}}\beta_{\mathrm{s}k}^2/(1 + K\rho_{\mathrm{p}}\beta_{\mathrm{s}k})$ and $\varepsilon_{\mathrm{s}k}^2 \triangleq \beta_{\mathrm{s}k} - \sigma_{\mathrm{s}k}^2$.

B. Destination Channel Estimation

To simplify the description, we separate the source data into two successive parts as $\mathbf{S} = [\mathbf{S}^{\mathrm{B}}\ \mathbf{S}^{\mathrm{C}}]$, where $\mathbf{S}^{\mathrm{B}} \in \mathbb{C}^{K\times K}$ is transmitted within phase B and $\mathbf{S}^{\mathrm{C}} \in \mathbb{C}^{K\times(T_{\mathrm{d}}-K)}$ is transmitted within phase C. Here, T_{d} is the total length of source data to transmit by each user within a coherence interval. Without loss of generality, we assume $T_{\mathrm{d}} > K$. During phase B, source data and destination pilots are transmitted simultaneously. The following elaboration reveals that $\mathbf{S}^{\mathrm{B}}$ can be exactly detected from the received signal with a large number of RS antennas and the contamination of destination channel estimation can be suppressed by applying source data cancellations.

At first, we consider the detection of source data from the interfered received signal at RS, which can be expressed as

$$\mathbf{R}^{\mathrm{B}} = \sqrt{\rho_{\mathrm{s}}}\mathbf{G}_{\mathrm{s}}\mathbf{S}^{\mathrm{B}} + \sqrt{K\rho_{\mathrm{p}}}\mathbf{G}_{\mathrm{d}}\boldsymbol{\Psi} + \mathbf{N}^{\mathrm{B}} \tag{4}$$

where $\boldsymbol{\Psi} \in \mathbb{C}^{K\times K}$ denotes the destination pilot matrix transmitted at power ρ_{p} per user and $\mathbf{N}^{\mathrm{B}}$ is the AWGN matrix consisting of $\mathcal{CN}(0, 1)$ RVs. By applying MRC processing, the combined signal can be expressed by

$$\tilde{\mathbf{S}}^{\mathrm{B}} = \hat{\mathbf{G}}_{\mathrm{s}}^{\mathrm{H}}\mathbf{R}^{\mathrm{B}}. \tag{5}$$

Therefore, by employing the law of large numbers [11], we can summarize the source data detection as Proposition 1.

Proposition 1. *With MRC processing, the source data $\mathbf{S}^{\mathrm{B}}$ can be exactly detected from $\tilde{\mathbf{S}}^{\mathrm{B}}$, if a large number of reception antennas are equipped at the RS, i.e., $M \to \infty$.*

Proof: See Appendix B. ■

By subtracting the source data interference from (4), we obtain the MMSE estimation of destination channels as follows:

$$\hat{\mathbf{G}}_{\mathrm{d}} = \frac{1}{\sqrt{K\rho_{\mathrm{p}}}}\left(\mathbf{R}^{\mathrm{B}} - \sqrt{\rho_{\mathrm{s}}}\hat{\mathbf{G}}_{\mathrm{s}}\mathbf{S}^{\mathrm{B}}\right)\boldsymbol{\Psi}^{\mathrm{H}}\tilde{\mathbf{D}}_{\mathrm{d}} = \mathbf{G}_{\mathrm{d}}\tilde{\mathbf{D}}_{\mathrm{d}} + \tilde{\mathbf{N}}^{\mathrm{B}}\tilde{\mathbf{D}}_{\mathrm{d}} \tag{6}$$

