

# DECONVOLUTION-SEGMENTATION FOR TEXTURED IMAGES

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## ABSTRACT

The paper tackles the problem of joint deconvolution and segmentation specifically for textured images. The images are composed of patches of textures that belong to a set of  $K$  possible classes. Each class of image is described by a Gaussian random field and the classes are modelled by a Potts field. The method relies on a hierarchical model and a Bayesian strategy to jointly estimate the labels, the textured images as well as the hyperparameters. An important point is that the parameter of the Potts field is also estimated. The estimators are designed in an optimal manner (marginal posterior maximizer for the labels and posterior mean for the other unknowns). They are computed based on a convergent procedure, from samples of the posterior obtained through an MCMC algorithm (Gibbs sampler including Perturbation-Optimization). A first numerical evaluation provides encouraging results despite the strong difficulty of the problem.

**Index Terms** — Deconvolution, segmentation, texture, Bayes, Potts, sampling, optimization.

## I. INTRODUCTION AND MOTIVATION

The paper addresses the tough problem of textured image segmentation from indirect (noisy and blurred) observations. Most existing approaches do not tackle the indirect observations issue and focus only on segmentation. To the best of our knowledge, the proposed method is a first attempt for a joint deconvolution-segmentation of textured images. In addition, it also performs hyperparameter estimation.

Image segmentation has been of great interest for decades, so the literature is extensive [1]. For instance, let us cite thresholding, growing methods, bottom-up aggregation, active contour, Markov approaches, level sets, curve evolution, watershed, graph cuts and random walker,...

For the specific case of textured images [2, 3], an important part of the literature relies on features based upon moments of the image, on wavelet or Fourier transform, on spatial or spectral histogram,... Let us cite some other approaches. A method providing texture edges [4] uses active contours and a patch based approach for texture analysis. A significant method based on both gray level (intervening contour framework) and texture (textons) is presented in [5]. [6] models a homogeneous textured region by a Gaussian distribution and the region boundaries by adaptive chain codes. Another approach [7] attempts to accurately characterize the textures by combining a collection of statistics and filter responses. Texel based image segmentation is achieved

in [8] by identifying the modes in the probability density function of region properties.

A significant class of segmentation methods relies on a probabilistic formulation. [9] presents a work for image partitioning into homogeneous regions and for locating edges based on disparity measures. In [10], an image segmentation method is developed based on Markov chain Monte Carlo and the  $K$  adventurers algorithm. [11] introduced a weighted Markov model that estimates the model parameters and thus performs unsupervised image segmentation. Let us also mention the popular graph partitioning approach [12, 13]. One of the most commonly used model for the labels in the probabilistic approaches is the Potts model to favour homogeneous regions. It is mostly used for piecewise constant or piecewise smooth images [14–16].

However, none of the aforementioned segmentation approaches is formulated in the context of indirect observations. Interesting works [17–23] are the Bayesian methods for image segmentation from indirect data (inversion-segmentation) also based a Potts model for the labels. Nevertheless, the existing developments are not adapted for textures. On the contrary, the present paper proposes joint deconvolution-segmentation specifically devoted to textured images. In addition, it also performs Potts parameter estimation whereas existing papers do not.

## II. PROBLEM STATEMENT

In this work,  $\mathbf{y}$  represents the blurred and noisy observation of an image  $\mathbf{i}$  that is a patchwork of textures. Each patch is extracted from an image  $\mathbf{x}_k$ , for  $k = 1..K$  and  $\ell$  represents the associated label.  $\mathbf{y}$ ,  $\mathbf{i}$ , the  $\mathbf{x}_k$  and  $\ell$  are column vectors of size  $P$  (the number of pixels).

**Label** — The labels are modelled by a Potts field, driven by a “correlation” parameter  $\beta$

$$\Pr[\ell|\beta] = \mathcal{C}_z(\beta)^{-1} \cdot \exp\left[\beta \sum_{r \sim s} \delta(\ell_r, \ell_s)\right] \quad (1)$$

where  $\sim$  stands for the four nearest neighbour relation between pixels and  $\delta$  is the Kronecker function. An important feature of the proposed method is the capability to estimate the parameter  $\beta$ . To this end, the partition function  $\mathcal{C}_z$  is a crucial function since it is involved in the likelihood of  $\beta$  attached to any configuration. Its analytical expression is unknown<sup>1</sup> and it is a huge summation over the  $K^P$  possible configurations. However, based on stochastic simulation, we have precomputed it for several image sizes and class

<sup>1</sup>except for the Ising field, i.e.  $K = 2$ , see [24] and also [25, 26].

numbers [27, 28]. The reader is invited to consult papers such as [29, 30] for alternatives.

