Event-based Particle Filtering with Point and Set-valued Measurements

Somayeh Davar, *Student Member, IEEE* and Arash Mohammadi, *Member, IEEE*
Concordia Institute for Information System Engineering (CIISE), Concordia University,
Montreal, QC, Canada H3G 1M8 Emails: {s_dava, arashmoh}@encs.concordia.ca

***Abstract*—The paper is motivated by recent and rapid growth of Cyber-Physical Systems (CPS) and the critical necessity for preserving restricted communication resources in their application domains. In this context, a distributed state estimation architecture is considered where a remote sensor communicates its measurements to the fusion centre (FC) in an event-based fashion. We propose a systematic and intuitively pleasing distributed state estimation algorithm which jointly incorporates point and set-valued measurements within the particle filtering framework. Referred to as the event-based particle filter (EBPF), point-valued measurements are incorporated in the estimation recursion via a conventional particle filter formulation, while set-valued measurements are incorporated by developing an observation update step similar in nature to quantized particle filtering approach. More specifically, in the absence of an observation (i.e., having a set-valued measurement), the proposed EBPF evaluates the probability that the unknown observation belongs to the event-triggering set based on its particles which is then used to update the corresponding particle weights. The simulation results show that the proposed EBPF outperforms its counterparts specifically in low communication rates, and confirms the effectiveness of the proposed hybrid estimation algorithm.**

***Index Terms*—Cyber-physical systems, Event triggering, Event-based estimation, Particle filtering, Set-valued measurements, Non-Gaussian state estimation.**

I. Introduction

The paper develops an event-based estimation framework, referred to as event-based particle filtering (EBPF), for distributed state estimation problems where the remote sensor communicates its measurements to the fusion centre (FC) in an event-based fashion, and the non-linear estimator, resided at the FC, jointly incorporates point and set-valued measurements to estimate the non-Gaussian posterior distribution. The Kalman filter (KF) [1] is considered as the classical state estimation approach in this context due its simple and efficient sequential formulation. In a conventional KF-based estimation scheme, each sensor samples and communicates its local measurements to the FC periodically using equidistance samples (referred to as a time-driven strategy). Recent developments and advancements of sensor technologies, cyber-physical systems (CPS) [2], and network control systems (NCS) [3] renders the above mentioned conventional approach to be impractical because of the following key reasons: (i) Measurements contain different information contents over time; (ii) Sensor nodes have restricted power supplies, therefore, can not afford to periodically transfer information to the FC as communication is the main source of power consumption, and; (iii) The channel bandwidth is limited, another barrier in implementing time-driven distributed estimation algorithms such as the conventional KF. These issues have resulted in a recent surge of interest in developing intelligent transmission, scheduling, and estimation schemes [4]–[12] to reduce the communication overhead of sensors in order to increase their practical applicability by improving their energy efficiency.

Recent Solution methodologies developed to reduce the aforementioned extra communication overhead, associated with distributed estimation, can be generally classified into: (i) State estimation based on offline scheduling schemes [13], [14] where the transmission schedule is designed in advance of employment, and; (ii) Event-based estimation (EBE) methodologies [15]–[20] where communication of sensor information is only triggered once the system meets a specific condition, which is identified using a triggering mechanism at the sensor level based on real-time local observations. While it is simpler to impalement algorithms belonging to the former category, a priori information regarding the physical system is required and their performance is typically unacceptable in practice, specially in hostile environments where the characteristics of the system constantly changes. This resulted in a recent surge of interest in designing/developing event-based implementations as they are capable of providing the possibility of maintaining the required estimation performance under strict communication constraints.

