Low-Complexity Non-Uniform Penalized Affine Projection Algorithms for Active Noise Control

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***Abstract*—This paper describes new algorithms that incorporates the non-uniform norm constraint into the zero-attracting and reweighted modified filtered-x affine projection or pseudo affine projection algorithms for active noise control. The simulations indicate that the proposed algorithms can obtain better performance for primary and secondary paths with various sparseness levels with insignificant numerical complexity increase. It is also shown that the version using a linear function instead of the reweighted term leads to the best results, particularly for combinations of sparse or semi-sparse primary and secondary paths.**

***Keywords*—active noise control, affine projection, zero-attracting algorithms, norm penalty.**

I. Introduction

Active noise control (ANC) is a technique for removing noise from a system by subtracting the effect of a noise-generating plant from a signal [1]. There are many ANC algorithms that take into account the additional electro-acoustic path between the filter output and measured error signal, known as the secondary path [1]. The filtered-x (FX) scheme was initially proposed for the least-mean square (LMS) algorithms [2] but later adapted for the family of the affine projection (AP) algorithm [3]. The Fx scheme eliminates the instability caused by the additional delay in the secondary path for the LMS [3] or affine family of algorithms [4]. The drawback of the Fx scheme is that the convergence speed is very slow because a small step size has to be used [5]. The modified filtered-x (MFx) approach [6] is adding filtering steps that approximate the instantaneous error signal and greatly improves the convergence speed.

A common situation in practical application is that the system to be identified has a certain degree of sparseness. Numerous algorithms were derived by incorporating sparsity penalties e.g. the zero-attracting (ZA) and reweighted zero-attracting (RZA) algorithms [7], [8]. It has been proved that their convergence performance is better when the system was sparse. These sparsity constraints were firstly incorporated into MFx affine projection algorithms in [9]. They were called the ZA-MFxAP and RZA-MFxAP algorithms. In [10] an approximation of the algorithms from [11] that led to their pseudo versions was introduced. They were called the zero-attracting MFx pseudo affine projection (ZA-MFxPAP) and reweighted zero-attracting MFx pseudo affine projection (RZA-MFxPAP). Using the same approximation firstly proposed in [11] for an echo cancellation example, the pseudo versions obtain almost identical performance with the original algorithms at a smaller numerical complexity [10].

In this paper the integration of a non-uniform norm constraint [12] into the cost function is proposed. Also, it is shown that the use of a segment linear function approximation [12] can lead to performance improvements in some cases for a very small additional computational cost. The non-uniform norm penalized (NNP) versions of ZA-MFx-AP, RZA-MFx-AP, ZA-MFx-PAP, RZA-MFx-PAP are derived in this paper. Also, the segment version that replace the reweighting function of the NNP-RZA-MFx-PAP (SNNP-RZA-MFx-PAP) is proposed.

The paper starts with an introduction, while Section II presents the equations of ZA-MFxAP, RZA-MFxAP algorithms and their pseudo versions (ZA-MFxPAP and RZA-MFxPAP respectively). The proposed algorithms are presented in Section III. In Section IV, the results of simulations for various primary and secondary echo paths using the considered algorithms are presented. Section V presents the conclusions.

II. The Zero-Attracting and Reweighted Zero-Attracting versions of MFxAP and MFxPAP algorithms

In broadband feedforward ANC the noise is reduced by subtracting from the acoustic signal a signal generated by using an error signal [10]. In FX algorithms the input signal $x(k)$ is filtered using an estimate of the secondary path, $\hat{\mathbf{s}}(k)$ and generates $x_f(k)$. In the MFx algorithms the instantaneous error signal $\hat{e}(k)$ is estimated [7]. It was shown that the convergence speed of the MFx structures is higher than that of the Fx structures. The MFxAP structure is shown in Fig. 1 and more details about its signals, variables, parameters, notations and structure can be found in [9].