where $\tilde{\mathbf{D}}_{\mathrm{d}} \triangleq \left(\mathbf{I}_K + \frac{\rho_{\mathrm{s}}\sum_{i=1}^{K}\varepsilon_{\mathrm{s}i}^2 + 1}{K\rho_{\mathrm{p}}}\mathbf{D}_{\mathrm{d}}^{-1}\right)^{-1}$ and $\tilde{\mathbf{N}}^{\mathrm{B}} \triangleq (\sqrt{\rho_{\mathrm{s}}}\mathcal{E}_{\mathrm{s}}\mathbf{S}^{\mathrm{B}} + \mathbf{N}^{\mathrm{B}})\boldsymbol{\Psi}^{\mathrm{H}}$. According to the property of MMSE estimation, we have the following relationship:

$$\mathbf{G}_{\mathrm{d}} = \hat{\mathbf{G}}_{\mathrm{d}} + \mathcal{E}_{\mathrm{d}} \tag{7}$$

where the kth ($k = 1, 2, \cdots, K$) columns of both matrices, $\hat{\mathbf{g}}_{\mathrm{d}k}$ and $\boldsymbol{\varepsilon}_{\mathrm{d}k}$, are mutually independent of each other following $\mathcal{CN}(\mathbf{0}, \sigma_{\mathrm{d}k}^2\mathbf{I}_M)$ and $\mathcal{CN}(\mathbf{0}, \varepsilon_{\mathrm{d}k}^2\mathbf{I}_M)$, respectively, and $\sigma_{\mathrm{d}k}^2 \triangleq K\rho_{\mathrm{p}}\beta_{\mathrm{d}k}^2/\left(\rho_{\mathrm{s}}\sum_{i=1}^{K}\varepsilon_{\mathrm{s}i}^2 + 1 + K\rho_{\mathrm{p}}\beta_{\mathrm{d}k}\right)$ and $\varepsilon_{\mathrm{d}k}^2 \triangleq \beta_{\mathrm{d}k} - \sigma_{\mathrm{d}k}^2$.

IV. Achievable Rate Analysis

In this section, we characterize the performance of the proposed scheme by evaluating the average achievable rates of the considered multipair massive MIMO relaying system as $\mathcal{R} = \frac{1}{\mathcal{L}T_{\mathrm{c}}}\sum_{\iota=1}^{\mathcal{L}}\sum_{k=1}^{K}\mathcal{R}_k$ where $\mathcal{R}_k = \min\{\mathcal{R}_k^{\mathrm{UL}}, \mathcal{R}_k^{\mathrm{DL}}\}$ with $\mathcal{R}_k^{\mathrm{UL}}$ and $\mathcal{R}_k^{\mathrm{DL}}$ denoting the uplink and downlink rate between user pair k and the RS, respectively. Moreover, we employ the technique developed by [4] to calculate the achievable rate for per-link communication, i.e., $\mathcal{R}_k^{\mathrm{UL}}$ and $\mathcal{R}_k^{\mathrm{DL}}$. The per-link rate is defined by $\mathcal{R}_k^{\mathrm{PL}} = \tau_{\mathrm{d}}\log_2(1 + \gamma_k^{\mathrm{PL}})$, where τ_{d} denotes the data transmission duration and the effective signal to noise ratio is defined by $\gamma_k^{\mathrm{PL}} \triangleq \mathcal{S}_k^{\mathrm{PL}}/(\mathcal{I}_k^{\mathrm{PL}} + \mathcal{N}_k^{\mathrm{PL}})$; $\mathcal{S}_k^{\mathrm{PL}}$, $\mathcal{I}_k^{\mathrm{PL}}$ and $\mathcal{N}_k^{\mathrm{PL}}$ represents the power of desired signal, uncorrelated interference and AWGN, respectively.