**Textures** — The textured images  $\mathbf{x}_k \in \mathbb{C}^P$ ,  $k = 1..K$  are described by simple zero-mean stationary Gaussian fields

$$f(\mathbf{x}_k | \mathbf{R}_k) = (2\pi)^{-P} \det(\mathbf{R}_k)^{-1} \exp\left(-\|\mathbf{x}_k\|_{\mathbf{R}_k^{-1}}^2\right)$$

with covariance  $\mathbf{R}_k$ . For notational convenience, it is defined through a scale parameter  $\gamma_k$  and a structure matrix  $\mathbf{\Lambda}_k$ :  $\mathbf{R}_k^{-1} = \gamma_k \mathbf{\Lambda}_k$ . Since  $\mathbf{x}_k$  is a stationary field,  $\mathbf{\Lambda}_k$  is a Toeplitz-block-Toeplitz (TbT) matrix and by Whittle approximation, it becomes Circulant-block-Circulant (CbC). Then the pdf becomes separable in the Fourier domain:

$$f(\mathbf{x}_k | \mathbf{R}_k) = \prod_p (2\pi)^{-1} \gamma_k \lambda_{k,p} \exp\left[-\gamma_k \lambda_{k,p} |\hat{x}_{k,p}|^2\right] \quad (2)$$

where, for  $p = 1..P$ , the  $\hat{x}_{k,p}$  are the Fourier coefficients of the image  $\mathbf{x}_k$  and the  $\lambda_{k,p}$  are the eigenvalues of  $\mathbf{\Lambda}_k$ . Thus  $\lambda_k^{-1}$  describes the Power Spectral Density (PSD) of  $\mathbf{x}_k$  in a discrete form, more specifically,  $\gamma_k \lambda_{k,p}$  is the inverse variance of  $\hat{x}_{k,p}$ . As an example, we have chosen a Laplacian parametric model, with known central frequency and width. Nevertheless, any other parametric form can be used for the PSD, e.g., Gaussian, Lorentzian,...

**Image** — The process of obtaining the image  $i$  containing the texture patches, starting from the full textures  $\mathbf{x}_k$  and the labels  $\ell$  is formalized as:

$$i = \sum_k \mathbf{S}_k(\ell) \mathbf{x}_k \quad (3)$$

the  $\mathbf{S}_k$  being diagonal matrices with entry 1 for pixel  $p$  in the class  $k$  and 0 elsewhere:  $\mathbf{S}_k(\ell) = \text{diag}\{\delta(\ell_p, k), p = 1..P\}$ . They are zero-forcing matrices that extract from the image  $\mathbf{x}_k$  the pixels with label  $k$  and nullify the others.

**Convolution filter** — The convolution matrix  $\mathbf{H}$  has a TbT structure. It becomes CbC by circulant approximation and its eigenvalues are the Fourier transfer coefficients. Any function could be introduced (Gaussian, Airy,...) and the considered one is an isotropic Gaussian centred in the null frequency with known spatial width.

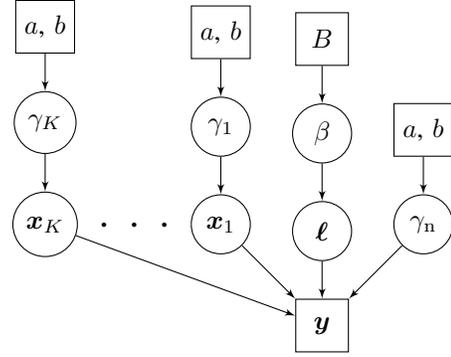
**Noise** — The errors are modelled as additive, zero-mean, white and Gaussian with inverse variance  $\gamma_n$ . So, the pdf for the data given the image and the noise level (likelihood) is:

$$f(\mathbf{y} | i, \gamma_n) = (2\pi)^{-P} \cdot \gamma_n^P \cdot \exp\left[-\gamma_n \|\mathbf{y} - \mathbf{H}i\|^2\right]. \quad (4)$$