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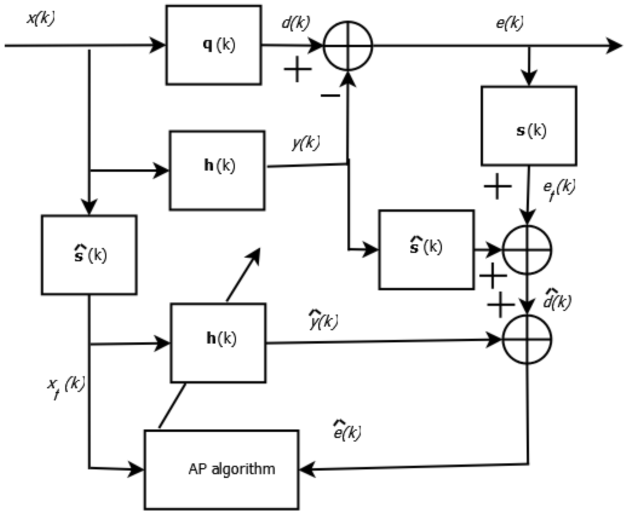

Fig. 1. Modified filtered-x AP structure [9].

It is well suited to ANC as it does not require the signal $\mathbf{d}(k)=\mathbf{V}(k)\mathbf{h}(k+1)$ be available, where $\mathbf{V}(k)=[\boldsymbol{\tau}(k)\ \boldsymbol{\tau}(k-1)\ ...\ \boldsymbol{\tau}(k-L)]$ is the regressor matrix, $\boldsymbol{\tau}(k)=[x(k),x(k-1),...x(k-O+1)]^T$, O is the projection order and L is the filter length [10]. The $\hat{\mathbf{d}}(k)=\mathbf{V}_f(k)\mathbf{h}(k+1)$ condition is set, where $\mathbf{V}_f(k)=[\boldsymbol{\tau}_f(k)\ \boldsymbol{\tau}_f(k-1)\ ...\ \boldsymbol{\tau}_f(k-L)]$, with $\boldsymbol{\tau}_f(k)=[x_f(k),x_f(k-1),...x_f(k-O+1)]^T$. Also we have $\hat{\mathbf{y}}(k)=\mathbf{V}_f(k)\mathbf{h}(k)$, $\hat{\mathbf{e}}(k)=\mathbf{V}_f(k)\mathbf{V}_f^H(k)\underline{\lambda}$, and $\underline{\lambda}=\left[\mathbf{V}_f(k)\mathbf{V}_f^H(k)\right]^{-1}\hat{\mathbf{e}}(k)$ [9]. The cost function used to derive the MFxAP algorithm was [9].

$$J(k)=\left\|\mathbf{h}(k+1)-\mathbf{h}(k)\right\|^2+ \\ +\mathrm{Re}\left\{\left|\hat{\mathbf{d}}(k)'-\mathbf{V}_f(k)\mathbf{h}(k+1)\right|^H\underline{\lambda}\right\} \tag{1}$$

A. Zero-attracting MFxAP (ZA-MFxAP) algorithm

The ZA-MFxAP uses a sparsity-inducing penalty, i.e. a ℓ_1-norm penalty as an approximation. If we denote by $\mathbf{V}_f^+(k)=\mathbf{V}_f^H(k)\left[\mathbf{V}_f(k)\mathbf{V}_f^H(k)+\delta\mathbf{I}\right]^{-1}$, the weight update recursion is [9]:

$$\mathbf{h}(k+1)=\mathbf{h}(k)+\mu\mathbf{V}_f^+(k)\hat{\mathbf{e}}(k)+ \\ +\rho\mathbf{V}_f^+(k)\mathbf{V}_f(k)\mathrm{sgn}\{\mathbf{h}(k)\}\text{-}\alpha\,\mathrm{sgn}\{\mathbf{h}(k)\} \tag{2}$$

where, μ is the step size, α is a constant, $\rho=\mu\alpha$ and $\mathbf{I}$ is an identity matrix.

B. Reweighted zero-attracting MFxAP (RZA-MFxAP) Algorithm

The RZA-MFxAP algorithm uses a log-sum penalty and its recursion is the following [9]:

$$\mathbf{h}(k+1)=\mathbf{h}(k)+\mu\mathbf{V}_f^+(k)\hat{\mathbf{e}}(k)+ \\ \rho'\mathbf{V}_f^+(k)\mathbf{V}_f(k)\boldsymbol{\Psi}(k)-\gamma\varepsilon\boldsymbol{\Psi}(k) \tag{3}$$

