

# Near Field Targets Localization Using Bistatic MIMO System with Spherical Wavefront Based Model

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**Abstract**—This paper proposes a subspace based near field targets localization method with bistatic MIMO system consisting of uniform linear transmitting and receiving arrays. The proposed method uses the spherical wavefront based exact model to avoid the systematic error introduced by the Fresnel approximation, which is usually made on the wavefront to simplify the signal model for a near field source in the existing literature on near field sources localization. By avoiding this approximation, we have significantly improved the estimation accuracy. Additionally, unlike most of the existing near field sources localization techniques, the proposed method works for the array with interelement spacing greater than a quarter of the carrier wavelength.

## I. INTRODUCTION

MIMO system has grabbed a lot of attention in radar applications. Based on the location of the transmitting and receiving antennas with respect to a target, MIMO system can be classified as colocated or distributed [1], [2]. In case of colocated MIMO system, the consecutive sensors in the transmitting and receiving arrays are separated by a distance less than or equal to one half of the carrier wavelength, however, the arrays may be located significantly apart. When the transmitting and receiving arrays are in the same place, the system is monostatic MIMO, and when they are in separate places, the system is bistatic MIMO. A large number of existing targets localization methods using bistatic MIMO system have been proposed for far field targets as compared to that for the near field targets [3]–[7]. Since a bistatic MIMO radar has colocated antennas, it provides high parameters identifiability, improved angular resolution, and transmit beampattern flexibility [8]–[10].

A point source emits spherical wavefront which leads to a nonlinear signal model. To simplify the signal model, the spherical wavefront is approximated as planar in far field situation and quadric in near field situation by using the first order and second order Taylor's expansions respectively [11]. However, this Fresnel approximation introduces systematic bias in the estimates of the location parameters like range

and directional angle. This bias becomes significant when the targets are close to the array. [12] shows the position estimation error in the near field region of a bistatic MIMO system due to the Fresnel approximation. From the perspective of precision, the error should be avoided if possible. There can be two approaches to avoid the error due to the Fresnel approximation. One approach can be to use a method which can directly deal with the spherical wavefront based exact model such as [13]. Other approach can be to mitigate the estimation error by a correction technique [14].

This paper proposes a novel method to locate near field targets using bistatic MIMO system with the exact wavefront model. The proposed method is inspired by the method in [7] which is proposed for the approximated model. We improve the method proposed in [7] by fully exploiting all the available information and adapting it to deal with the exact wavefront based signal model. The method in [7] uses five transmitting antennas and four cross-covariance matrices between the output data blocks to estimate the location parameters, thus, leaves the remaining six cross-covariance matrices unexploited. In [7], if the number of transmitting antennas increases, the unused portion of the covariance matrix also increases. In the proposed method, we try to use all the submatrices of the covariance matrix. Due to the use of submatrices in the method in [7] and the proposed method, the maximum number of localizable targets is limited by the number of receiving antennas. Like most of the existing near field sources localization methods, the method in [7] works only with the arrays whose interelement spacing is less than or equal to a quarter of the carrier wavelength. The proposed method can avoid such constraint, and support the interelement spacing greater than a quarter of the carrier wavelength. Along with this paper, we have also proposed another method in [15] which is based on the near field model with Fresnel approximation like [7].

The remainder of the paper is organized as follows. Section II formulates the spherical wavefront based signal model for the near field region of a bistatic MIMO system with uniform linear transmitting and receiving arrays. Section III describes the proposed method to estimate the location parameters of

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targets viz. angles of departure and arrival, distance between the targets and the reference point of the transmitting array, and distance between the targets and the reference point of the receiving array. Section IV shows the performance of the proposed method.