**Hierarchical model** — The dependencies between the variables are described by the proposed hierarchy presented in Fig. 1. The joint law then reads:

$$f(\mathbf{y}, \ell, \mathbf{x}_{1..K}, \gamma_n, \gamma_{1..K}, \beta) = f(\mathbf{y} | \gamma_n, \ell, \mathbf{x}_{1..K}) \cdot f(\ell | \beta) \cdot \prod_k f(\mathbf{x}_k | \gamma_k) \cdot f(\gamma_n) \cdot f(\beta) \cdot \prod_k f(\gamma_k). \quad (5)$$

**Hyperparameter models** — Regarding the precision parameters  $\gamma_n$  and  $\gamma_k$ ,  $k = 1..K$ , one can see in the model (2) for the textures and (4) for the observation that they are precision parameters in Gaussian laws, hence the Gamma



**Fig. 1:** Hierarchical model: the round nodes / the square ones show the estimated / fixed quantities.

laws are conjugate forms:  $\mathcal{G}(\gamma; \alpha, \beta)$ . Furthermore, little prior information is available on these parameters, so diffuse priors are used by setting  $(\alpha, \beta)$  to small values.

When it comes to  $\beta$ , a conjugate prior is not available, given the expression of the partition. A uniform prior on an interval  $[0, B]$  is considered as a reasonable choice:

$$p(\beta) = \mathcal{U}_{[0, B]}(\beta)$$

where  $B$  is defined as the maximum possible value of  $\beta$ .

### III. BAYESIAN FORMULATION

#### III-A. Design of the estimators

Given the joint law (5), optimal estimation function can then be deduced, as Bayesian risk minimizer.

- Regarding discrete parameters: the labels  $\ell$  are estimated as a Marginal Posterior Maximizer;
- Regarding continuous parameters:  $\gamma_n$ , the  $\gamma_k$  and the textures  $\mathbf{x}_k$  are estimated as the Posterior Mean.

They rely on the posterior that is fully specified by the image formation (3), the distribution for the labels (1), for the textures (2), for the observations (4), and the hyperpriors:

$$f(\ell, \mathbf{x}_{1..K}, \gamma_n, \gamma_{1..K} | \mathbf{y}) \propto \mathcal{C}_y \exp\left[-\gamma_n \|\mathbf{y} - \mathbf{H} \sum_k \mathbf{S}_k(\ell) \mathbf{x}_k\|^2\right] \cdot \mathcal{C}_{\gamma_n} \gamma_n^{\alpha_n + P - 1} \cdot \exp(-\gamma_n \beta_n) \cdot \mathcal{C}_z(\beta) \exp\left[\beta \sum_{r \sim s} \delta(\ell_r, \ell_s)\right] \mathcal{U}_{[0, B]}(\beta) \cdot \prod_k [\mathcal{C}_x \det(\mathbf{\Lambda}_k)^{-1} \exp(-\gamma_k \|\mathbf{i}\|_{\mathbf{\Lambda}_k}^2)] \cdot \prod_k [\mathcal{C}_{\gamma_k} \gamma_k^{\alpha_k + P - 1} \cdot \exp(-\gamma_k \beta_k)] \quad (6)$$

It summarizes the information contained by the data and the priors about the unknowns.

#### III-B. Computing the Estimators

Due to the intricate form of the posterior, estimates cannot be theoretically calculated. They must be extracted using

numerical methods and stochastic samplers are especially adequate. More precisely, the Gibbs sampler is employed since it splits the global problem in simpler ones. It requires to sequentially sample the *a posteriori* conditional laws.

- The noise parameter  $\gamma_n$  has a Gamma conditional

$$\gamma_n \sim \gamma_n^{\alpha_n + P - 1} \exp -\gamma_n \left[ \|\mathbf{y} - \mathbf{H} \sum_k \mathbf{S}_k \mathbf{x}_k\|^2 + \beta_n \right]$$

so its sampling does not pose any difficulties.