where $\boldsymbol{\Psi}(k)=\mathrm{sgn}\{\mathbf{h}(k)\}/\left(1+\varepsilon\,\mathrm{sgn}\{\mathbf{h}(k)\}\right)$. The strength of the zero-attraction is $\rho'=\mu\gamma\varepsilon$, where γ is a constant and ε is the shrinkage magnitude. In [10] an approximation was used and the "pseudo" versions of ZA-MFx-AP and RZA-MFx-AP were proposed. They involved an approximation of the second term of both Eq. (2) and Eq. (3) when the step size is close to 1. The weight recursion of the algorithm called the Zero-Attracting Modified Filtered-X Pseudo Affine Projection (ZA-MFxPAP) is the following [10]:

$$\mathbf{h}(k+1)=\mathbf{h}(k)+\mu\mathbf{V}_{f1}^+(k)e(k)+ \\ +\rho\mathbf{V}_f^+(k)\mathbf{V}_f(k)\mathrm{sgn}\{\mathbf{h}(k)\}\text{-}\alpha\,\mathrm{sgn}\{\mathbf{h}(k)\} \tag{4}$$

where the first column of $\mathbf{V}_f^+(k)$ is denoted as $\mathbf{V}_{f1}^+(k)$. The weight recursion of the Reweighted Zero-Attracting Modified Filtered-X Pseudo Affine Projection (RZA-MFxPAP) is the following [10]:

$$\mathbf{h}(k+1)=\mathbf{h}(k)+\mu\mathbf{V}_{f1}^+(k)e(k)+ \\ +\rho'\mathbf{V}_f^+(k)\mathbf{V}_f(k)\boldsymbol{\Psi}(k)-\gamma\varepsilon\boldsymbol{\Psi}(k) \tag{5}$$

It was shown in [10] that ZA-MFxPAP and RZA-MFxPAP has with $O^2L+6L(O-1)-2O$ less multiplications than the original ZA-MFxAP and RZA-MFxAP algorithms. It can be noticed that this complexity reduction is linearly proportional with L and proportional with O^2.

III. THE PROPOSED ALGORITHMS

We propose to derive new algorithms by integrating a non-uniform norm constraint into the cost function of the conventional algorithm as in [12]

$$J'(k)=J(k)+\gamma_{PAP}\left\|\mathbf{h}(k)\right\|_p^p \tag{6}$$

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where γ_{PAP} is a small positive value used to balance the estimation error and the sparse p-norm-like penalty $\|\mathbf{h}(k)\|_p^p$ [10]

$$\|\mathbf{h}(k)\|_p^p = \sum_{i=1}^{L} |h_i|^p \tag{7}$$

where $0 \le p \le 1$.

Following the same steps shown in [10] the weight recursion of the non-uniform norm penalized ZA-MFx-AP (NNP-ZA-MFxAP) algorithm is the following:

$$\begin{aligned}\mathbf{h}(k+1) &= \mathbf{h}(k) + \mu \mathbf{V}_f^{+}(k)\hat{\mathbf{e}}(k) + \\ &+\rho \mathbf{V}_f^{+}(k)\mathbf{V}_f(k)\mathbf{f}\,\mathrm{sgn}\{\mathbf{h}(k)\} \text{-} \alpha \mathbf{f}\,\mathrm{sgn}\{\mathbf{h}(k)\}\end{aligned} \tag{8}$$

where $\mathbf{f} = [f_0 \, f_1 \, \dots f_{L-1}]$ and

$$f_i = \frac{\mathrm{sgn}\left[E\left[h_i(k)\right] - |h_i(k)|\right] + 1}{2}, \quad \forall 0 \le i \le L-1 \tag{9}$$

Using the same approximation of [10] the weight recursion of the non-uniform norm penalized ZA-MFx-PAP (NNP-ZA-MFxPAP) algorithm is the following:

$$\begin{aligned}\mathbf{h}(k+1) &= \mathbf{h}(k) + \mu \mathbf{V}_{f1}^{+}(k) e(k) + \\ &+\rho \mathbf{V}_f^{+}(k)\mathbf{V}_f(k)\mathbf{f}\,\mathrm{sgn}\{\mathbf{h}(k)\} \text{-} \alpha \mathbf{f}\,\mathrm{sgn}\{\mathbf{h}(k)\}\end{aligned} \tag{10}$$