### Notations

$\square$  denotes the Khatri-Rao product.  $\mathfrak{D}\{\mathbf{v}\}$  denotes the diagonal matrix with components of vector  $\mathbf{v}$  along its diagonal.  $\mathcal{E}\{\bullet\}$  implies the expected value.  $[\bullet]^*$ ,  $[\bullet]^T$ ,  $[\bullet]^H$ , and  $[\bullet]^\dagger$  respectively represent the conjugate, transpose, Hermitian transpose, and pseudo inverse of a matrix.  $[\bullet]^{-1}$  denotes the inverse of a square matrix.  $\mathbb{R}$  and  $\mathbb{C}$  are the sets of all real and complex numbers respectively.  $\angle$  denotes the principal value of the argument of a complex number.  $\mathbf{I}_G$  signifies an identity matrix of dimension  $G \times G$  where  $G$  belongs to the set of positive integers.

## II. SIGNAL DATA MODEL

Assume a narrowband bistatic MIMO system with  $M$  transmitting and  $N$  receiving omnidirectional sensors. Let  $d_e$  and  $d_r$  be the interelement spacings in the transmitting and receiving uniform linear arrays respectively. In addition, we suppose that each transmitter emits temporally orthogonal signals with same bandwidth and carrier frequency which impinge on  $P$  near field point targets and their reflections are intercepted by the receiving array. At the reception, the orthogonal signals are separated by using matched filters. The received matched filtered signal data at time  $t$  can be written as [3]–[6]

$$\mathbf{y}(t) = (\mathbf{A} \square \mathbf{B}) \mathbf{s}(t) + \mathbf{n}(t) \quad (1)$$

where  $\mathbf{y}(t) \in \mathbb{C}^{MN \times 1}$  is the received signal data vector.  $\mathbf{s}(t)$  contains the reflection coefficients of  $P$  targets which follow Swerling model II [16].  $\mathbf{n}(t)$  is the noise vector with spatially and temporally white complex Gaussian components with zero mean and covariance matrix  $\mathcal{E}\{\mathbf{n}(t) \mathbf{n}^H(t)\} = \sigma^2 \mathbf{I}_{MN}$  where  $\sigma^2$  is the noise variance.  $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_P] \in \mathbb{C}^{M \times P}$  and  $\mathbf{B} = [\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_P] \in \mathbb{C}^{N \times P}$  are the directional matrices of  $P$  targets with  $\mathbf{a}_p = [1, a_{(2,p)}, \dots, a_{(M,p)}]^T$  and  $\mathbf{b}_p = [1, b_{(p,2)}, \dots, b_{(p,N)}]^T$  respectively being the directional vectors of departure and arrival of the  $p$ th target having their first element as their reference point where  $a_{(m,p)} = e^{-j2\pi \delta_{e(m,p)}/\lambda}$ ,  $b_{(p,n)} = e^{-j2\pi \delta_{r(p,n)}/\lambda}$ ,  $m \in \{1, 2, \dots, M\}$ ,  $n \in \{1, 2, \dots, N\}$ ,  $\lambda$  is the wavelength of the carrier wave,

$$\delta_{e(m,p)} = \sqrt{\rho_{e_p}^2 + (m-1)^2 d_e^2 - 2(m-1)d_e \rho_{e_p} \cos(\theta_{e_p})} - \rho_{e_p} \quad (2)$$

is the difference between the distance traveled by the signal emitted by the  $m$ th transmitting antenna to reach the  $p$ th target and the distance traveled by the signal emitted by the reference transmitting antenna to reach the same target,

$$\delta_{r(p,n)} = \sqrt{\rho_{r_p}^2 + (n-1)^2 d_r^2 - 2(n-1)d_r \rho_{r_p} \cos(\theta_{r_p})} - \rho_{r_p} \quad (3)$$

is the difference between the distance traveled by the reflected signal from the  $p$ th target to the  $n$ th receiving antenna and the distance traveled by it to reach the reference receiving antenna,  $\rho_{e_p}$  and  $\rho_{r_p}$  are respectively the distances from the reference transmitting and receiving antennas to the  $p$ th target,  $\theta_{e_p}$  is the angle of departure measured at the reference transmitting antenna with respect to the axis of the transmitting array, and  $\theta_{r_p}$  is the angle of arrival of the  $p$ th target measured at the reference receiving antenna with respect to the axis of the receiving array [11].  $\theta_{e_p}$  and  $\theta_{r_p}$  can vary from 0 to  $\pi$  rad.