The event-based concept emerged by the seminal work of Astrom and Bernhardsson [21] where it was shown that Lebesgue sampling is superior for state estimation purposes in some dynamical systems. References [22], [23] are among the early event-based methodologies and proposed the send-on-delta (SOD) triggering mechanism where the transmission is triggered only when the difference between the current measurement and the previously transmitted one is greater than a pre-defined threshold (delta). In such event-based estimation scenarios and in the absence of an observation (i.e., the triggering conditions are not satisfied) the estimator still has access to side information, i.e., the measurement belongs to the set characterized by the triggering mechanism. Incorporation of the side information from the event-triggering mechanism during non-event iterations results in a hybrid update strategy, i.e., state estimation with joint set-valued and point-valued measurements which is first considered in [24] and

This work was partially supported by the Natural Sciences and Engineering Research Council (NSERC) of Canada through the NSERC Discovery Grant RGPIN-2016-049988.

Fig. 1. Block diagram of the open-loop event-based estimation architecture.

has recently been extended in Reference [8]. In this context the mechanism used to trigger an event at the sensor side dictates the nature of the posterior distribution at the remote estimator and consequently mandates the proper (possibly optimal) form of the estimation/fusion algorithm at the FC. In the conventional time-driven scenario with point-valued measurements the simple and efficient formulation of the KF comes from the Gaussianity of conditional posterior distribution. In the hybrid scenarios described above, however, due to joint incorporation of set and point valued measurements, the posterior distribution becomes non-Gaussian, therefore, the conventional KF is no longer applicable.

To overcome non-Gaussianity of the posterior distribution, some efforts have been recently considered specially by imposing a Gaussian assumption on the posterior distribution, e.g., using single Gaussian approximation [7], [8], Gaussian sum approximation [9], and non-linear filtering scenarios [10]. However, while Gaussian-based approximation of the event-based posterior has been investigated extensively, application of non-Gaussian filtering using particle filters [25]–[28] is still in its infancy. To the best of our knowledge, only very recently, EBE using non-Gaussian particle filter approximation is considered in [11] and [12], where in the latter simply the number of particles belonging to the triggering set is used to update particle weights, while the former uses stochastic triggering [19] which results in having a Gaussian posterior. The paper addresses this gap. In particular, the paper proposes a systematic and intuitively pleasing mechanism to jointly incorporate point and set-valued measurements within the particle filter framework. More specifically, we capitalize on the fact that in particle filtering framework the observations's nature (being point or set-valued) will mainly affect the likelihood function which is used to update each particle's weight. In presence of an observation (point-valued measurements), the likelihood function can exactly be evaluated for each particle. In absence of an observation (set-valued measurement case), the proposed EBPF evaluates the probability that the unknown observation belongs to the event-triggering set based on its particles which is then used to update the corresponding particle weights. Intuitively speaking, the proposed EBPF utilizes the set-valued information similar in nature to the way that particle filter utilizes quantized observations [29]–[31]. In other words, point-valued measurements are incorporated in the estimation via a conventional particle filter while set-valued measurements are incorporated in the state estimates using a filter similar in nature to quantized particle filter.

The rest of the paper is organized as follows: Section II formulates the event-based estimation problem. In Section III, we introduce the proposed EBPF framework. Simulation results are provided in Section IV. Finally Section V concludes the paper.

II. Problem Formulation

We consider an estimation problem represented by the following linear state-space model

$$\boldsymbol{x}_k = \boldsymbol{F}_k \boldsymbol{x}_{k-1} + \boldsymbol{w}_k \quad (1)$$
$$\boldsymbol{z}_k = \boldsymbol{H}_k \boldsymbol{x}_k + \boldsymbol{v}_k, \quad (2)$$

where k denotes iteration index; $\boldsymbol{x}_k \in \mathbb{R}^{n_x}$ denotes the state vector; $\boldsymbol{z}_k \in \mathbb{R}^{n_z}$ denotes the sensor's measurement; functions $\boldsymbol{F}_k$ and $\boldsymbol{H}_k$ represent the state and observation models, respectively, and; terms $\boldsymbol{w}_k$ and $\boldsymbol{v}_k$ represent the uncertainties in the state and observation models, respectively. It is assumed that $\boldsymbol{w}_k$ and $\boldsymbol{v}_k$ are mutually uncorrelated white Gaussian noises with covariances $\boldsymbol{Q}_k > 0$, and $\boldsymbol{R}_k > 0$.

