

A Novel Hybrid Analog-Digital Transmitter for Multi-antenna Base Stations

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Abstract—A new hybrid Analog-Digital (hybrid A-D) precoder is proposed for multi-antenna base stations in massive Multiple-Input Multiple-Output (MIMO) which allows a trade-off between the number of required RF-chains and the update rate of the analog part. It is shown that the number of RF-chains can be reduced even below the number of eigenmodes of the channel, thereby closing the gap between the standard hybrid A-D and the single-RF MIMO. This is achieved by dividing the input data streams into blocks and jointly optimizing the digital and the analog precoder parts for each block. The analog part of the precoder needs to be updated once per block and remains static over each block interval. Out of band radiation due to switching is resolved by inserting a short guard interval between blocks. It is shown the number of RF-chains can be any arbitrary positive integer to obtain zero distortion at the user terminals if the update rate is high enough. The proposed precoder offers a significant performance gain at the expenses of data dependent precoding and higher update rates of the analog part.

I. INTRODUCTION

Massive Multiple-Input Multiple-Output (MIMO) is considered as one of the main technologies for the next generation of wireless networks referred to as 5G [1]. In massive MIMO, central base stations with many antennas serve multiple users utilizing high multiplexing gains offered by the MIMO nature of the system in dense multi-path wireless channels [2]. While increasing the number of antennas at massive MIMO base stations is crucial for accurate beamforming and hardening of the channel [2], high RF-cost of such huge stations may limit the number of antennas [3].

Hybrid Analog-Digital (A-D) structures are among the solutions to reduce the RF cost and complexity of massive MIMO base stations [4]. In hybrid A-D, part of precoding is performed in analog domain to save on the number of RF-chains. A tunable analog network is employed in which signal transformation is done using some RF components such as phase shifters. This can significantly reduce the RF cost since the number of power amplifiers and digital-to-analog converters decreases. In Time Division Duplex (TDD) massive MIMO, based on the reciprocity property of wireless channels, in receive mode hybrid A-D structures use the same hybrid configuration to first map received signals onto a reduced dimensional space and then sample them. This reduces the number of analog-to-digital converters in receive mode.

Several implementation ideas for the analog part of hybrid A-D have been proposed in literature, including analog phase shifter networks [5]–[7] and Butler matrices [8]. The main problem in implementing the analog part is that RF operations

such as phase shifters and combiners normally lead to RF loss. Furthermore, since tuning of analog precoder networks in frequency is a non-trivial task, they are usually designed for a single frequency band. This makes multi-band communications very difficult. Note that in higher frequency bands such as mmWave, wireless channels tend to have a strong line-of-sight component, therefore hybrid A-D becomes more attractive. In line-of-sight communications, an analog precoder can simply form multiple beams towards the user terminals.

Hybrid A-D precoders usually are data-independent and are updated based on channel state information. In this paper, we propose a data dependent hybrid A-D precoder which is updated by a higher rate than the rate at which the channel changes. This allows us to reduce the number of RF-chains even further compared to state-of-the-art hybrid A-D precoders. For a fixed number of RF-chains, we show that our data dependent precoding structure can significantly improve the performance in a downlink multi-user MIMO system. This is achieved at the expense of complexity in baseband domain which is much more easier to handle than the analog domain. Utilizing this hybrid A-D precoding, the number of users can be even larger than the number of RF-chains. This is in contrast to the common believe that the number of RF-chains must be at least the number of eigenmodes of the channel.

The rest of this paper is organized as follows. The proposed hybrid structure is explained in Section II. A short review on the current analog precoder technologies is given in Section III. Section IV presents the numerical and simulation results and finally the paper is concluded in Section V.