## III. PROPOSED LOCALIZATION APPROACH

As in [7], the received signal vector is subdivided into  $M$  subvectors belonging to each transmitting antenna as  $\mathbf{y}(t) = [\mathbf{y}_1^T(t), \mathbf{y}_2^T(t), \dots, \mathbf{y}_M^T(t)]^T$ . The subvector corresponding to the  $m$ th transmitter can be written as

$$\mathbf{y}_m(t) = \check{\mathbf{y}}_m(t) + \mathbf{n}_m(t) \quad (4)$$

where  $\mathbf{n}_m(t)$  is the corresponding noise subvector and  $\check{\mathbf{y}}_m(t) = \mathbf{B} \mathbf{D}_m \mathbf{s}(t)$  where

$$\mathbf{D}_m = \mathfrak{D} \{ [a_{(m,1)}, a_{(m,2)}, \dots, a_{(m,P)}] \} \quad (5)$$

is the diagonal matrix with the elements of the  $m$ th row of  $\mathbf{A}$  as its diagonal components. As defined above,  $\mathbf{B} = [\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_P]$  is the directional matrix of  $P$  targets of the receiving array.

The eigendecomposition of the covariance matrix  $\mathbf{R} = \mathcal{E}\{\mathbf{y}(t) \mathbf{y}^H(t)\} \in \mathbb{C}^{MN \times MN}$  can be written as

$$\mathbf{R} = \mathbf{U} \mathbf{\Lambda} \mathbf{U}^H \quad (6)$$

where the columns of  $\mathbf{U}$  are the eigenvectors and the diagonal elements of  $\mathbf{\Lambda}$  are the corresponding eigenvalues. If the eigenvalues are sorted in descending order, then the  $q$ th eigenvalue can be expressed as

$$\mu_q = \begin{cases} \check{\mu}_q + \sigma^2 & \text{if } q \in \{1, 2, \dots, P\} \\ \sigma^2 & \text{if } q \in \{P+1, P+2, \dots, MN\} \end{cases} \quad (7)$$

where  $\check{\mu}_q$  is the  $q$ th eigenvalue among the  $P$  nonzero eigenvalues of  $(\mathbf{A} \square \mathbf{B}) \mathbf{R}_s (\mathbf{A} \square \mathbf{B})^H$  arranged in descending order with  $\mathbf{R}_s = \mathcal{E}\{\mathbf{s}(t) \mathbf{s}^H(t)\}$ . We assume that  $P$  is known. The following subspace based approach can be used to remove the additive noise, in which the noiseless covariance matrix is reconstructed from  $\mathbf{U}$  and  $\mathbf{\Lambda}$  of  $\mathbf{R}$  as [17]

$$\check{\mathbf{R}} = \check{\mathbf{U}} \mathfrak{D} \{ [\mu_1 - \sigma^2, \dots, \mu_P - \sigma^2] \} \check{\mathbf{U}}^H \quad (8)$$

where  $\check{\mathbf{U}}$  contains the columns of  $\mathbf{U}$  corresponding to the  $P$  largest eigenvalues of  $\mathbf{R}$ . When a finite number of samples is used, then  $\mu_{P+1} \neq \mu_{P+2} \neq \dots \neq \mu_{MN} \neq \sigma^2$ . To overcome it, we estimate the noise variance by  $\hat{\sigma}^2 = (\mu_{P+1} + \mu_{P+2} + \dots + \mu_{MN}) / (MN - P)$ .