We consider a distributed estimation architecture (Fig. 1) where the sensor communicates its observation $\boldsymbol{z}_k$ to the FC which recursively update the posterior distribution $P(\boldsymbol{x}_k|\boldsymbol{Z}_k)$ based on the collective set of observations $\boldsymbol{Z}_k = \{\boldsymbol{z}_1, \ldots, \boldsymbol{z}_k\}$ received up to and including the current iteration (k). By considering the state-space model given by Eqs. (1)-(2) and its statistical properties, the posterior follows a Gaussian distribution, i.e., $P(\boldsymbol{x}_k|\boldsymbol{Z}_k) \sim \mathcal{N}(\hat{\boldsymbol{x}}_{k|k}, \boldsymbol{P}_{k|k})$ with $\hat{\boldsymbol{x}}_{k|k} = \mathbb{E}\{\boldsymbol{x}_k|\boldsymbol{Z}_k\}$, and $\boldsymbol{P}_{k|k} = \mathbb{E}\{(\boldsymbol{x}_k - \hat{\boldsymbol{x}}_{k|k})(\boldsymbol{x}_k - \hat{\boldsymbol{x}}^T_{k|k})\}$. The KF provides the optimal solution for this Gaussian case as

Prediction Step:

$$\hat{\boldsymbol{x}}_{k|k-1} = \boldsymbol{F}_k \hat{\boldsymbol{x}}_{k-1|k-1} \quad (3)$$
$$\boldsymbol{P}_{k|k-1} = \boldsymbol{F}_k \boldsymbol{P}_{k-1|k-1} \boldsymbol{F}_k^T + \boldsymbol{Q}_k. \quad (4)$$

Update Step:

$$\hat{\boldsymbol{x}}_{k|k} = \hat{\boldsymbol{x}}_{k|k-1} + \boldsymbol{K}_k \big(\boldsymbol{z}_k - \boldsymbol{H}_k \hat{\boldsymbol{x}}_{k|k-1}\big) \quad (5)$$
$$\boldsymbol{K}_k = \boldsymbol{P}_{k|k-1} \boldsymbol{H}_k^T \big(\boldsymbol{H}_k \boldsymbol{P}_{k|k-1} \boldsymbol{H}_k^T + \boldsymbol{R}_k\big)^{-1} \quad (6)$$
$$\boldsymbol{P}_{k|k} = \boldsymbol{P}_{k|k-1} - \boldsymbol{K}_k \boldsymbol{H}_k^T \boldsymbol{P}_{k|k-1}. \quad (7)$$

While the sensor has limited power sources, the FC has adequate power to perform complex estimation algorithms. In the event-based communication/fusion framework, after making each measurement the sensor decides on keeping or sending its observation to the remote estimator. The local decisions are governed by a binary triggering criteria denoted by γ_k which is defined as follows

$$\begin{cases} \gamma_k = 1: & \text{Event occurs, communication is triggered.} \\ \gamma_k = 0: & \text{Idle case, no communication.} \end{cases}$$

Based on the above triggering mechanism, the collective set of observations up to and including iteration k at the FC is defined as $\tilde{\boldsymbol{Z}}_k = \{\gamma_1 \boldsymbol{z}_1, \ldots, \gamma_k \boldsymbol{z}_k\}$. When the event-triggering condition is satisfied (i.e., $\gamma_k = 1$), the exact value of the sensor measurement $\boldsymbol{z}_k$ is known at the FC, referred to as "point-valued observation information". On the other hand, when the event-triggering condition is violated (i.e., $\gamma_k = 0$), some information contained in the event-triggering sets is known to the estimator instead, referred to as "set-valued information". The main issue here comes from the non-Gaussianity of the a posteriori distribution due to joint incorporation of point and set-valued measurements, i.e., $P(\boldsymbol{x}|\tilde{\boldsymbol{Z}}_k)$ no longer follows a Gaussian distribution. Next, we present the proposed EBPF implementation which systematically uses point and set-valued observation to perform the estimation task.