A. Downlink Analysis

By applying the MRT processing to the downlink data $\mathbf{X} \in \mathbb{C}^{K\times T_{\mathrm{d}}}$ and sending it into destination channels with power ρ_{d}, we can obtain the received signal at destination users, particularly, for user k ($k = 1, 2, \cdots, K$), as

$$\mathbf{y}_k = \sqrt{\rho_{\mathrm{d}}}\alpha\mathbf{g}_{\mathrm{d}k}^{\mathrm{H}}\hat{\mathbf{G}}_{\mathrm{d}}\mathbf{X} + \mathbf{z}_k \tag{8}$$

where $\mathbf{z}_k \in \mathbb{C}^{1\times T_{\mathrm{d}}}$ is the AWGN vector consisting of $\mathcal{CN}(0, 1)$ RVs and α is the factor to normalize the average transmit power, i.e., $\mathbb{E}\{\|\alpha\hat{\mathbf{G}}_{\mathrm{d}}\|_{\mathrm{F}}^2\} = 1$. To separate the desired signal from interference and noise, (8) can be rewritten as

$$\mathbf{y}_k = \underbrace{\sqrt{\rho_{\mathrm{d}}}\alpha\mathbb{E}\left\{\mathbf{g}_{\mathrm{d}k}^{\mathrm{H}}\hat{\mathbf{g}}_{\mathrm{d}k}\right\}\mathbf{x}_k}_{\text{desired signal}} + \underbrace{\breve{\mathbf{z}}_k}_{\text{effective noise}} \tag{9}$$

where $\breve{\mathbf{z}}_k$ is the effective noise.Therefore, we can obtain the effective SINR at the kth destination user as

$$\gamma_k^{\mathrm{DL}} = \frac{\rho_{\mathrm{d}}\alpha^2\left|\mathbb{E}\left\{\mathbf{g}_{\mathrm{d}k}^{\mathrm{H}}\hat{\mathbf{g}}_{\mathrm{d}k}\right\}\right|^2}{\rho_{\mathrm{d}}\alpha^2\mathbb{V}\mathrm{ar}\left\{\mathbf{g}_{\mathrm{d}k}^{\mathrm{H}}\hat{\mathbf{g}}_{\mathrm{d}k}\right\} + \mathrm{MI}_k^{\mathrm{DL}} + 1} \tag{10}$$

where the power of downlink multipair interference (MI) is

$$\mathrm{MI}_k^{\mathrm{DL}} \triangleq \rho_{\mathrm{d}}\alpha^2\mathbb{E}\left\{\left|\sum_{i=1, i\neq k}^{K}\mathbf{g}_{\mathrm{d}k}^{\mathrm{H}}\hat{\mathbf{g}}_{\mathrm{d}i}\right|^2\right\}. \tag{11}$$

Theorem 1. *By employing the MRT processing, the achievable rate of the downlink data forwarding to the destination user k ($k = 1, 2, \cdots, K$) for a finite RS antenna number M can be characterized by*

$$\mathcal{R}_k^{\mathrm{DL}} = T_{\mathrm{d}}\log_2\left(1 + \gamma_k^{\mathrm{DL}}\right) \tag{12}$$

where $\gamma_k^{\mathrm{DL}} = \frac{M\sigma_{\mathrm{d}k}^4}{(\beta_{\mathrm{d}k} + 1/\rho_{\mathrm{d}})\sum_{i=1}^{K}\sigma_{\mathrm{d}i}^2}$.

B. Uplink Analysis

In the following descriptions, we first introduce the rate analysis of phase B, and then perform the phase C analysis.

1) Analysis of uplink phase B: We highlight the kth row of $\tilde{\mathbf{S}}^{\mathrm{B}}$ in (5) and rewrite it as follows:

$$\tilde{\mathbf{s}}_k^{\mathrm{B}} = \underbrace{\sqrt{\rho_{\mathrm{s}}}\mathbb{E}\left\{\hat{\mathbf{g}}_{\mathrm{s}k}^{\mathrm{H}}\mathbf{g}_{\mathrm{s}k}\right\}\mathbf{s}_k^{\mathrm{B}}}_{\text{desired signal}} + \underbrace{\breve{\mathbf{n}}_k^{\mathrm{B}}}_{\text{effective noise}} \tag{13}$$

where $\breve{\mathbf{n}}_k^{\mathrm{B}}$ is the effective noise.Therefore, the effective SINR of the received signal during phase B can be expressed by