The weight recursion of the non-uniform norm penalized RZA-MFx-AP (NNP-RZA-MFxAP) algorithm is the following:

$$\begin{aligned}\mathbf{h}(k+1) &= \mathbf{h}(k) + \mu \mathbf{V}_f^{+}(k)\hat{\mathbf{e}}(k) + \\ &+\rho' \mathbf{V}_f^{+}(k)\mathbf{V}_f(k)\mathbf{f}\boldsymbol{\Psi}(k) - \gamma\varepsilon \mathbf{f}\boldsymbol{\Psi}(k)\end{aligned} \tag{11}$$

Integrating the same concept and the segment linear function that replace the re-weighting term $1/\left(1+\varepsilon\,\mathrm{sgn}\{\mathbf{h}(k)\}\right)$ (Eq. 12)

$$\mathbf{l}(h_i) = \begin{cases} 350|h_i|, & |h_i| < 0.005 \\ \delta, & \text{elsewhere} \end{cases} \tag{12}$$

the weight recursion of the segment non-uniform norm penalized RZA-MFx-AP (SNNP-RZA-MFxPAP) algorithm is the following:

$$\begin{aligned}\mathbf{h}(k+1) &= \mathbf{h}(k) + \mu \mathbf{V}_{f1}^{+}(k) e(k) + \\ &+\rho' \mathbf{V}_f^{+}(k)\mathbf{V}_f(k)\mathbf{fl}\,\mathrm{sgn}\{\mathbf{h}(k)\} - \gamma\varepsilon \mathbf{fl}\,\mathrm{sgn}\{\mathbf{h}(k)\}\end{aligned} \tag{13}$$

The constant 350 and threshold 0.005 were obtained as in [12], from the reweighting term in RNNP-AP algorithm. The segment approximation has been also used for other algorithms [13]. It can be noticed that the weight update equation of SNNP-RZA-MFxPAP algorithm is more similar now with the equation update for the NNP-ZA-MFxPAP algorithm after the use of segment linear function.

The sparse p-norm like penalty lead to an increase with L multiplications for the reweighted zero-attracting version. It is obvious that the added numerical complexity cost is insignificant if compared even with the savings obtained by the pseudo versions of [10] compared with the original algorithms ($L \ll O^2 L + 6L(O-1) - 2O$). There is no increase in terms of multiplications for the zero-attracting NNP versions. The added numerical complexity in terms of additions is also insignificant for both zero-attracting and reweighted zero-attracting based algorithms.

IV. SIMULATION RESULTS

This section compares the results of simulations of the proposed algorithms with those investigated in [9] and [10]. For all the algorithms, the input signal is a white noise, SNR = 40 dB, the paths from [9] were used and the parameters were tuned to the same values as in [10]. For each primary path the algorithms were run for 120,000 iterations with the secondary path set as sparse at the start of the experiment, changed to partially-sparse at iteration 15,000 and to non-sparse at iteration 55,000. For all simulations the following parameters were used: $\rho = 10^{-7}$, $\delta = 0.002$ and $\varepsilon = 10$. The performance of the algorithms has been figured by the mean-square deviation (MSD) convergence curves. Different step sizes were used in order have the same initial convergence speed for each plant type.

Fig. 2 shows a comparison of the RZA-MFxAP, ZA-MFxAP and SNNP-RZA-MFxAP algorithms for an order O = 4 and a sparse plant.

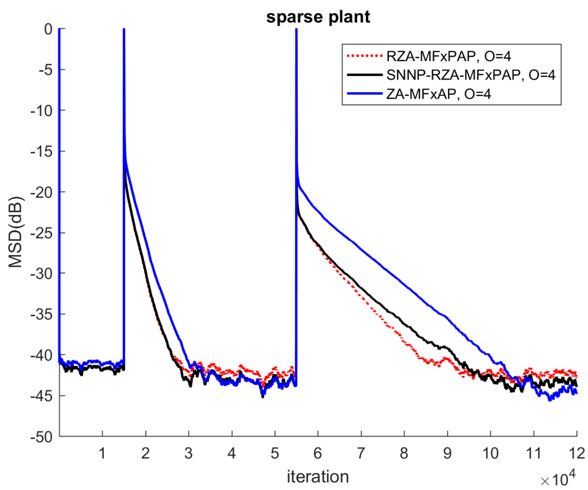

Fig. 2. MSD results of ZA-MFxAP, RZA-MFxPAP and SNNP-RZA-MFxAP algorithms for a sparse plant and a projection order O = 4.