Theoretically,  $\check{\mathbf{R}} = (\mathbf{A} \square \mathbf{B}) \mathbf{R}_s (\mathbf{A} \square \mathbf{B})^H$  can be viewed as a block matrix with submatrices

$$\begin{aligned} \check{\mathbf{R}}_{(m,m')} &= \mathcal{E}\{\check{\mathbf{y}}_m(t) \check{\mathbf{y}}_{m'}^H(t)\} \in \mathbb{C}^{N \times N} \\ &= \mathbf{B} \mathbf{D}_m \mathbf{R}_s \mathbf{D}_{m'}^* \mathbf{B}^H \end{aligned} \quad (9)$$

where  $m' \in \{1, 2, \dots, M\}$ . When  $m = 1$ , we have

$$\check{\mathbf{R}}_{(1, m')} = \mathbf{B} \mathbf{R}_s \mathbf{D}_{m'}^* \mathbf{B}^H \quad (10)$$

because  $\mathbf{D}_1$  is the identity matrix. Using (10), (9) can be rewritten as

$$\check{\mathbf{R}}_{(m, m')} = \mathbf{B} \mathbf{D}_m \mathbf{B}^\dagger \check{\mathbf{R}}_{(1, m')}. \quad (11)$$

On rearranging, we can get

$$\check{\mathbf{R}}_{(m, m')} \check{\mathbf{R}}_{(1, m')}^+ = \mathbf{B} \mathbf{D}_m \mathbf{B}^\dagger \quad (12)$$

where  $[\bullet]^+$  denotes the inverse operation using singular value decomposition which is explained below.

As mentioned in [7], the inverse of  $\check{\mathbf{R}}_{(1, m')}$  should be calculated by using the singular values and vectors corresponding to the  $P$  largest singular values to improve the robustness in a noisy environment. It is because, when  $P < N$ ,  $\check{\mathbf{R}}_{(1, m')}$  should theoretically be a non-invertible square matrix. Thus, we use the following method to obtain its inverse and use  $\check{\mathbf{R}}_{(1, m')}^+$  instead of  $\check{\mathbf{R}}_{(1, m')}^{-1}$  to represent its inverse. Let  $\check{\mathbf{R}}_{(1, m')} = \mathbf{U}_{m'} \mathbf{S}_{m'} \mathbf{V}_{m'}^H$  where  $\mathbf{U}_{m'} \in \mathbb{C}^{N \times N}$ ,  $\mathbf{S}_{m'} \in \mathbb{R}^{N \times N}$ , and  $\mathbf{V}_{m'} \in \mathbb{C}^{N \times N}$  are the matrices containing the left singular vectors, singular values, and right singular vectors respectively. Then,  $\check{\mathbf{R}}_{(1, m')}^+ = \check{\mathbf{V}}_{m'} \check{\mathbf{S}}_{m'}^{-1} \check{\mathbf{U}}_{m'}^H$  where  $\check{\mathbf{U}}_{m'} \in \mathbb{C}^{N \times P}$ ,  $\check{\mathbf{S}}_{m'} \in \mathbb{R}^{P \times P}$ , and  $\check{\mathbf{V}}_{m'} \in \mathbb{C}^{N \times P}$  are the matrices that contain the left singular vectors associated with the  $P$  largest singular values, the corresponding  $P$  singular values and right singular vectors respectively. In practice, the remaining  $(N - P)$  singular values will be close to zero. Therefore,  $\check{\mathbf{R}}_{(1, m')}$  will not be rank deficient and the conventional way of inverting a square matrix may introduce numerical inaccuracy. Thus, this step improves the numerical accuracy of the calculation of inverse.

