Quaternion Adaptive Line Enhancer

S. Sanei[1], C. Cheong Took[1], S. Enshaeifar[1], and T. K. M. Lee[2]

[1]Faculty of Engineering and Physical Sciences, University of Surrey, UK.
[2]School of Electrical and Electronic Engineering, Singapore Polytechnic, Singapore.

***Abstract*—The recovery of periodic signals from their noisy single channel mixtures has made wide use of the adaptive line enhancer (ALE). The ALE, however, is not designed for detection of two- (2-D) or three-dimensional (3-D) periodic signals such as tremor in an unconstrained hand motion. An ALE which can perform restoration of 3-D periodic signals is therefore required for such purposes. These signals may not exhibit periodicity in a single dimension. To address and solve this problem a quaternion adaptive line enhancer (QALE) is introduced in this paper for the first time which exploits the quaternion least mean square (QLMS) algorithm for the detection of 3-D (extendable to 4-D) periodic signals.**

***Index Terms*—ALE, quaternion adaptive line enhancer, QLMS, quaternion shift.**

I. Introduction

Many signals and time series, which are often noisy in nature, contain periodic or cyclostationary components. As an example, most of the signals recorded from the human body are periodic, quasiperiodic, or cyclostationary (i.e. some order statistics of the data are periodic) [1]. These signals are often buried in noise or mixed with other periodic or aperiodic signals. Extraction of such cyclic signal components is very important for monitoring the status of patients undergoing medical treatment.

The adaptive line enhancer (ALE) was introduced by Widrow et al. [2] and widely used for the separation of a generally weak sinusoid, periodic, or narrowband signal from strong broad-band noise. This has been a classical problem in the field of nonlinear and adaptive signal processing.

The general block diagram of an ALE is depicted in Fig. 1. The input $s(n)$ is assumed to be the sum of a narrow-band signal $x(n)$ and a broad-band signal $v(n)$ which is considered as noise. $e(n)$ is the error signal between $s(n)$ and the estimated signal $\hat{x}(n)$. The vector of prediction filter parameters $\mathbf{w}$ are iteratively and automatically adjusted based on $e(n)$ so that the statistical mean squared error (MSE), $E[e^2(n)]$, where $E[\cdot]$ stands for statistical expectation, is minimized.

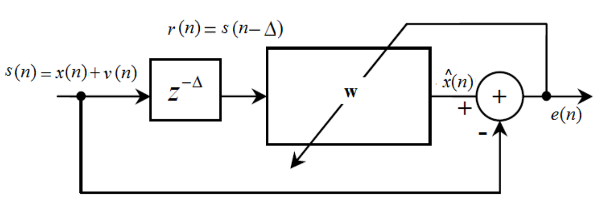

Fig. 1: A single channel (one dimensional traditional) adaptive line enhancer.

The ALE operates by virtue of the difference between the correlation lengths of $x(n)$ and $v(n)$. The correlation length refers to the interval within which the correlation between the first and last sample is not zero and therefore, shows the system order. The delay parameter Δ should be chosen larger than the correlation length of $v(n)$, but smaller than the correlation length of $x(n)$. In this case, it is possible for $\mathbf{w}$ to make a Δ-step ahead prediction of $x(n-\Delta)$ based on the present and past samples of $s(n-\Delta)$. However, $\mathbf{w}$ is not able to predict $v(n)$ from the knowledge about present and past samples of $v(n-\Delta)$. As a result, after the parameters of $\mathbf{w}$ have converged toward their optimal values, $e(n)$ is approximately equal to $v(n)$ and the ALE output $\hat{x}(n)$ is approximately equal to $x(n)$.

The ALE has applications in many areas such as communications, sound, vibration, and biomedical signal processing. There are, however, limitations for the traditional ALE, since it is designed to be used for single channel (one dimensional) signals and its application is limited to narrowband signals and Gaussian noise. As such, it is not applicable in scenarios where the signal is in three dimensional (3-D) space, the noise is not white Gaussian, or the artefact signals have temporally correlated components. Sanei et al. [3] proposed an ALE which incorporates singular spectrum analysis to alleviate the problems of one dimensional (1-D) ALE. In this paper, the focus is therefore, on developing a new ALE which works for denoising and recovery of 3-D periodic signals.