The subscript s indicates the value for the sparse case, ss the value for the semi-sparse case while ns the value for the non-sparse situation. We used $\mu_s = 1$, $\mu_{ss} = 0.6$ and $\mu_{ns} = 0.4$. For all the NNP and Pseudo based versions the step size is one. It can be seen that the performance of the proposed SNNP-RZA-MFxAP algorithm is the best among the considered algorithms in terms of convergence speed and steady state MSD for very sparse and semi-sparse secondary paths. The gain is up to about 2 dB. The simulation also confirms the superior performance of the re-weighted versions over the ZA-MFxAP algorithms for the sparse secondary paths [9].

The next figure examines the MSD performance of the proposed non-uniform norm re-weighted versions for a variable projection order in case of sparse plant.

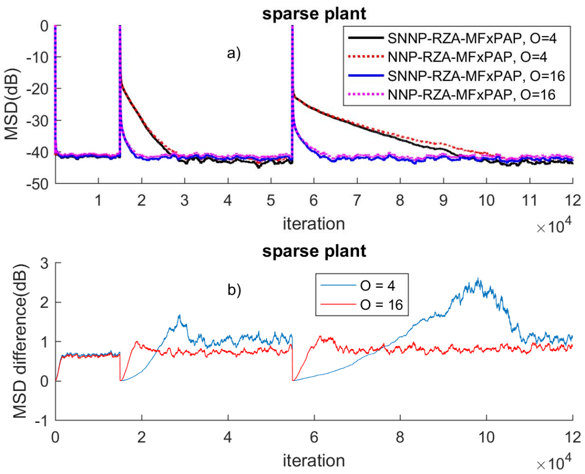

Fig. 3. a) MSD results of the proposed NNP-RZA-MFxPAP based algorithms for sparse plant and two projection orders. b) MSD difference between the proposed NNP-RZA-MFxAP version and its segment based approximated version for sparse plant.

As expected, it can be seen from Fig. 3a that using a higher projection order leads to a faster convergence especially for semi-sparse and non-sparse secondary paths. Also, it can be noticed from Fig. 3b that the use of the segment version increases the MSD performance for both projection order values. The improvement is about 1 dB for O = 16 and over 2 dB for O = 4. Therefore, it is convenient to use the segment version in order to increase the performance at the same time with reducing the numerical complexity associated with the penalty term of the RZA based MFxAP algorithms.

The performance of the proposed re-weighted algorithms for different projection orders when the plant is partially sparse can be seen in Fig. 4. The same conclusion as above can be drawn in terms of both MSD convergence speed and steady-state performance when the projection order increases (Fig. 4a). Also, it can be noticed from Fig. 4b that the use of the segment version increases the MSD performance for both projection order values, especially for a non-sparse secondary path and a high projection order (it is around 4 dB for O = 16).

The results of running the algorithms for a non-sparse plant and a projection order O = 4 can be seen in Fig. 5. It was shown in [10] that, in this case, the performance of the pseudo affine projection algorithms was almost identical to that of the original affine projection algorithms.

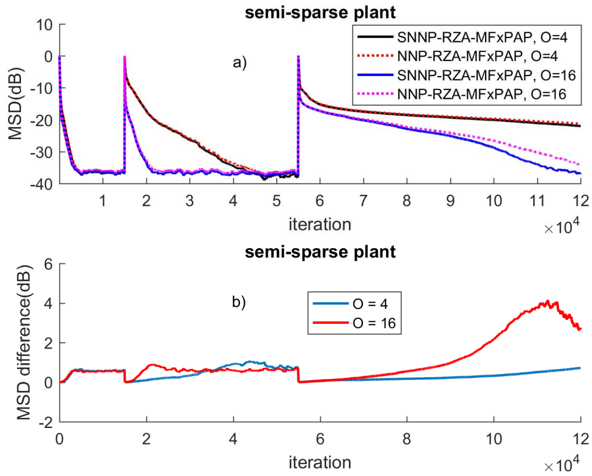

Fig. 4. a) MSD results of the proposed NNP-RZA-MFxPAP algorithms for semi-sparse plant and two projection orders. b) MSD difference between the proposed NNP-RZA-MFxPAP version and its segment based approximated version for semi-sparse plant.