The submatrices with the same index  $m$  have the same  $\mathbf{D}_m$ , thus we can add them together as

$$\bar{\mathbf{R}}_m = \frac{1}{M} \sum_{m'=1}^M \check{\mathbf{R}}_{(m, m')} \check{\mathbf{R}}_{(1, m')}^+ = \mathbf{B} \mathbf{D}_m \mathbf{B}^\dagger. \quad (13)$$

(13) can also be expressed as

$$\bar{\mathbf{R}}_m \mathbf{B} = \mathbf{B} \mathbf{D}_m \quad (14)$$

where the columns of  $\mathbf{B}$  and diagonal elements of the diagonal matrix  $\mathbf{D}_m$  (5) are, by definition, respectively the eigenvectors and eigenvalues of  $\bar{\mathbf{R}}_m$ . Let  $\mathbf{u}_{(m, p)}$  and  $\gamma_{(m, p)}$  be respectively the  $p$ th eigenvector and eigenvalue of  $\bar{\mathbf{R}}_m$  ( $\gamma_{(m, p)}$  is one of the  $P$  largest eigenvalues). Further, we can write  $\mathbf{u}_{(m, p)} = \alpha_{(m, p)} e^{j\phi_{(m, p)}} \mathbf{b}_p$  and  $\gamma_{(m, p)} = a_{(m, p)}$  where  $\alpha_{(m, p)}$  and  $\phi_{(m, p)}$  respectively represent the scaling factor and phase shift introduced during the eigendecomposition. Since the eigendecomposition of  $M - 1$  matrices  $\bar{\mathbf{R}}_m$  (with  $m \in \{2, 3, \dots, M\}$ ) is performed independently, therefore an additional step is required to pair all the  $M - 1$  sets of the eigenvalues and eigenvectors. Classically, the pairing can be done by comparing the inner product of the eigenvectors from the fact that the inner product of two aligned vectors is greater than that of two nonaligned vectors [7].

$\rho_{r_p}$  and  $\theta_{r_p}$  can be estimated from the angular part of  $\mathbf{u}_{(m, p)}$ . The argument of a complex number can only be calculated in its principal form. Therefore, phase unwrapping is necessary [18]. Phase unwrapping also introduces phase shift. As we know that  $\angle b_{(p, 1)} = 0$ , therefore the unwrapped phase at  $n = 1$  is the phase shift introduced during the eigendecomposition as well as unwrapping. Thus, subtracting this phase shift from all the remaining components of the unwrapped phase vector gives an estimation of the true phases. The eigendecompositions of  $M - 1$   $\bar{\mathbf{R}}_m$  matrices provide  $M - 1$  eigenvectors associated to the signal reflected by the  $p$ th target. Thus,  $M - 1$  estimates of the true phases can be obtained for each target. At high SNR each estimation will be the same, however, when SNR is low, it is better to combine them by averaging. Let  $\hat{\delta}_{r_{(p, n)}}$  be the estimated value of  $\delta_{r_{(p, n)}}$  obtained from the averaged estimate of the true phase by dividing it by  $-2\pi/\lambda$ .

Rearranging (3) leads to

$$2(n-1)d_r \rho_{r_p} \cos(\theta_{r_p}) + 2\hat{\delta}_{r_{(p, n)}} \rho_{r_p} = (n-1)^2 d_r^2 - \hat{\delta}_{r_{(p, n)}}^2 \quad (15)$$

which can be used to construct a system of linear equations in  $\rho_{r_p} \cos(\theta_{r_p})$  and  $\rho_{r_p}$  for all the values of  $n$ . Total least squares method given in [19] can be used to solve this system. To obtain the total least squares estimates, let  $[v_{1_p}, v_{2_p}, v_{3_p}]^T$  be the right-singular-vector associated with the smallest singular value of

$$\begin{bmatrix} 2d_r & 2\hat{\delta}_{r_{(p, 2)}} & d_r^2 - \hat{\delta}_{r_{(p, 2)}}^2 \\ 4d_r & 2\hat{\delta}_{r_{(p, 3)}} & 4d_r^2 - \hat{\delta}_{r_{(p, 3)}}^2 \\ 6d_r & 2\hat{\delta}_{r_{(p, 4)}} & 9d_r^2 - \hat{\delta}_{r_{(p, 4)}}^2 \\ \vdots & \vdots & \vdots \\ 2(N-1)d_r & 2\hat{\delta}_{r_{(p, N)}} & (N-1)^2 d_r^2 - \hat{\delta}_{r_{(p, N)}}^2 \end{bmatrix}$$