$$\gamma_k^{\mathrm{B}} = \frac{\rho_{\mathrm{s}}\left|\mathbb{E}\left\{\hat{\mathbf{g}}_{\mathrm{s}k}^{\mathrm{H}}\mathbf{g}_{\mathrm{s}k}\right\}\right|^2}{\rho_{\mathrm{s}}\mathbb{V}\mathrm{ar}\left\{\hat{\mathbf{g}}_{\mathrm{s}k}^{\mathrm{H}}\mathbf{g}_{\mathrm{s}k}\right\} + \mathrm{MI}_k^{\mathrm{UL}} + \mathrm{PI}_k^{\mathrm{UL}} + \mathrm{AN}_k^{\mathrm{UL}}} \tag{14}$$

where the power of uplink MI, destination pilot interference (PI) and AWGN are respectively defined as follows:

$$\mathrm{MI}_k^{\mathrm{UL}} \triangleq \rho_{\mathrm{s}} \mathbb{E}\left\{\left|\sum_{i=1, i \neq k}^{K} \hat{\mathbf{g}}_{\mathrm{s}k}^{\mathrm{H}} \mathbf{g}_{\mathrm{s}i}\right|^2\right\} \tag{15}$$

$$\mathrm{PI}_k^{\mathrm{UL}} \triangleq \rho_{\mathrm{p}} \mathbb{E}\left\{\left\|\hat{\mathbf{g}}_{\mathrm{s}k}^{\mathrm{H}} \mathbf{G}_{\mathrm{d}}\right\|^2\right\} \tag{16}$$

$$\mathrm{AN}_k^{\mathrm{UL}} \triangleq \mathbb{E}\left\{\left\|\hat{\mathbf{g}}_{\mathrm{s}k}\right\|^2\right\}. \tag{17}$$

2) Analysis of uplink phase C: During phase C, only MI interferes the uplink data transmission. Similar to phase B analysis, we can obtain the effective SINR of the received signal during phase C as shown by

$$\gamma_k^{\mathrm{C}} = \frac{\rho_{\mathrm{s}} \left|\mathbb{E}\left\{\hat{\mathbf{g}}_{\mathrm{s}k}^{\mathrm{H}} \mathbf{g}_{\mathrm{s}k}\right\}\right|^2}{\rho_{\mathrm{s}} \mathbb{V}\mathrm{ar}\left\{\hat{\mathbf{g}}_{\mathrm{s}k}^{\mathrm{H}} \mathbf{g}_{\mathrm{s}k}\right\} + \mathrm{MI}_k^{\mathrm{UL}} + \mathrm{AN}_k^{\mathrm{UL}}} \tag{18}$$

where $\mathrm{MI}_k^{\mathrm{UL}}$ and $\mathrm{AN}_k^{\mathrm{UL}}$ are defined by (15) and (17), respectively. Finally, we summarize the uplink achievable rates of block-shifted relaying systems as Theorem 2.

Theorem 2. *With MRC processing, the uplink achievable rate of source user k ($k = 1, 2, \cdots K$) for a finite antenna number M can be characterized by*

$$\mathcal{R}_k^{\mathrm{UL}} = K \log_2(1 + \gamma_k^{\mathrm{B}}) + (T_{\mathrm{d}} - K) \log_2(1 + \gamma_k^{\mathrm{C}}) \tag{19}$$

where $\gamma_k^{\mathrm{B}} = \frac{M\sigma_{\mathrm{s}k}^2}{\sum_{i=1}^{K} \beta_{\mathrm{s}i} + \left(\rho_{\mathrm{p}} \sum_{i=1}^{K} \beta_{\mathrm{d}i} + 1\right)/\rho_{\mathrm{s}}}$ *and* $\gamma_k^{\mathrm{C}} = \frac{M\sigma_{\mathrm{s}k}^2}{\sum_{i=1}^{K} \beta_{\mathrm{s}i} + 1/\rho_{\mathrm{s}}}$.