ISBN 978-0-9928626-7-1 © EURASIP 2017

III. THE PROPOSED EBPF FRAMEWORK

In the proposed EBPF framework, the remote estimator at the FC computes the state estimate based on the event-triggered measurements it receives from the remote sensor. Without loss of generality and for simplicity of the presentation, we consider the practical "send on delta" triggering criteria/condition [22]. In an open-loop scenario, in order to decide whether or not to send new measurements, the sensor computes the distance between its current measurement and the previously transmitted measurement based on the following event-triggering schedule

$$\gamma_k = \begin{cases} 1, & \text{if } |\boldsymbol{z}_k - \boldsymbol{z}_{\tau_k}| \geq \Delta \\ 0, & \text{otherwise}, \end{cases} , \tag{8}$$

where τ_k denotes the time of last communication from the sensor to the FC, and Δ denotes the triggering threshold. Based on the above triggering mechanism, we define the hybrid observation vector as $\boldsymbol{Y}_k = \{\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots, \boldsymbol{y}_k\}$ where

$$\boldsymbol{y}_k = \begin{cases} \boldsymbol{z}_k & \text{if } \gamma = 1 \\ \{\boldsymbol{z}_k : \boldsymbol{z}_k \in (\boldsymbol{z}_{\tau_k} - \Delta, \boldsymbol{z}_{\tau_k} + \Delta)\} & \text{if } \gamma = 0 \end{cases} .$$

As stated previously, the posterior distribution $P(\boldsymbol{x}_k|\boldsymbol{Y}_k)$ based on collective set of hybrid observations is no longer Gaussian, eliminating the application of linear filters such as the KF. In such a non-Gaussian scenario, the optimal Bayesian filtering recursion for iteration k is given by

$$P(\boldsymbol{x}_k|\boldsymbol{Y}_k) = \frac{P(\boldsymbol{y}_k|\boldsymbol{x}_k)P(\boldsymbol{x}_k|\boldsymbol{Y}_{k-1})}{P(\boldsymbol{y}_k|\boldsymbol{Y}_{k-1})}, \tag{9}$$

where

$$P(\boldsymbol{x}_k|\boldsymbol{Y}_{k-1}) = \int P(\boldsymbol{x}_{k-1}|\boldsymbol{Y}_{k-1}) f(\boldsymbol{x}_k|\boldsymbol{x}_{k-1}) d\boldsymbol{x}_{k-1}. \tag{10}$$

In order to compute the non-Gaussian posterior distribution given by Eq. (9) jointly based on point and set-valued measurements, we develop the EBPF which approximates the filtering distribution $P(\boldsymbol{x}_k|\boldsymbol{Y}_k)$ using a set of samples (particles) $\{\mathbb{X}_k^i\}_{i=1}^{N_s}$ derived from a proposal distribution $q(\boldsymbol{x}_k|\boldsymbol{Y}_k)$ with normalized weights $W_k^i = \frac{P(\mathbb{X}_k^i|\boldsymbol{Y}_k)}{q(\mathbb{X}_k^i|\boldsymbol{Y}_k)}$ associated with the vector particles. Note that N_s denotes the number of particles used by the filter. The EBPF implements the filtering recursions by propagating the particles $\mathbb{X}_k^i$ and associated weights W_k^i, $(1 \leq i \leq N_s)$, as follows

$$\mathbb{X}_k^i \sim q(\mathbb{X}_k^i|\mathbb{X}_{k-1}^i, \boldsymbol{Y}_{k-1}) \tag{11}$$

$$W_k^i \propto W_{k-1}^i \frac{P(\boldsymbol{y}_k|\mathbb{X}_k^i)P(\mathbb{X}_k^i|\mathbb{X}_{k-1}^i)}{q(\mathbb{X}_k^i|\mathbb{X}_{k-1}^i, \boldsymbol{Y}_k)}. \tag{12}$$