which is constructed by using the coefficients of (15). The range and angle of arrival of the  $p$ th target can respectively be calculated by  $\hat{\rho}_{r_p} = -v_{2_p}/v_{3_p}$  and  $\hat{\theta}_{r_p} = \cos^{-1}(v_{1_p}/v_{2_p})$  where  $\hat{\rho}_{r_p}$  and  $\hat{\theta}_{r_p}$  denote the estimated values of  $\rho_{r_p}$  and  $\theta_{r_p}$  respectively.

The angle of the eigenvalue  $\gamma_{(m, p)} = e^{-j2\pi\delta_{e(m, p)}/\lambda}$  can be used to estimate the transmitter side ranges and angles of departure of the targets. Being a complex entity, the unwrapped phase of  $\gamma_{(m, p)}$  should be estimated, which is followed by the phase shifting with respect to the phase of reference component like before to provide the true unwrapped phase vector. Since, (2) and (3) have the same expression, the similar steps are conducted as in the case of the estimation of the receiver side location parameters. The estimated value of  $\delta_{e(m, p)}$  is obtained from the true unwrapped phase and then used to build a system of linear equations whose  $m$ th equation is given by

$$2(m-1)d_e \rho_{e_p} \cos(\theta_{e_p}) + 2\hat{\delta}_{e(m, p)} \rho_{e_p} = (m-1)^2 d_e^2 - \hat{\delta}_{e(m, p)}^2 \quad (16)$$

with  $\hat{\delta}_{e(m,p)}$  being the estimated value of  $\delta_{e(m,p)}$ . Total least squares is used to estimate the unknowns, viz.  $\rho_{e_p} \cos(\theta_{e_p})$  and  $\rho_{e_p}$ , of the system of linear equations. Similar to the receiver side estimation, let  $[w_{1p}, w_{2p}, w_{3p}]^T$  be the right-singular-vector linked with the smallest singular value of the following matrix constructed from the coefficients of (16)

$$\begin{bmatrix} 2d_e & 2\hat{\delta}_{e(2,p)} & d_e^2 - \hat{\delta}_{e(2,p)}^2 \\ 4d_e & 2\hat{\delta}_{e(3,p)} & 4d_e^2 - \hat{\delta}_{e(3,p)}^2 \\ 6d_e & 2\hat{\delta}_{e(4,p)} & 9d_e^2 - \hat{\delta}_{e(4,p)}^2 \\ \vdots & \vdots & \vdots \\ 2(M-1)d_e & 2\hat{\delta}_{e(M,p)} & (M-1)^2 d_e^2 - \hat{\delta}_{e(M,p)}^2 \end{bmatrix}.$$

The range from the transmitter to the  $p$ th target and angle of departure can respectively be given by  $\hat{\rho}_{e_p} = -w_{2p}/w_{3p}$  and  $\hat{\theta}_{e_p} = \cos^{-1}(w_{1p}/w_{2p})$  where  $\hat{\rho}_{e_p}$  and  $\hat{\theta}_{e_p}$  denote the estimated values of  $\rho_{e_p}$  and  $\theta_{e_p}$  respectively.

Even though the proposed method is inspired by the method in [7], it doesn't come across any step which can pose any constraint on the interelement spacings of the transmitting and receiving arrays. Therefore, the proposed method supports interelement spacing of  $\lambda/2$ .

#### IV. SIMULATION RESULTS

Consider a bistatic MIMO system consisting of  $M = 8$  transmitting and  $N = 9$  receiving antennas. The interelement spacing in both the transmitting and receiving uniform linear arrays is  $\lambda/2$ .