C. Asymptotic Analysis

In this section, we analyze the asymptotic performance of the proposed scheme at extremely high and low SNR regions under the assumption that a large number of user pairs are served by large-scale RS antennas. In following asymptotic derivations, we assume that the transmit power of all data and pilot is identical, i.e., $\rho_{\mathrm{s}} = \rho_{\mathrm{d}} = \rho_{\mathrm{p}} = \rho$.

Firstly, we analyze the asymptotic performance at the high SNR region with the proposed pilot scheme. By setting $\rho \to \infty$, the effective SINRs can be approximated as $\lim_{\rho\to\infty} \gamma_k^{\mathrm{B}} = \frac{M\beta_{\mathrm{s}k}}{\sum_{i=1}^{K} \beta_{\mathrm{s}i} + \sum_{i=1}^{K} \beta_{\mathrm{d}i}}$, $\lim_{\rho\to\infty} \gamma_k^{\mathrm{C}} = \frac{M\beta_{\mathrm{s}k}}{\sum_{i=1}^{K} \beta_{\mathrm{s}i}}$ and $\lim_{\rho\to\infty} \gamma_k^{\mathrm{DL}} = \frac{M\beta_{\mathrm{d}k}}{\sum_{i=1}^{K} \beta_{\mathrm{d}i}}$. On the other hand, we consider the low SNR region by assuming $\rho \to 0$. It is not difficult to obtain the effective SINRs of uplink and downlink data transmission at low SNR as $\lim_{\rho\to 0} \gamma_k^{\mathrm{B}} = \lim_{\rho\to 0} \gamma_k^{\mathrm{C}} = MK\rho^2\beta_{\mathrm{s}k}^2$ and $\lim_{\rho\to 0} \gamma_k^{\mathrm{DL}} = \frac{MK\rho^2\beta_{\mathrm{d}k}^4}{\sum_{i=1}^{K} \beta_{\mathrm{d}i}^2}$.

By respectively substituting above asymptotic effective SINRs into (12) and (19), we obtain the asymptotic per-link achievable rates, and hence raise asymptotic overall system rates by summing them up.

By comparing the asymptotic achievable rates of the proposed and conventional pilot schemes, Corollary 1 can be concluded to confirm advantages of the proposed scheme.

Corollary 1. *With a large number of served user pairs, the proposed block-shifted pilot scheme outperforms the conventional orthogonal one in multipair half-duplex massive MIMO relaying systems for both high and low SNRs.*

Remark 2. According to Corollary 1, it is noticeable that the conclusion is independent of the coherence interval length T_{c} and the number of successive coherence intervals used for continuous communications $\mathcal{L}$. Therefore, the proposed scheme is always superior to, at least not worse than, the conventional scheme for both low and high SNRs, regardless of concerned coherence intervals.

V. Numerical Results

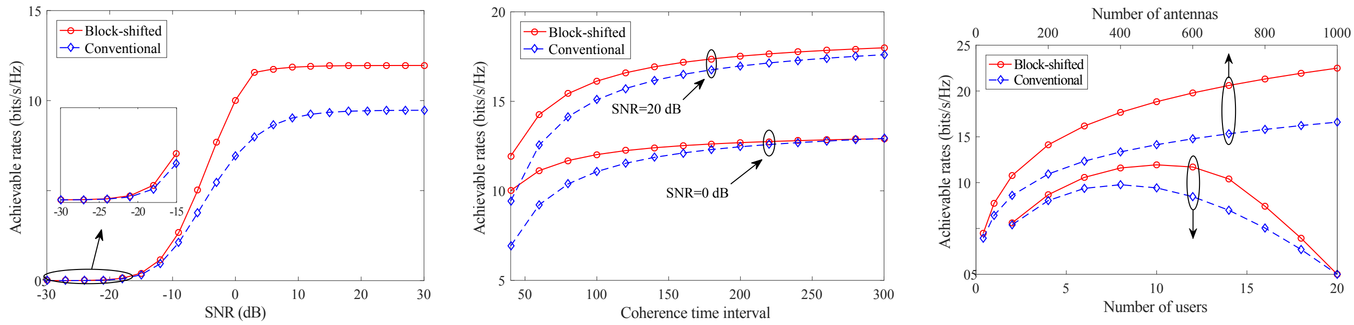
In this section, we verify the performance improvements of the proposed block-shifted pilot scheme by presenting numerical results. In following evaluations, if it is not specified, the RS is equipped with 128 antennas serving 10 pairs of users, the coherence time interval is set to be 40 time slots, and the transmission power of pilots, source data and forwarding data satisfies $\rho_{\mathrm{p}} = \rho_{\mathrm{s}} = \rho_{\mathrm{d}} = 20$ dB.