- The PSD scale parameters  $\gamma_k$  also have Gamma forms

$$\gamma_k \sim \gamma_k^{\alpha_k + P - 1} \exp -\gamma_k \left[ \|\mathbf{x}_k\|_{\Lambda_k}^2 + \beta_k \right]$$

which can be straightforwardly sampled.

- The correlation parameter  $\beta$  follows an intricate pdf

$$\beta \sim \mathcal{C}_z(\beta)^{-1} \exp \left[ \beta \sum_{p \sim q} \delta(\ell_p; \ell_q) \right] \mathcal{U}_{[0, B]}(\beta),$$

that cannot be straightforwardly sampled. The procedure is identical to the one presented in our papers [27, 28]. It relies on the inverse cumulative distribution function and it takes advantage of the pre-computations of the partition function.

- The conditional posterior pdf of the texture  $\mathbf{x}_k$  reads

$$\mathbf{x}_k \sim \exp - \left[ \gamma_n \|\mathbf{y} - \mathbf{H} \sum_k \mathbf{S}_k \mathbf{x}_k\|^2 + \gamma_k \|\mathbf{x}_k\|_{\Lambda_k}^2 \right] \quad (7)$$

that is a Gaussian law, and it is easy to show that the mean  $\boldsymbol{\mu}_k$  and the covariance  $\boldsymbol{\Sigma}_k$  write

$$\begin{aligned} \boldsymbol{\Sigma}_k^{-1} &= \gamma_n \mathbf{S}_k^\dagger \mathbf{H}^\dagger \mathbf{H} \mathbf{S}_k + \gamma_k \Lambda_k \\ \boldsymbol{\mu}_k &= \boldsymbol{\Sigma}_k \cdot \gamma_n \mathbf{S}_k^\dagger \mathbf{H}^\dagger \bar{\mathbf{y}}_k \end{aligned}$$

where  $\bar{\mathbf{y}}_k = \mathbf{y} - \mathbf{H} \sum_{l \neq k} \mathbf{S}_l \mathbf{x}_l$ . However, its practical sampling is a very difficult task due to the high dimension of the variable. Usually, sampling a Gaussian law requires handling the covariance or the precision, *e.g.*, factorization, diagonalisation or inversion, which are impossible here. Nevertheless, the literature accounts for a sampling algorithm [31] by Perturbation-Optimization (PO) and it is applicable here.

- 1) The **Perturbation** phase consists in drawing a realization of: (1) a Gaussian white noise and (2) the prior model for  $\mathbf{x}_k$  (computed by FFT).
- 2) The **Optimization** step relies on preconditioned gradient descent or preconditioned conjugate gradient descent (they perform similarly). The computations are efficiently achieved through FFT and zero-forcing.

- The labels have a multidimensional categorical law:

$$\ell \sim \exp \left[ -\gamma_n \|\mathbf{y} - \mathbf{H} \sum_k \mathbf{S}_k(\ell) \mathbf{x}_k\|^2 + \beta \sum_{r \sim s} \delta(\ell_r, \ell_s) \right]$$

that is complicated and non separable, so, its sampling is not an easy task. A solution is to successively sample each  $\ell_p$  conditionally on the others (and on the rest of the variables), in a Gibbs scheme.

To this end, let us introduce the notation  $\mathbf{x}_k^p$  for the image with all its pixels identical to  $\mathbf{i}$  except for pixel  $p$ . The pixel  $p$  in  $\mathbf{x}_k^p$  is the pixel  $p$  of  $\mathbf{x}_k$ . Let us note  $\mathcal{E}_{p,k} = \|\mathbf{y} - \mathbf{H} \mathbf{x}_k^p\|^2$

that quantifies the adequation of pixel  $p$  of class  $k$  with the data. Sampling a label  $\ell_p$  requires the probabilities:

$$\Pr(\ell_p = k | *) \propto \exp \left[ -\gamma_n \mathcal{E}_{p,k} + \beta \sum_{r: r \sim p} \delta(\ell_r, k) \right]$$

for  $k = 1..K$ . To compute these probabilities, we must evaluate the two terms of the energy. The second term is the contribution of the prior and it can be easily computed for each  $k$  by counting the neighbours of pixel  $p$  with label  $k$ .