In the case where the periodic data is 3-D, such as the one in Fig. 2, successive application of 1-D ALE for each dimension in 3-D space is not effective in general for the recovery of such signals from their noisy versions as the signal may not exhibit periodicity in any one of the three dimensions. This can be seen in Fig. 2, where none of the components of the 3-D signal in any one of the three dimensions is periodic. The proposed algorithm in this article therefore, aims at recovering a 3-D periodic wave by developing an ALE which can operate in 3-D. For this purpose a quaternion based ALE (QALE) algorithm is proposed. Often, least mean square (LMS) algorithm is used for solving the conventional ALE problem [2]. In order to develop the QALE, an effective quaternion-based LMS is used.

Recently, a class of quaternion least mean square (QLMS) stochastic gradient adaptive filtering algorithms has been designed in [4] for filtering of hyper-complex processes. Such a system can be applied to both circular and noncircular signals and therefore, exploits the correlation between the real and complex components of a quaternion-valued signal. Their analysis has shown that for circular data in the quaternion

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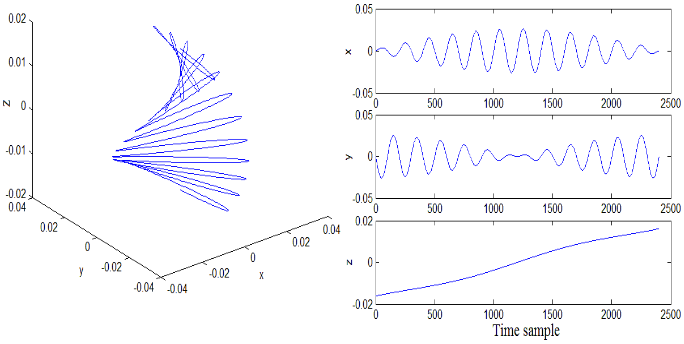

Fig. 2: The noise free 3-D sinusoid; (left) its 3-D illustration and (right) its variation along each axis.

(Hamiltonian, $\mathbb{H}$) domain the pseudocovariance $E\{\mathbf{x}\mathbf{x}^T\}$ does not disappear as it does in the complex domain $\mathbb{C}$. Further, it has been shown that operating in the quaternion domain allows for the fusion of heterogenous data sources. Inclusion of pseudocovariance however, has proved to be very effective in improving the performance of QLMS.

Quaternions, used for more than 150 years (conceived by Hamilton in 1843), can be regarded as a noncommutative extension of complex numbers, and comprise of at most four variables [5]. A quaternion variable $q \in \mathbb{H}$ which has a real/scalar part $\Re(q)$ (here, denoted by subscript a), and a vector part $\Im(q)$ comprising of three imaginary parts (denoted by subscripts b, c, and d), can be expressed as

$$\begin{aligned} q &= [\Re(q), \Im(q)] = [q_a, \mathbf{q}] \in \mathbb{H} \\ &= [q_a, (q_b, q_c, q_d)] \\ &= q_a + \imath q_b + \jmath q_c + \kappa q_d \qquad \{q_a, q_b, q_c, q_d\} \in \mathbb{R} \end{aligned} \tag{1}$$

where $\imath$, $\jmath$, and κ are the orthogonal unit vectors and have the properties $\imath\jmath = \kappa$, $\jmath\kappa = \imath$, $\kappa\imath = \jmath$, and $\imath\jmath\kappa = \imath^2 = \jmath^2 = \kappa^2 = -1$. Quaternions have found applications in computer graphics, for the modelling of three-dimensional (3-D) rotations [6], in robotics [7], molecular modelling [8], processing colour images [9], hyper-complex digital filters [10], texture segmentation [11], source separation [12], watermarking [13], spectrum estimation [14] quaternion singular value decomposition and in the MUSIC algorithm to process polarized waves [15], [16], quaternion least squares [8], [17], and quaternion singular spectrum analysis [18]. In [4] the formulation for a quaternion LMS adaptive filtering has also been provided and used for the processing of quaternion valued signals.

Just as important as many of the above applications, detection and extraction of periodic 3-D signals buried in noise, artefacts, and undesired periodic or aperiodic signals is required for many applications. Detection of weak underwater tone signals based on line spectrum extraction and tremor signals of a patient suffering from stroke or Parkinsons, from their 3-D trajectory of hand movement, are some examples of such applications.

II. METHODOLOGY

The conventional LMS algorithm minimises $E[ee^*]$ where $e(n) = d(n) - \mathbf{w}^T(n)\mathbf{x}(n)$, $d(n)$ is the desired or target signal, $\mathbf{x}(n)$ is the input signal, $\mathbf{w}(n)$ is the vector of filter parameters, and $(\cdot)^*$, $(\cdot)^H$, and $(\cdot)^T$ refer to conjugate, conjugate transpose, and transpose operations for a vector respectively.