II. THE PROPOSED HYBRID A-D PRECODER

A multi-user MIMO system is considered in which a central base station with N antennas serves K single-antenna users. Inter cell interference is neglected. It is assumed that $N \geq K$ holds, which is a common assumption in massive MIMO systems. We consider a frequency-flat channel which is modeled in discrete-time domain as

$$\mathbf{y}_k = \mathbf{H}\mathbf{x}_k + \mathbf{n}_k, \quad (1)$$

where $\mathbf{y}_k = [y_{1,k} \cdots y_{K,k}]^T$ is the received vector at the user terminals at the k -th time instance, $\mathbf{H} \in \mathbb{C}^{K \times N}$ is the channel matrix, $\mathbf{x}_k \in \mathbb{C}^{N \times 1}$ is the signal vector on the transmit antennas at the k -th time instance and \mathbf{n}_k is additive white Gaussian noise with variance σ_n^2 .

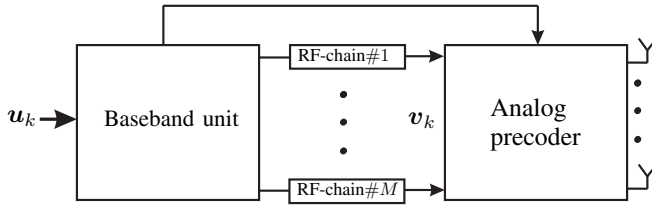


Fig. 1: The block diagram of the proposed hybrid A-D.

The base station uses the hybrid A-D transmitter structure shown in Fig. 1 which consists of a baseband block in digital domain, M RF-chains and an analog precoder network. Let $\mathbf{u}_k \in \mathbb{C}^{K \times 1}$ and $\mathbf{v}_k \in \mathbb{C}^{M \times 1}$ be the users information vector and the output signal vector of the RF-chains at the k -th time instance, respectively. The analog precoding block in Fig. 1 can be implemented using any of the proposals in [5]–[8]. It transforms the vector \mathbf{v}_k to $\mathbf{x}_k \in \mathbb{C}^{N \times 1}$ in analog domain, and is updated at rate R_a .

The baseband unit stacks L input vectors to form block $\mathbf{U}_i \triangleq [\mathbf{u}_{Li+1}, \mathbf{u}_{Li+2}, \dots, \mathbf{u}_{Li+L}]$ where $i \in \{0, 1, 2, \dots\}$ represents the block indices and L is a positive integer. The analog precoder is updated at the beginning of each block and kept fixed during the block interval. Let \mathbf{A}_i be the analog precoder matrix for the i th block. The baseband unit updates \mathbf{A}_i based on the rule

$$\{\mathbf{A}_i, \mathbf{V}_i\} = \underset{\mathbf{A} \in \mathbb{A}, \mathbf{V} \in \mathbb{C}^{M \times L}}{\operatorname{argmin}} \operatorname{tr} \left[(\mathbf{H}\mathbf{A}\mathbf{V} - \mathbf{U}_i)(\mathbf{H}\mathbf{A}\mathbf{V} - \mathbf{U}_i)^\dagger + \lambda \mathbf{V}\mathbf{V}^\dagger \right], \quad (2)$$

where $\mathbf{V}_i \triangleq [\mathbf{v}_{Li+1}, \mathbf{v}_{Li+2}, \dots, \mathbf{v}_{Li+L}]$ and λ is a positive constant. The first term in the right-hand-side of (2) is a measure for multi-user interference at the user terminals. The second term represents the total transmit power constraint with the coefficient λ .

Note that the proposed hybrid precoder differs from Standard Hybrid Precoder (SHP) in which the analog precoder matrix depends on the channel matrix alone and is data independent. In SHP, M is equal to or larger than K which allows us to update the analog part at the rate of $1/T_c$, where T_c is the coherence time of channel. However, in the proposed structure M can be any arbitrary positive integer and the update rate of the analog part is $R_a = \frac{1}{LT_s + \tau}$, where T_s is the symbol time and τ is a guard interval between blocks used for updating the analog precoder. At the updating time, the base station remains silent in order to reduce out-of-band radiation. In current wireless communication systems, the channel coherence time is in the order of milliseconds [9]. On the other hand, modern RF switching provides switching rates at about 100Mswitch/s. Thus, updating the analog precoder with rate $\frac{1}{LT_s + \tau}$ for $L \propto 10$ is easily possible without any significant rate loss caused by the guard interval. Note that the guard interval should be large enough such that the pulse shaping filter effect vanishes to avoid out of band radiation.