Let us now focus on the first term,  $\mathcal{E}_{p,k}$ . To write this term in a more convenient form, we introduce:

- A vector  $\mathbf{1}_p \in \mathbb{R}^P$  with null entries except for the  $p$ -th that is equal to 1.
- The difference between the  $p$ -th pixel of the image  $\mathbf{x}_k$  and the one of  $\mathbf{i}$ :  $\Delta_{k,p} = x_p - x_{k,p}$ .

We then have  $\mathbf{x}_k^p = \mathbf{i} - \Delta_{k,p} \mathbf{1}_p$ , so:

$$\mathcal{E}_{p,k} = \bar{\mathbf{y}}^\dagger \bar{\mathbf{y}} + \Delta_{k,p}^2 \mathbf{1}_p^\dagger \mathbf{H}^\dagger \mathbf{H} \mathbf{1}_p - 2 \Delta_{k,p} \mathbf{1}_p^\dagger \mathbf{H}^\dagger \bar{\mathbf{y}} \quad (8)$$

where  $\bar{\mathbf{y}} = \mathbf{y} - \mathbf{H} \mathbf{i}$ . These quantities can be easily computed based on FFT and zero-forcing. Moreover, the residue  $\bar{\mathbf{y}}$  can be updated from iteration to iteration and it allows for efficient computations.

## IV. RESULTS

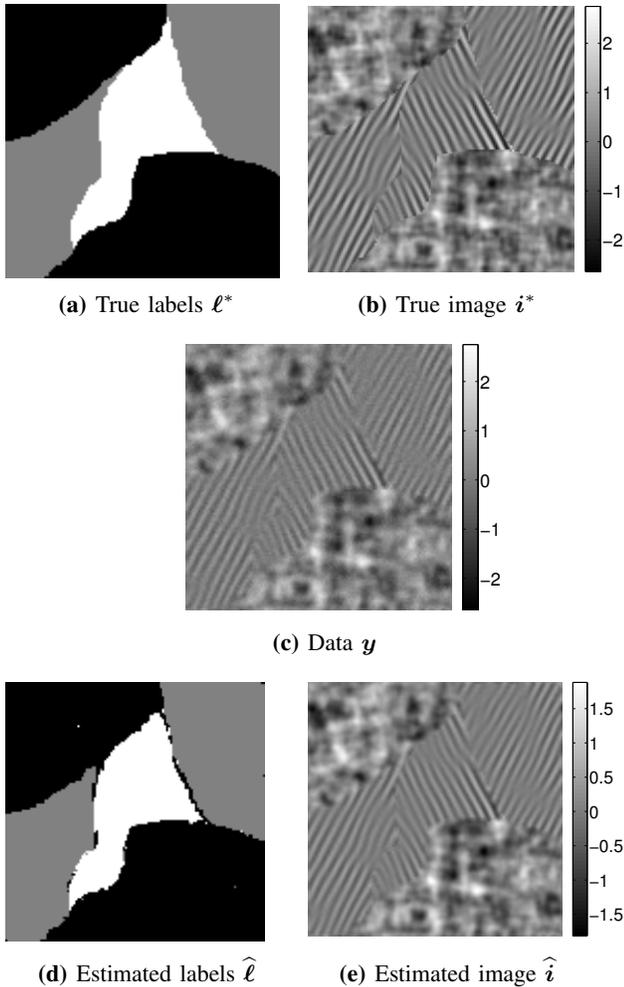
The problem of texture segmentation is challenging, especially in the present case where (1) data are affected by blur and noise and (2) estimation of hyperparameters including Potts parameter is desired. The previous section proposes a method for deconvolution-segmentation in this context and this section gives a first numerical evaluation.

The algorithm has been run in different scenarios, several times, from identical and different initializations: it has shown consistent qualitative and quantitative behaviours. Usually, it is iterated 100 times and Markov chains seem stable after about 20 iterations (burn-in period). The estimates for the labels are computed as the empirical marginal posterior maximizers and for the other parameters as empirical posterior means. Computation time is about five minutes.