In an ALE however, $d(n) = x(n - \Delta)$ as mentioned in Section I and $x(n)$ is a periodic noisy signal where the 1-D time delay $\Delta = mP$, P is the signal period and m is an integer. When the noise is white, m can be as small as unity.

In 3-D applications there is need for a quaternion delay along the signal base-line trajectory. This is naturally a shift equivalent to an integer multiple of the signal cycle period in the 3-D space.

In our application the quaternion input signal is defined as

$$x_q(n) = x_a(n) + \imath x_b(n) + \jmath x_c(n) + \kappa x_d(n) \tag{2}$$

where $x_a(n)$, $x_b(n)$, $x_c(n)$, and $x_d(n)$ are the four signals in four orthogonal directions. For a 3-D case, an example can be the hand movement in the x-y-z coordinates.

In the augmented quaternion LMS proposed in [4] similar to original LMS, we have:

$$\mathbf{J}(n) = e(n)e(n)^* = e_a^2(n) + e_b^2(n) + e_c^2(n) + e_d^2(n) \tag{3}$$

In order for the QLMS to cater for general quaternion processes, a quaternion-valued semi-widely linear model can be employed [19];

$$\mathbf{y}(n) = \mathbf{w}^T(n)\mathbf{x}(n) + \mathbf{g}^T(n)\mathbf{x}^H(n) \tag{4}$$

This model incorporates the information contained in both the covariance, $\mathbf{C}_{xx} = E[\mathbf{x}\mathbf{x}^H]$, and pseudocovariance, $\mathbf{P}_{xx} = E[\mathbf{x}\mathbf{x}^T]$. According to [4], using QLMS the update equations for $\mathbf{w}$ and $\mathbf{g}$ are obtained as:

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu[2e(n)\mathbf{x}^*(n) - \mathbf{x}^*(n)e^*(n)] \tag{5}$$

$$\mathbf{g}(n+1) = \mathbf{g}(n) + \mu[2e(n)\mathbf{x}(n) - \mathbf{x}(n)e^*(n)] \tag{6}$$

In order to have a unified and combined update equation, as often considered for augmented quaternion signals and systems, the above two filters are augmented to form

$$\mathbf{h}^a(n) = [\mathbf{w}^T(n) \quad \mathbf{g}^T(n)]^T \tag{7}$$

In that case the weight update of the QLMS is expressed as

$$\mathbf{h}^a(n+1) = \mathbf{h}^a(n) + \mu[2e^a(n)\mathbf{x}^{a*}(n) - \mathbf{x}^{a*}(n)e^{a*}(n)] \tag{8}$$

where the augmented error is given by

$$e^a(n) = d(n) - \mathbf{h}^{aT}(n)\mathbf{x}^a(n) \tag{9}$$

where $\mathbf{x}^a(n) = [\mathbf{x}^T(n) \quad \mathbf{x}^H(n)]^T$. In the proposed 3-D quaternion-based ALE (QALE), depicted in Fig. 3,

$$\mathbf{x}^a(n) = \mathbf{x}_s^a(n) + \mathbf{v}^a(n) \tag{10}$$

is the augmented input noisy signal and the target signal for the QLMS filter, is a quaternion shift of the input signal i.e.

$$d(n) = \mathbf{r}^a(n) = \mathbf{x}^a(n - \Delta_q) \tag{11}$$

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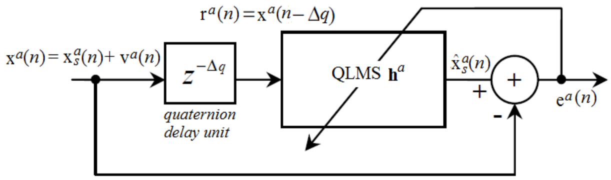

Fig. 3: Block diagram of the proposed QALE.

vice versa. Therefore, the output $\mathbf{y}^a(n) = \hat{\mathbf{x}}_s^a(n)$ is an estimation of the noise free signal $\mathbf{x}_s^a(n)$.