The proposed transmitter reduces to the single-RF MIMO transmitter proposed in [10] by setting $M = L = 1$ and $\tau = 0$.

The optimization problem (2) can be solved in terms of \mathbf{V} by keeping \mathbf{A} fixed. Letting the derivative with respect to \mathbf{V} to zero results in [11]

$$\mathbf{V}_i = \begin{cases} \left(\mathbf{A}_i^\dagger \mathbf{H}^\dagger \mathbf{H} \mathbf{A}_i + \lambda \mathbf{I} \right)^{-1} \mathbf{A}_i^\dagger \mathbf{H}^\dagger \mathbf{U}_i & M \leq K \\ \mathbf{A}_i^\dagger \mathbf{H}^\dagger \left(\mathbf{H} \mathbf{A}_i \mathbf{A}_i^\dagger \mathbf{H}^\dagger + \lambda \mathbf{I} \right)^{-1} \mathbf{U}_i & M > K \end{cases}. \quad (3)$$

Note that for $\lambda > 0$, the two terms in (3) are equal which can be shown by the matrix inversion lemma. However, we split the two cases since they are different for $\lambda = 0$. For brevity, we only consider the case of $M \leq K$ in this section. The same argumentation applies to the case $M \geq K$. Substituting (3) in (2), the optimization problem then becomes

$$\mathbf{A}_i = \underset{\mathbf{A} \in \mathbb{A}}{\operatorname{argmin}} \operatorname{tr} \left[\left(\mathbf{I} - \mathbf{H} \mathbf{A} \left(\mathbf{A}^\dagger \mathbf{H}^\dagger \mathbf{H} \mathbf{A} + \lambda \mathbf{I} \right)^{-1} \mathbf{A}^\dagger \mathbf{H}^\dagger \right) \mathbf{U}_i \mathbf{U}_i^\dagger \right] \quad (4)$$

which should be solved based on the knowledge about \mathbb{A} imposed by the analog precoder network.

In SHP, the analog precoder is data independent and the output of digital precoder at the k -th time instance is updated based on the data vector \mathbf{u}_k alone. Therefore, taking the expectation with respect to the data vector, one can reach to

$$\check{\mathbf{A}} = \underset{\mathbf{A} \in \mathbb{A}}{\operatorname{argmin}} \operatorname{tr} \left[\left(\mathbf{I} - \mathbf{H} \mathbf{A} \left(\mathbf{A}^\dagger \mathbf{H}^\dagger \mathbf{H} \mathbf{A} + \lambda \mathbf{I} \right)^{-1} \mathbf{A}^\dagger \mathbf{H}^\dagger \right) \right] \quad (5)$$

as the precoder optimization in SHP. In this case, $\check{\mathbf{A}}$ is valid for the whole coherence time duration.

The proposed block-based strategy in this paper can be applied to any type of analog precoder. In the next section, some analog precoders reported in the literature are explained. In the remaining parts of this paper, for the sake of simplicity in notation we omit the block index i . Furthermore, we assume that $\beta \triangleq N/M$ is an integer.

We define the distortion measure D as

$$D \triangleq \frac{1}{KL} \operatorname{E} \operatorname{tr} \left[(\mathbf{H}\mathbf{A}\mathbf{V} - \mathbf{U})(\mathbf{H}\mathbf{A}\mathbf{V} - \mathbf{U})^\dagger \right], \quad (6)$$

where the expectation is over the channel matrix and the input data. In the following lemma, we show that for $\lambda = 0$, $N \geq K$, $L \leq M$ and $\mathbf{A} \in \mathbb{C}^{N \times M}$ the precoder can be designed such that the distortion becomes zero. The result of Lemma 1 holds even for $M \leq K$ which means that the number of RF-chains may be less than the number of K (or the number of eigenmodes in the channel).

Lemma 1: For $\mathbf{A} \in \mathbb{C}^{N \times M}$, there exists at least one set of solution for $\{\mathbf{A}, \mathbf{V}\}$ resulting in

$$\operatorname{tr} \left[(\mathbf{H}\mathbf{A}\mathbf{V} - \mathbf{U})(\mathbf{H}\mathbf{A}\mathbf{V} - \mathbf{U})^\dagger \right] = 0, \quad (7)$$

which means zero interference at the user terminals, if $N \geq K$ and $L \leq M$.