It can be noted that when the plant is non-sparse, the segment approximation of the non-uniform norm penalized MFxPAP version leads only to minor MSD improvements (see Fig. 5b). The non-uniform norm penalized ZA-MFxAP version has a different behavior depending on the sparseness of the secondary path. While there is a minor MSD performance loss for the semi-sparse secondary path (less than 0.5 dB), there is a noticeable MSD improvement for the non-sparse plant up to around 2 dB. Last figure examines the MSD performance of the proposed non-uniform norm re-weighted versions for different projection orders in case of non-sparse plant and the following step size values: $\mu_s = 0.7$, $\mu_{ss} = 1$ and $\mu_{ns} = 0.9$.

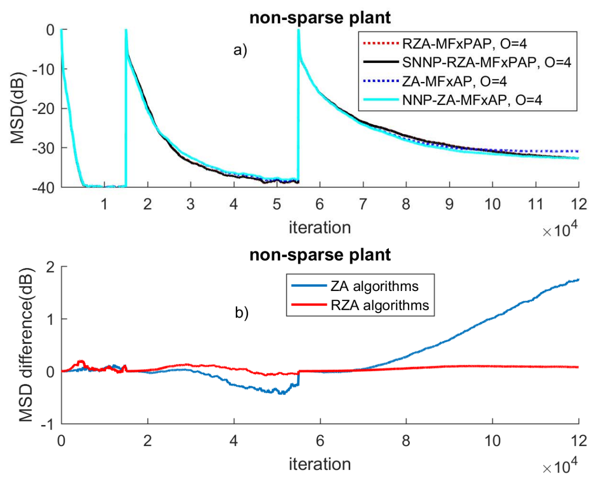

Fig. 5. a) MSD results of RZA-MFxPAP, ZA-MFxAP, NNP-ZA-MFxAP and SNNP-RZA-MFxPAP algorithms for non-sparse plant and a projection order O = 4; b) MSD difference between ZA-MFxAP and NNP-ZA-MFxAP algorithms, RZA-MFxAP and SNNP-ZA-MFxAP respectively, for non-sparse plant and O = 4.

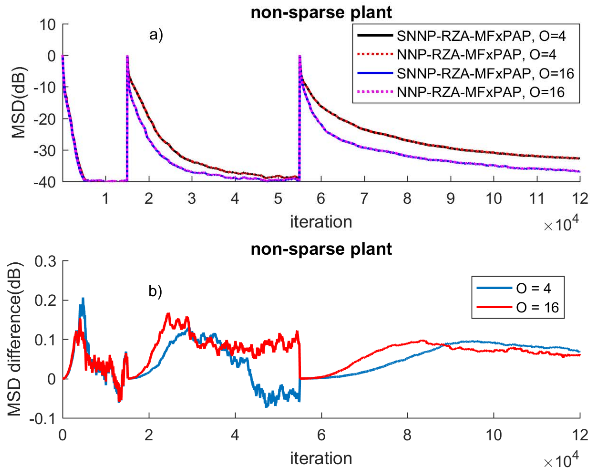

Fig. 6. a) MSD results of the proposed RZA-MFxPAP algorithms for non-sparse plant and two projection orders. b) MSD difference between the proposed NNP-RZA-MFxAP version and its segment based approximated version for non-sparse plant.

It can be noticed that the MSD convergence speed and steady-state performance is improved when the projection order increases, especially for semi-sparse and sparse plants (Fig. 5a). However, there is not a clear MSD improvement when the segment version is used (the maximum gain is about 0.2 dB for both projection order values).

Finally, it can be noticed from Figs. 2-6 that, for all considered primary paths, the proposed non-uniform norm penalized algorithms have slightly better performance than the original algorithms when the secondary path is sparse. Further work might incorporate variable step-size or projection versions [14]-[15], block-sparse versions [16] or using the approach from [17] in order to develop new MFX-based active noise control algorithms.

V. CONCLUSIONS

This paper has proposed several non-uniform norm penalized adaptive algorithms of the affine projection algorithm based on modified filtered-x scheme for active noise control. The simulation results demonstrate that MSD improvements can be obtained for various sparseness in the primary or secondary path with a very small complexity increase over the previously published ZA-MFxPAP and RZA-MFxPAP algorithms.

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