Firstly, we evaluate the system achievable rates under different SNRs and compare the performance between the proposed and conventional schemes. The comparisons are shown in Fig. 2(a) where the SNR varies from -30 dB to 30 dB. The result figure alongside with the embedded one depicts that the proposed scheme outperforms the conventional one in both high and low SNRs, which verifies Corollary 1, where about 2.5 bits/s/Hz improvement is observed in the high SNR region.

Moreover, we elaborate the performance comparisons by increasing the number of RS antennas. As depicted by Fig. 2(c), it is obvious that the gap of rates between the proposed and conventional schemes is increasing as the number of RS antennas grows. It verifies the fact that with more RS antennas, the source and destination channels are closer to be orthogonal to each other, thus less interference resides in the combined signal. On the other hand, albeit as few as 20 antennas are deployed on RS, the proposed scheme still outperforms the conventional one, which implies that it is feasible and superior even in a moderate size massive MIMO relaying system.

As claimed in previous discussions, the performance improvements of the proposed scheme is obtained by balancing the tradeoff between pilot overhead and channel estimation accuracy. Here, we examine the performance improvements of the proposed scheme with respect to the length of coherence intervals. According to Fig. 2(b), the achievable rate of the proposed scheme always outperforms the conventional one in the corresponding communication mode from 20 to 300 time slots of the coherence interval at both SNRs, which verifies the conclusion in Remark 2. Nonetheless, the gap between two schemes is decreasing as the increasing of interval length with a fixed number of user pairs, which is due to the reduction of pilot transmission overhead within a longer coherence interval.

In practice, the network operator would prefer to serve more users to improve overall rates when a large number of antennas are deployed. Therefore, the performance of the



(a) Different SNRs (b) Coherence time interval (c) Number of user pairs / RS antennas

Fig. 2: Achievable rate performance comparisons between the proposed and conventional schemes under different conditions.

proposed scheme is examined by varying the number of user pairs at a fixed coherence interval. As shown by Fig. 2(c), 12 bits/s/Hz is achieved when 10 user pairs are communicating simultaneously by utilizing the proposed scheme while about 9.7 bits/s/Hz is achieved when only 8 user pairs are served with the conventional scheme. This shows about 24% improvements of achievable rate performance and 25% increments on the serving users with the proposed scheme. Therefore, it explicitly reveals the superiority of the proposed scheme to the conventional one.

VI. Conclusion

In this paper, a design of block-shifted pilot scheme is proposed in the half-duplex multipair massive MIMO relaying system. The proposed pilot scheme improves the achievable rate performance by decreasing the pilot transmission overhead. The asymptotic orthogonality of massive MIMO channels is utilized to suppress pilot-data interference while detecting source data and estimating destination channels. The performance improvement and superiority of the proposed scheme is proved theoretically and verified numerically.

Appendix

A. Preliminary

Lemma 1. *(The law of large numbers) [11] Let* $\mathbf{p}$ *and* $\mathbf{q}$ *are two mutually independent* $L \times 1$ *random vectors consisting of i.i.d.* $\mathcal{CN}(0, \sigma_p^2)$ *and* $\mathcal{CN}(0, \sigma_q^2)$ *RVs, respectively. Then* $\lim\limits_{L\to\infty} \mathbf{p}^{\mathrm{H}}\mathbf{p}/L \overset{\text{a.s.}}{=} \sigma_p^2$ *and* $\lim\limits_{L\to\infty} \mathbf{p}^{\mathrm{H}}\mathbf{q}/L \overset{\text{a.s.}}{=} 0$.