Proof: To prove this lemma, we show that under the given conditions, the matrix equation

$$\left(\mathbf{H}\mathbf{A} \left(\mathbf{A}^\dagger \mathbf{H}^\dagger \mathbf{H}\mathbf{A} \right)^{-1} \mathbf{A}^\dagger \mathbf{H}^\dagger - \mathbf{I} \right) \mathbf{U}\mathbf{U}^\dagger = \mathbf{0} \quad (8)$$

has at least one solution set. For $M = K$, this holds for every matrix \mathbf{U} if the inverse in (8) exists. For $M < K$, the matrix $\mathbf{H}\mathbf{A} \left(\mathbf{A}^\dagger \mathbf{H}^\dagger \mathbf{H}\mathbf{A} \right)^{-1} \mathbf{A}^\dagger \mathbf{H}^\dagger - \mathbf{I}$ has M eigenvalues equal to 0 and $K - M$ eigenvalues equal to 1. On the other hand, for $L \leq K$, the matrix $\mathbf{U}\mathbf{U}^\dagger$ has least $K - L$ zero eigenvalues. Therefore, one solution is obtained if the space spanned by the eigenvectors corresponding to the non-zero eigenvalues of the matrix $\mathbf{H}\mathbf{A} \left(\mathbf{A}^\dagger \mathbf{H}^\dagger \mathbf{H}\mathbf{A} \right)^{-1} \mathbf{A}^\dagger \mathbf{H}^\dagger - \mathbf{I}$, is the same space spanned by the eigenvectors of $\mathbf{U}\mathbf{U}^\dagger$ which correspond to the zero eigenvalues. Therefore, one sufficient condition is $L \leq M$. ■

Based on the result in Lemma 1, the total distortion can be zero even for $M \leq K$, if $L \leq M$. One can reduce the number of RF-chains by simply updating the analog part at a higher rate. As an example, for $N = 200$ and $K = 40$ we can use only 10 RF-chains and update the analog part for each block with length $L = 10$. Note that the result of Lemma 1 is only true when the analog precoder matrix \mathbf{A} can be freely chosen which is not the case in practice. For realistic analog precoders, there is some loss due to some hardware restrictions.

To compare different scenarios in the numerical results section, we use a lower bound for the average achievable rate of the users which is given in the following lemma.

Lemma 2: The average rate per user is bounded by

$$\bar{R} \triangleq \frac{1}{K} \sum_{k=1}^K R_k \geq \frac{LT_s}{\tau + LT_s} \log \left(1 + \frac{\sigma_u^2}{\sigma_n^2 + D} \right), \quad (9)$$

where R_k is the rate of the k -th user.

Proof: For Gaussian distributed input signals, the worst case happens when the interference becomes independent and Gaussian distributed which leads to

$$\bar{R} \geq \frac{LT_s}{\tau + LT_s} \frac{1}{K} \sum_{k=1}^K \log \left(1 + \frac{\sigma_u^2}{\sigma_n^2 + I_k} \right), \quad (10)$$

where I_k is the power of interference at the k -th user terminal and the factor $\frac{LT_s}{\tau + LT_s}$ represents the rate loss due to the guard interval. Using the Jensens inequality and the fact that the function $f(x) = \log \left(1 + \frac{\sigma_u^2}{\sigma_n^2 + x} \right)$ is a convex function, we obtain [12]

$$\begin{aligned} \bar{R} &\geq \frac{LT_s}{\tau + LT_s} \log \left(1 + \frac{\sigma_u^2}{\sigma_n^2 + \frac{1}{K} \sum_{i=1}^K I_i} \right) \\ &= \frac{LT_s}{\tau + LT_s} \log \left(1 + \frac{\sigma_u^2}{\sigma_n^2 + D} \right). \end{aligned} \quad (11)$$

We use the lower bound in Lemma 2 in the numerical results section as the main performance measure. To have a fair

comparison, we fix the total transmit power which is defined as

$$P_t = E \text{tr}(\mathbf{V}\mathbf{V}^\dagger), \quad (12)$$

where the expectation should be taken over the data inputs and the channel realizations.