Fig. 1 and Fig. 2 respectively show the performance of the proposed method with respect to SNR in terms of RMSE in the estimation of the ranges and directional angles of two targets located at  $(9.72\lambda, 1.83 \text{ rad}, 11.85\lambda, 1.19 \text{ rad})$  and  $(11.51\lambda, 1.08 \text{ rad}, 9.47\lambda, 1.63 \text{ rad})$  which indicate  $(\rho_{e_p}, \theta_{e_p}, \rho_{r_p}, \theta_{r_p})$ . The signal data with  $L = 10^3$  samples are put under  $K = 10^3$  Monte Carlo trials to calculate the RMSE. The RMSE is calculated by using the following equation

$$\epsilon(\eta_p) = \sqrt{\frac{1}{K} \sum_{k=1}^K (\hat{\eta}_p(k) - \eta_p)^2} \quad (17)$$

where  $\eta_p \in \{\rho_{e_p}, \theta_{e_p}, \rho_{r_p}, \theta_{r_p}\}$  and  $\hat{\eta}_p(k) \in \{\hat{\rho}_{e_p}(k), \hat{\theta}_{e_p}(k), \hat{\rho}_{r_p}(k), \hat{\theta}_{r_p}(k)\}$  is the estimated value in the  $k$ th trial.

Fig. 1 and Fig. 2 indicate that the proposed method is able to estimate the location parameters of near field targets without using the Fresnel approximation.

In addition, assume another MIMO system with  $M = 5$  transmitting and  $N = 9$  receiving antennas with interelement spacing of  $\lambda/4$ . Using  $L = 10^3$  samples,  $K = 10^3$  Monte Carlo trials, and two targets at  $(0.71\lambda, 1.9 \text{ rad}, 2.62\lambda, 1.04 \text{ rad})$  and  $(1.68\lambda, 1.01 \text{ rad}, 3.11\lambda, 0.94 \text{ rad})$ , we have compared the RMSE of the location parameters estimated by the method in [7] and the proposed method in Fig. 3 and Fig. 4.

Fig. 3 and Fig. 4 show the expected results. The RMSE of the location parameters estimated by the proposed method is

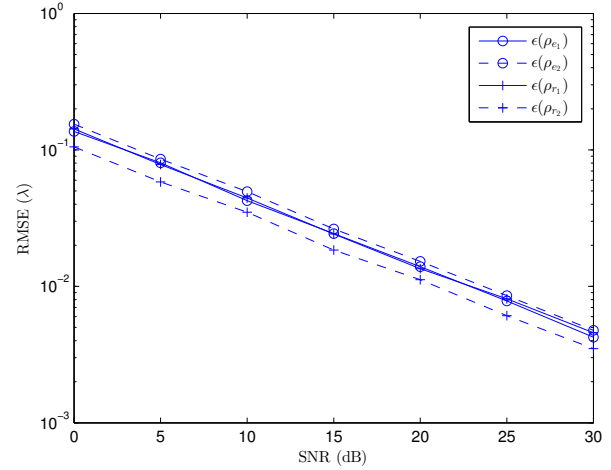


Fig. 1. RMSE in ranges estimation versus SNR;  $d_e = d_r = \lambda/2$ ,  $K = 10^3$ ,  $L = 10^3$ ,  $M = 8$ ,  $N = 9$ , and  $P = 2$ .

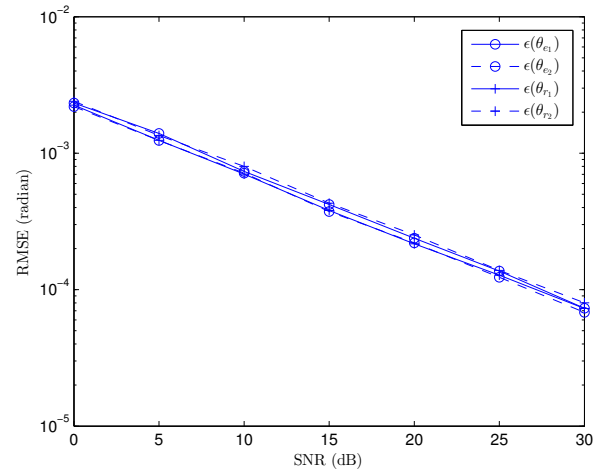


Fig. 2. RMSE in angles estimation versus SNR;  $d_e = d_r = \lambda/2$ ,  $K = 10^3$ ,  $L = 10^3$ ,  $M = 8$ ,  $N = 9$ , and  $P = 2$ .

less than that of the method in [7] because of the absence of the systematic bias introduced by the Fresnel approximation. In the figures, at high SNR, when the Fresnel approximation error surpasses the error due to the additive noise, the RMSE plot of [7] experiences a floor effect.