Consequently, the EBPF computes a particle-based approximation of the conditional posterior $p(\boldsymbol{x}_k|\boldsymbol{Y}_k)$ as

$$p(\boldsymbol{x}_k|\boldsymbol{Y}_k) = \sum_{i=1}^{N_s} W_k^{(i)} \delta\big(\boldsymbol{x}_k - \mathbb{X}_k^{(i)}\big), \tag{13}$$

The minimum mean square error (MMSE) estimate at iteration $(k \geq 1)$ is defined as the expected value of the posterior distribution $p(\boldsymbol{x}_k|\boldsymbol{Y}_k)$, i.e., $\hat{\boldsymbol{x}}_{k|k} \triangleq \mathbb{E}\{\boldsymbol{x}_k|\boldsymbol{Y}_k\}$, and $\boldsymbol{P}_{k|k} \triangleq \mathbb{E}\{(\boldsymbol{x}_k - \hat{\boldsymbol{x}}_{k|k})(\boldsymbol{x}_k - \hat{\boldsymbol{x}}_{k|k})^T|\boldsymbol{Y}_k\}$. The EBPF computes the state estimate and its associated covariance matrix based on the particles as follows

$$\hat{\boldsymbol{x}}_{k|k} = \frac{1}{N_s} \sum_{i=1}^{N_s} \mathbb{X}_k^{(i)} \tag{14}$$

$$\text{and } \boldsymbol{P}_{k|k} = \frac{1}{N_s} \sum_{i=1}^{N_s} \big(\mathbb{X}_k^{(i)} - \hat{\boldsymbol{x}}_{k|k}\big)\big(\mathbb{X}_k^{(i)} - \hat{\boldsymbol{x}}_{k|k}\big)^T. \tag{15}$$

The required terms for computing Eqs. (11)-(15) at each iteration is the particle set $\{\mathbb{X}_k^{(i)}, W_k^{(i)}\}$ for which we need to define the proposal distribution and form $P(\boldsymbol{y}_k|\mathbb{X}_k^i)$ to compute the weight equation. The EBPF generates N_s random particles from the transitional density, i.e., $\mathbb{X}_k^{(i)} \sim P(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})$ which is considered as the conventional choice for the proposal distribution $(q(\mathbb{X}_k^i|\mathbb{X}_{k-1}^i, \boldsymbol{Y}_k))$. Choice of the transitional density as the proposal results in the weight update equation (Eq. (12)) to become

$$W_k^i \propto W_{k-1}^i P(\boldsymbol{y}_k|\mathbb{X}_k^i). \tag{16}$$

The second step to implement EBPF is to evaluate the weight update equation which depends on whether or not the current sensor measurement has been communicated.

(i) Update based on Set-valued Measurements ($\gamma_k = 0$): In the absence of the sensor measurement and based on the triggering mechanism defined in Eq. (8), the estimator has the following side information

$$\boldsymbol{z}_k \in (\boldsymbol{z}_{\tau_k} - \Delta, \boldsymbol{z}_{\tau_k} + \Delta), \tag{17}$$

where $\boldsymbol{z}_{\tau_k}$ is the previously communicated observation. In this case, the likelihood function can be specified as follows

$$P\big(\boldsymbol{y}_k|\boldsymbol{x}_k, \gamma_k = 0\big) = P\big(\boldsymbol{z}_{\tau_k} - \Delta \leq \boldsymbol{z}_k \leq \boldsymbol{z}_{\tau_k} + \Delta\big), \tag{18}$$