Note that in Lemma 1, we assume that the analog part can be freely designed which is not true in practice where \mathbf{A} can only have some special forms due to hardware limitations. Therefore, distortion may not completely vanish. Note that in practice, it is sufficient to keep the distortion level smaller than thermal noise power at the user terminals.

III. ANALOG PRECODERS: IMPLEMENTATION IDEAS

The main challenge in designing hybrid A-D precoders is the analog part in which arithmetic operations such as phase shift, addition and multiplication may lead to losses or high power consumption of the control circuits. One of the initial ideas to implement analog precoding is to use a $1 \times \beta$ power splitter per RF-chain and connect each output port to one of the antennas via a phase shifter [5]. For such an analog network, \mathbf{A} can be modeled as

$$\mathbf{A} = \mathbf{\Phi}\mathbf{B}, \quad (13)$$

where $\mathbf{B} = \frac{1}{\sqrt{\beta}} \text{diag}(\mathbf{1}_\beta, \dots, \mathbf{1}_\beta)$ is a $N \times M$ matrix and $\mathbf{\Phi} = \text{diag}(e^{j\phi_1}, \dots, e^{j\phi_N})$. The matrix \mathbf{B} represents the power splitters and ϕ_i models the phase shift imposed by the i th phase shifter. This structure is called Direct Phase Shift (DPS) in this paper.

While DPS has low power loss due to its simple structure, it does not offer many degrees of freedom since \mathbf{A} can only have some special forms and does not allow for an arbitrary matrix structure. One way to increase the degrees of freedom is to use $1 \times N$ power splitters and after changing the phase of the branches, combine them using an $N \times 1$ power combiner at each antenna. This is called Fully-Connected Architecture (FCA) in the literature [5], [8]. The main problem in FCA is the huge loss of the power combiners and phase shifters. In fact a power combiner with N inputs has a power loss factor of $\frac{1}{N}$ since it does not add the input signals coherently [13]. Furthermore, FCA needs MN phase shifters which increases the total power consumption and complexity. Note that in a typical massive MIMO system NM is in order of 1000.

Using a network of hybrid couplers is another idea suggested in [6] and [7]. In these structures, the number of phase shifters is reduced and hybrid couplers are used to combine the signals. Another idea for analog beamforming is to use Butler matrices which implement Discrete Fourier Transform (DFT) operation [8], [14].

In general, there is a trade-off between power loss and degrees of freedom in analog precoders. One can use more phase shifters and other RF components to increase the degrees of freedom at the expense of power loss and complexity. ■ The standard reflectarray [15] is another idea to simplify the combining circuit. The M RF-chains can be connected to M horn antennas which are directed toward the reflectarray. The

main benefit of the reflectarray is the simple hardware structure compared to the other structures. Multiple beams can be easily formed by tuning the phases of the patches. In fact, the combining is done at air. An appropriate reflect array for our application is a flat surface on which many patch antennas are implemented. Each patch antenna is loaded by a tunable circuit to adjust the phases. Pin diodes, varactors, Micro-Electro-Mechanical Systems (MEMS) switches and micro-machined motors are among the popular phase tuning technologies for reflectarray antennas. One of the challenges in reflectarrays is the derivation of the appropriate phases. In general, the patch antennas cannot be considered as individual elements and they affect each other depending on the target beams. Normally, full-wave analyses such as finite element method are required to analyze a reflectarray.

In this paper, we consider the DPS structure, although the general framework explained in the last section works with any analog precoder. In fact, the main message of this paper is to show that for any analog precoder, the strategy introduced in the last section can improve the performance significantly. Thus, we use a simple DPS precoder in the numerical results section.