An example is given in Fig. 2. It consists of an image composed of 5 regions and containing  $K = 3$  classes of texture. True label and image are shown in Fig. 2a and Fig. 2b. The observed image is given in Fig. 2c.

The algorithm produces a label configuration Fig. 2d very similar to the true one, with only 1.2% of miss-labeled pixels, despite the degradation of the image. The blur and the noise are eliminated and the resulting textured patchwork Fig. 2e strongly resembles the original image.

One of the main advantages of Bayesian approaches is that they not only provide estimates for the unknowns, but also coherent uncertainties. Fig. 3 illustrates our analysis on the label estimates and their probability. Fig. 3a gives the probabilities of the selected labels  $\hat{\ell}$  (*i.e.*, the maximum of the three probability fields shown in Fig. 3c). These probabilities are small at certain locations and it is safe to



**Fig. 2:** Segmentation and reconstructed images.

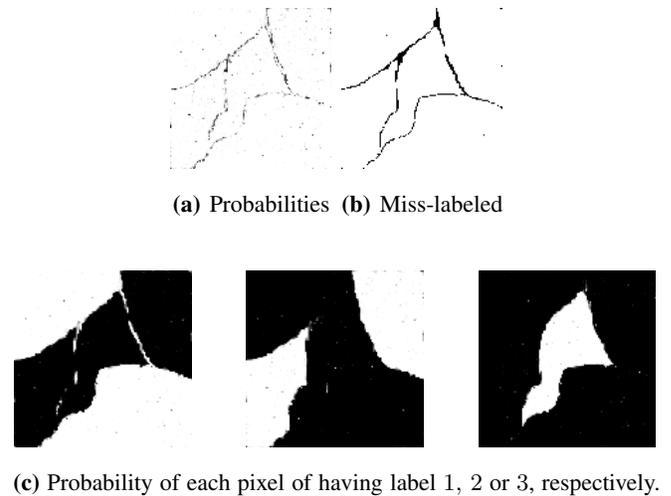
assume that at these locations there is a smaller chance of selecting a correct label.

This analysis can be done without the knowledge of the true labels. In order to verify if indeed the method is more prone to error in the area with small posterior probability, we have compared the estimation to the true labels in a case where they are known. We can immediately notice in Fig. 3b that all of the miss-labeled pixels are positioned in the areas of weaker posterior probability, shown in Fig. 3a. This reinforces our statement concerning the interest of the approach, due to its ability to anticipate errors.

## V. CONCLUSION AND PERSPECTIVES

The paper proposes a tool for joint deconvolution-segmentation dedicated to textured images. It is a very difficult problem due to the large amount of unknowns and their intricate dependencies. The paper proposes a twofold contribution.

- 1) To the best of our knowledge, it is a first attempt for a joint deconvolution-segmentation of textured images.



**Fig. 3:** Link between the probability of the selected label and the estimation error.

- 2) An important contribution is to also perform estimation of the Potts parameter whereas existing papers devoted to inversion-segmentation do not.

Moreover, our previous papers [25–28] include neither deconvolution nor textured images.

Theoretically, the proposed method relies on a Bayesian strategy and on optimal estimation functions. The numerical calculations rely on a convergent scheme: the algorithm produces samples of the posterior distribution and the estimates are computed from the samples. Nevertheless, the sampling process for the full textures has also proved to be challenging and has demanded an advanced sampling tool based on Perturbation-Optimization. The proposed methodological and algorithmic original aspects have contributed to developing a method that is both theoretically sound and numerically efficient for solving this difficult problem.

The previous section has presented a first numerical evaluation. These results have shown that the method is able to accurately segment the image, provide a good estimation for the hyperparameters including the Potts parameter and thus restore the original image.

From a methodological, practical and numerical standpoint, the study leads us to several future developments.

- A first one relies on a non-Gaussian model for the textures [32]. This would add an extra set of variables to the texture model and sampling stage. See also [33].
- The second development is devoted to myopic deconvolution [34, 35]: estimating the width of the convolution filter along with the rest of the parameters.
- A third one presents a method based on a Swendsen-Wang form [16, 36]. It allows for graph clustering and graph flipping (instead of single pixel flipping as in the case of the Gibbs sampler).

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