The method in [7] is based on the approximated wavefront model. Therefore, they have used the signal data generated from the approximated wavefront model to analyze the performance of their method. However, in reality, the received data should be ruled by the exact spherical wavefront based signal model, which is the reason that the performance of the method in [7] is rather poor in our simulations.

#### V. CONCLUSION

An improvement of the subspace based method in [7] has been proposed in this paper to localize targets in the near field region of a bistatic MIMO system without making the

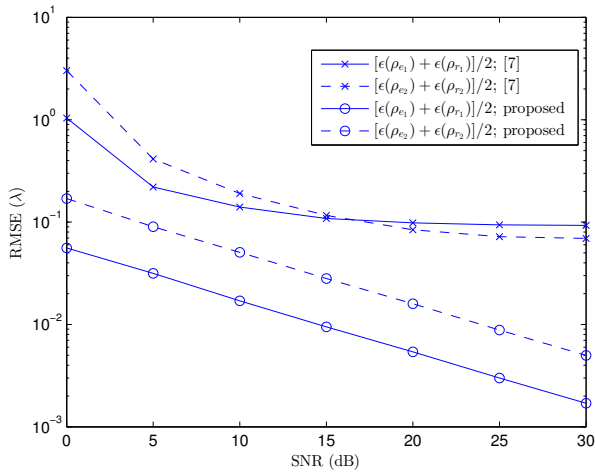


Fig. 3. RMSE in ranges estimated by the method in [7] and the proposed method versus SNR;  $d_e = d_r = \lambda/4$ ,  $K = 10^3$ ,  $L = 10^3$ ,  $M = 5$ ,  $N = 9$ , and  $P = 2$ .

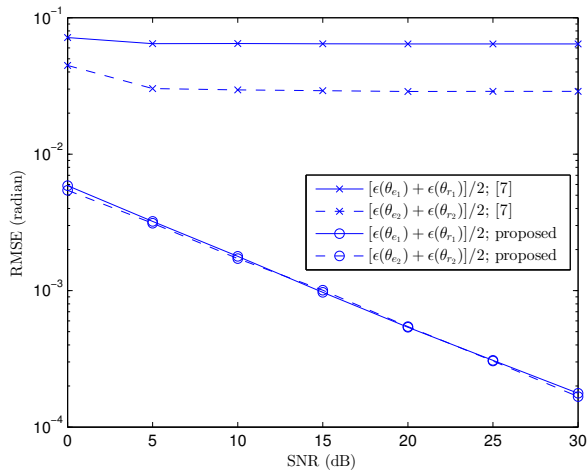


Fig. 4. RMSE in angles estimated by the method in [7] and the proposed method versus SNR;  $d_e = d_r = \lambda/4$ ,  $K = 10^3$ ,  $L = 10^3$ ,  $M = 5$ ,  $N = 9$ , and  $P = 2$ .

Fresnel approximation. The proposed method has much better performance in terms of RMSE of the estimated location parameters than the method proposed in [7]. Thanks to the capacity of directly dealing with the exact wavefront model and the exploitation of all the available information contained in the covariance matrix of the received signal. In addition, contrary to the most of the existing approximated model based methods, the proposed method works even for the array with interelement spacing equal to  $\lambda/2$ .

The performance analysis of the proposed method with the advanced covariance matrix estimators [20]–[22] and real radar data can be done as a future work.

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