IV. NUMERICAL RESULTS

In this section, we present some numerical results for the proposed hybrid precoding structure. A single-cell with $K = 10$ users is considered. The base station has $N = 50$ antennas and inter-cell-interference is neglected. The channel matrix is assumed to be iid Gaussian, i.e., $h_{i,j} \in \mathcal{CN}(0,1)$. Additive white Gaussian noise with variance 0.1 is considered at all the user terminals. Furthermore, the input symbols, i.e., the entries of \mathbf{U}_i are assumed to be iid Gaussian with the average power 1. To have a fair comparison, we limit the total transmit power at the base station by K .

DPS structure is used in the analog part and investigating the other structures introduced in Section III is left for the extended version of this paper. The phase shifters and the power splitters are assumed to be loss-less. Furthermore, it is assumed that the phase shifters are able to change the phases arbitrarily. The optimization problems are numerically solved using the MATLAB optimization toolbox. The lower bound introduced in Lemma 2 is used as the performance measure. Note that the distortion D in Lemma 2 is calculated by taking average over many samples.

First, we fix $M = 5$ and $\tau = T_s$ and investigate the effect of λ on the achievable rate. Note that λ here has a similar role as in regularized zero-forcing precoding [16] which balances between the amount interference and noise to maximize the signal to interference plus noise ratio. Fig. 2 shows that there is an optimum λ for which the achievable rate is maximum. Analytical track of optimum λ is an interesting future work. In the remaining figures in this section, we set $\lambda = 0.5$.

Next, the achievable rate versus the number of RF-chains, i.e., M , is plotted in Fig. 3 for various L and $\tau = T_s$. The achievable rate for the standard hybrid A-D shown by SHP is also plotted. It is observed that the proposed method

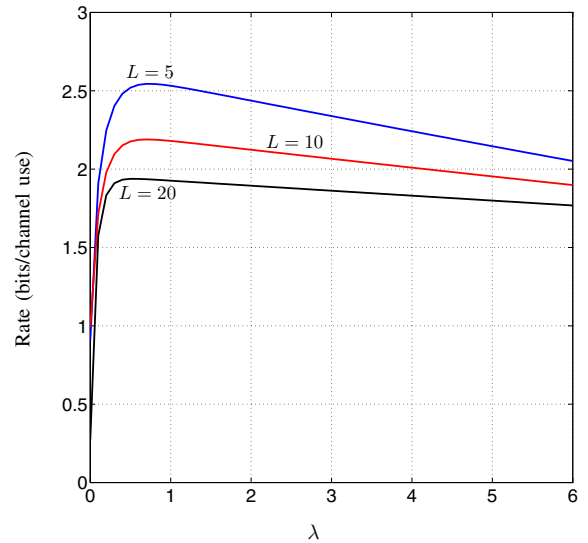


Fig. 2: The achievable rate versus λ for $M = 5$ and $\tau = T_s$.

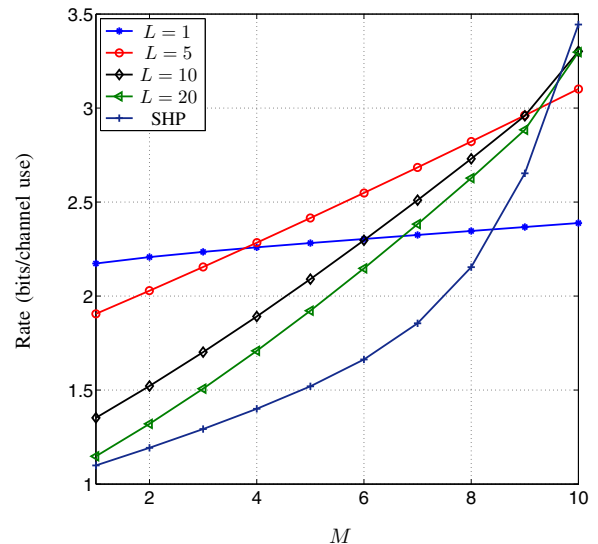


Fig. 3: The achievable rate versus M for various L and $\tau = T_s$. As a reference, the performance of the state-of-the-art hybrid A-D shown by SHP is also plotted.

outperforms SHP if the number of RF-chains is small. For $M = 10$, SHP method performs better since the rate loss in the proposed scheme due to guard interval cannot be compensated by the gain obtained from block processing. To observe the effect of block length L , the achievable rate versus L is plotted in Fig. 4 for $M = 2, 5, 10$. Note that for small L , the penalty of guard interval becomes considerable. Furthermore, for large L , the performance decreases since the matrix \mathbf{A} needs to be optimized for more input vectors. Therefore, it is observed that there is an optimum block length for which the rate is maximized.