B. Proof of Proposition 1

By normalizing its power, the kth $(k = 1, 2, \cdots, K)$ row of $\tilde{\mathbf{S}}^{\mathrm{B}}$ can be rewritten as follows:

$$\begin{aligned}\hat{\mathbf{s}}_k^{\mathrm{B}} &= \frac{\sqrt{\rho_{\mathrm{s}}}\hat{\mathbf{g}}_{\mathrm{s}k}^{\mathrm{H}}\mathbf{G}_{\mathrm{s}}\mathbf{S}^{\mathrm{B}} + \sqrt{K\rho_{\mathrm{p}}}\hat{\mathbf{g}}_{\mathrm{s}k}^{\mathrm{H}}\mathbf{G}_{\mathrm{d}}\mathbf{\Psi} + \hat{\mathbf{g}}_{\mathrm{s}k}^{\mathrm{H}}\mathbf{N}^{\mathrm{B}}}{\sqrt{\rho_{\mathrm{s}}}\,\|\hat{\mathbf{g}}_k\|^2} \\ &\overset{(\mathrm{a})}{=} \frac{\left(\hat{\mathbf{g}}_{\mathrm{s}k}^{\mathrm{H}}\hat{\mathbf{g}}_{\mathrm{s}k} + \hat{\mathbf{g}}_{\mathrm{s}k}^{\mathrm{H}}\boldsymbol{\varepsilon}_{\mathrm{s}k}\right)\mathbf{s}_k^{\mathrm{B}}}{\|\hat{\mathbf{g}}_{\mathrm{s}k}\|^2} + \frac{\sum_{i=1, i\neq k}^{K}\hat{\mathbf{g}}_{\mathrm{s}k}^{\mathrm{H}}\mathbf{g}_{\mathrm{s}i}\mathbf{s}_i^{\mathrm{B}}}{\|\hat{\mathbf{g}}_k\|^2} \\ &\quad + \frac{\sqrt{K\rho_{\mathrm{p}}}\sum_{i=1}^{K}\hat{\mathbf{g}}_{\mathrm{s}k}^{\mathrm{H}}\mathbf{g}_{\mathrm{d}i}\boldsymbol{\psi}_i}{\sqrt{\rho_{\mathrm{s}}}\,\|\hat{\mathbf{g}}_k\|^2} + \frac{\hat{\mathbf{g}}_{\mathrm{s}k}^{\mathrm{H}}\mathbf{N}^{\mathrm{B}}}{\sqrt{\rho_{\mathrm{s}}}\,\|\hat{\mathbf{g}}_k\|^2} \overset{\text{a.s.}}{=} \mathbf{s}_k^{\mathrm{B}}\end{aligned} \tag{20}$$

where $\mathbf{s}_i^{\mathrm{B}}$ and $\boldsymbol{\psi}_i$ is the ith row of $\mathbf{S}^{\mathrm{B}}$ and $\mathbf{\Psi}$, respectively. The derivation $\overset{(\mathrm{a})}{=}$ attributes to (3). Because $\hat{\mathbf{g}}_{\mathrm{s}k}^{\mathrm{H}}$ is independent of $\boldsymbol{\varepsilon}_{\mathrm{s}k}$, $\mathbf{g}_{\mathrm{s}i}$ $(i \neq k)$ and each column of $\mathbf{N}^{\mathrm{B}}$, by applying Lemma 1, the convergence in the last equality holds due to $M \to \infty$. Therefore, $\mathbf{s}_k^{\mathrm{B}}$ can be exactly detected from $\tilde{\mathbf{s}}_k^{\mathrm{B}}$.

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