Although in some cases such a 3-D shift, Δ_q, for at least one signal cycle is practically easy, such as those for a prescribed hand movement trajectory in an action research arm test (ARAT) [20], in theory, it may pose a challenging problem. An effective option, used in this study, is to apply quaternion singular spectrum analysis (QSSA) introduced in [18] and use the signal obtained from the first eigentriple (i.e. the eigenvalue and eigenvectors of the corresponding augmented quaternion singular value decomposition within the QSSA operation) as the 3-D baseline. In those rare cases where the dominant eigentriple is related to the periodic signal, a couple of identical eigenvalues can be seen in the eigenspace of the SSA. In such cases the trajectory eigentriple will be that corresponding to the 3rd eigenvalue. Given such smooth trajectory the points (x_{pi}, y_{pi}, z_{pi}), $i = 1, \ldots, N$, where N is the signal segment length, in one segment, say p, can be translated (shifted) to the points (x_{fi}, y_{fi}, z_{fi}) in another segment, say f, in the 3-D space. More importantly, a direction should be associated with each segment sample to be aligned with a similar sample of the other segment during the 3-D shift process. For low noise signals this can be achieved by incorporating the x-y-z values. At the presence of high level noise there will be ambiguity in the sample directions too. A complete solution to this problem however, is under research and will be the agenda of another paper.

III. Experiment

The performance of QALE was assessed for the synthetic signal shown in Fig. 4. This signal is constructed by adding white Gaussian 3-D noise to the signal in Fig. 2. Evidently, it is very difficult to realise the periodic behaviour of the data in 3-D from this figure as the noise severly perturbs the 3-D shape of the signal. The following equations can be used to create such signals.

$$x = \sin(\alpha n)\cos(6\beta n) + \Gamma_x(n)$$
$$y = \sin(\alpha n)\sin(6\beta n) + \Gamma_y(n)$$
$$z = \gamma[n + \sin(\frac{n}{3})] + \Gamma_z(n)$$

where $\Gamma(n)$ is the white Gaussian noise with different noise levels. The constants α, β, and γ can be changed; in this application they are set respectively to 3, 0.02, and 1. The QLMS target signal is also another later segment of the same signal with an interval Δ_q (equivalent to an integer number of signal cycles) which has been shifted forward along the 3-D direction (Fig. 5). It is evident that with the added noise the signal in 3-D is not recognisable.

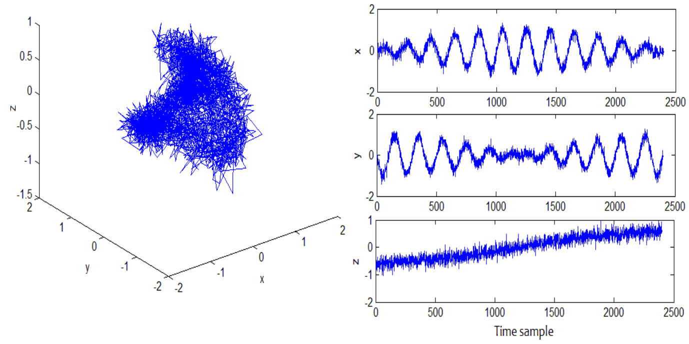

Fig. 4: The noisy 3-D sinusoid; (left) its 3-D illustration and (right) its variation along each axis; SNR=5 dB.

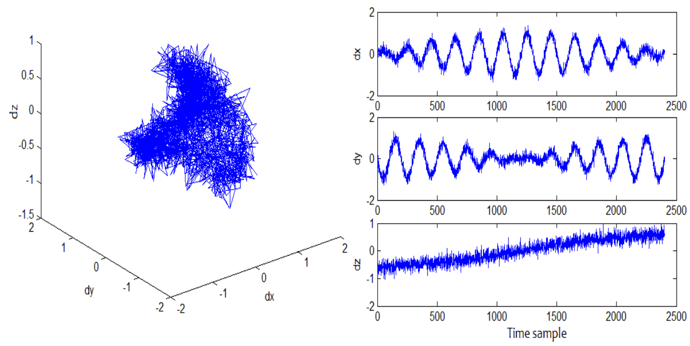

Fig. 5: Shifted, along the 3-D signal trajectory, noisy 3-D sinusoid of Fig. 4; (left) its 3-D illustration and (right) its variation along each axis; SNR=5 dB.