Next, we investigate the rate loss caused by the guard

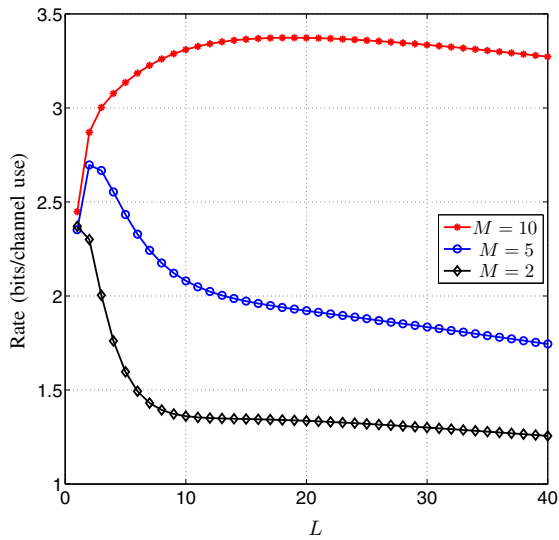


Fig. 4: The achievable rate versus L for $M = \{2, 5, 10\}$ and $\tau = T_s$.

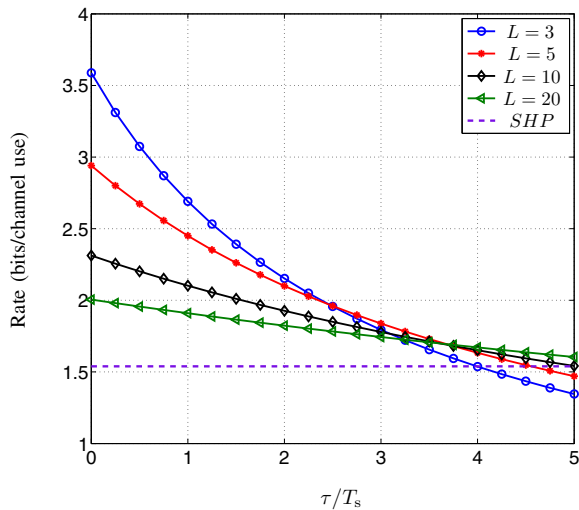


Fig. 5: The achievable rate versus τ/T_s for $L = \{3, 5, 10, 20\}$ and $M = 5$.

interval. We set $M = 5$. Fig. 5 shows the achievable rate versus τ/T_s for $L = \{3, 5, 10, 20\}$. It is observed that for short guard intervals, the gain of the proposed precoding is significant. For $\tau/T_s = 2$, the proposed scheme achieves about 2.15 bits/channel use for $L = 3$ while SHP achieves about 1.54 bits/channel use.

V. CONCLUSIONS

In this paper, hybrid A-D transmitters for massive MIMO base stations in downlink channels were investigated. In such base stations, the number of antennas is large and it is not economical to implement fully digital precoding since it needs a number of RF-chains equal to the number of antennas. Therefore, hybrid A-D precoding is attractive due to its low RF-cost and complexity. We considered a new structure in

which the analog precoder is updated for each block of input data. It was shown that higher update rates in the current hybrid A-D transmitters are very beneficial to reduce the RF-cost further. The number of RF-chains in the proposed structure can be even less than the number of users. For a fixed number of RF-chain, we showed that the block-based precoding outperforms standard hybrid A-D precoding at the expense of complexity.

In this paper, we consider a simple Gaussian iid channel model, and extending the results to channels with path loss and shadowing effect is our next future work. Furthermore, developing low complexity algorithms to calculate \mathbf{A} is an interesting future work. This is especially of interest since the analog part should be updated for every block, the calculation should be done at the guard interval time.

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