The proposed QALE algorithm was applied to the above signal for different noise levels. The results are depicted in Fig. 6. In our attempt, we considered that the baseline of movement and the sample directions were known a priori, so, the 3-D shift could be performed accurately. It is also seen that the noise effect in obscuring the signals is more obvious in 3-D cases as compared to 1-D cases. In some cases where the periodicity is along two dimensions, applying a simple post processing method to the signal in the third dimension can significantly reduce this effect. In our experiment, for example, the z component of the signal can be post processed to enhance the 3-D signal. This is however, out of the scope of this letter. It is also clear that by applying the traditional ALE successively to each of the x, y, and z dimensions, no conclusive result is expected. This is because, in general, the 3-D periodic signals are not expected to be periodic in each of the above directions. However, In the above example for the sake of comparison, we applied ALE to the signal in y direction and depicted the results in Fig. 7. By decreasing the signal-to-noise ratio (SNR), the performance of the algorithm deteriorates. The performance was evaluated in terms of mean square error (MSE) defined as:

$$\text{MSE} = \frac{\|\mathbf{x}_s^a(n) - \hat{\mathbf{x}}_s^a(n)\|^2}{\|\mathbf{x}_s^a(n)\|^2} \qquad (12)$$

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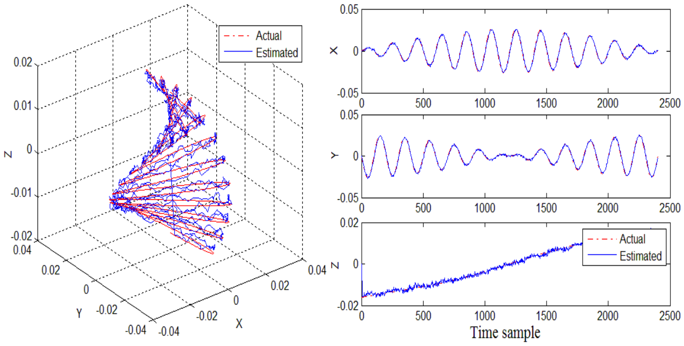

Fig. 6: The result of applying the proposed QALE to the 3-D signal of Fig. 4 and its comparison with the noise-free signals of Fig. 2; (left) 3-D illustration and (right) variation along each axis.

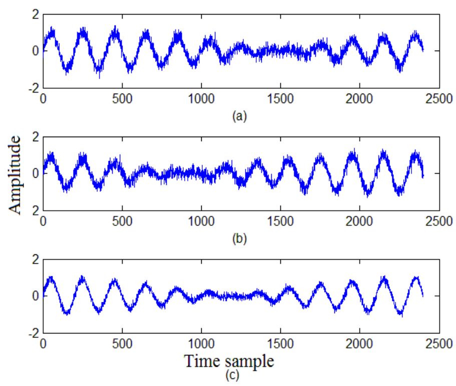

Fig. 7: The result of applying the traditional ALE to the simulated signals in y direction; (a) noisy input, (b) the shifted signal, and (c) the ALE output.

where $\|\cdot\|^2$ refers to Euclidean norm and each term can be expanded to sum square of its quaternion components, e.g.

$$\|\mathbf{x}_s^a(n)\|^2 = \|\mathbf{x}_{s_a}^a(n)\|^2 + \|\mathbf{x}_{s_b}^a(n)\|^2 + \|\mathbf{x}_{s_c}^a(n)\|^2 + \|\mathbf{x}_{s_d}^a(n)\|^2 \tag{13}$$

The calculated MSE for both ALE and QALE are illustrated in Fig. 8.

IV. Conclusion

A novel quaternion-based ALE algorithm has been developed to cater for the recovery of 3-D periodic signals from their noisy counterparts. The results demonstrate that the proposed QALE is effective for 3-D signls as the ALE is for 1-D signals. In the design of proposed QALE we used the QLMS algorithm. For rigour, the performance of the algorithm has been evaluated in terms of MSE and compared with that of the original ALE bearing in mind that the traditional 1-D ALE is not meant to work for the 3-D data.

There are many applications in nature for this technique. One example can be the recovery of hand tremor moving freely in an unconstrained 3-D motion. This can happen in patients suffering from stroke, Parkinsons, or alcoholic patients. Another example can be the detection of carrier signals in a polarised communication noisy waveform. More research however, has to be undertaken to enable the quaternion shift for real world applications of QALE where the 3-D trajectory is not fully known.

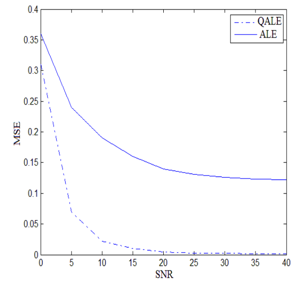

Fig. 8: Comparison between the performances of ALE and QALE. For ALE, we considered the signal changes in the y direction only. This performance therefore varies depending on which direction and how many cycles the signal is shifted through that direction.

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