which by substituting for $\boldsymbol{z}_k$ from Eq. (2), we have

$$\begin{aligned} &P\big(\boldsymbol{y}_k|\boldsymbol{x}_k, \gamma_k = 0\big) \\ &\quad = P\big(\boldsymbol{z}_{\tau_k} - \Delta \leq \boldsymbol{H}_k\boldsymbol{x}_k + \boldsymbol{v}_k \leq \boldsymbol{z}_{\tau_k} + \Delta\big) \\ &\quad = P\big([\boldsymbol{z}_{\tau_k} - \Delta - \boldsymbol{H}_k\boldsymbol{x}_k] \leq \boldsymbol{v}_k \leq [\boldsymbol{z}_{\tau_k} + \Delta - \boldsymbol{H}_k\boldsymbol{x}_k]\big). \end{aligned} \tag{19}$$

Note that in the third line of Eq. (19), we kept the noise in the middle and moved other terms to the sides in order to be able to compute the likelihood function based on the probability distribution of the noise. As the observation noise $\boldsymbol{v}_k$ has a zero-mean Gaussian distribution with variance $\boldsymbol{R}_k$, i.e., $\boldsymbol{z}_k \sim \mathcal{N}(0, \boldsymbol{R}_k)$, the likelihood function $P(\boldsymbol{y}_k|\boldsymbol{x}_k, \gamma_k = 0)$ is given

$$\begin{aligned} &P(\boldsymbol{y}_k|\boldsymbol{x}_k, \gamma_k = 0) \\ &\quad = \frac{1}{\sqrt{2\pi R_k}} \int_{\boldsymbol{z}_\tau - \Delta - \boldsymbol{H}_k\boldsymbol{x}_k}^{\boldsymbol{z}_\tau + \Delta - \boldsymbol{H}_k\boldsymbol{x}_k} \exp\left\{\frac{-t}{2R_k}\right\} dt \\ &\quad = \underbrace{\Phi\left(\frac{\boldsymbol{z}_\tau + \Delta - \boldsymbol{H}_k\boldsymbol{x}_k}{\sqrt{R_k}}\right) - \Phi\left(\frac{\boldsymbol{z}_\tau - \Delta - \boldsymbol{H}_k\boldsymbol{x}_k}{\sqrt{R_k}}\right)}_{\boldsymbol{h}(\boldsymbol{x}_k)}, \end{aligned} \tag{20}$$

where $\Phi(\cdot)$ is the cumulative Gaussian distribution with zero mean and variance 1 as follows

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\big(-\frac{t^2}{2}\big) dt. \tag{21}$$

ISBN 978-0-9928626-7-1

Algorithm 1 EBPF IMPLEMENTATION

Input: $\{\mathbb{X}_{k-1}^{(i)}, W_{k-1}^{(i)}\}_{i=1}^{N_s}$, γ_k, and $\boldsymbol{y}_k$.
Output: $\{\mathbb{X}_{k}^{(i)}, W_{k}^{(i)}\}_{i=1}^{N_s}$, $\hat{\boldsymbol{x}}_{k|k}$ and $\boldsymbol{P}_{k|k}$.

At iteration k, EBPF updates its particle set as follows:

S1. *Predictive Particle Generation*: Sample *predicted particle* from the proposal distribution i.e., $\mathbb{X}_k^{(i)} \sim P(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})$.

S2. *Hybrid Likelihood Computation*:
- **If** $\gamma_k = 0$: Compute $P(\boldsymbol{y}_k|\mathbb{X}_k^{(i)})$ using Eq. (20).
- **If** $\gamma_k = 1$: Compute $P(\boldsymbol{y}_k|\mathbb{X}_k^{(i)})$ using Eq. (22).

S3. *Weight Update*: Compute the weights associated with $\mathbb{X}_k^{(i)}$ using Eq. (16).

S4. *State Estimates*: Approximate the state estimate and its corresponding error covariance $\boldsymbol{P}_{k|k}$ from $\{\mathbb{X}_{k}^{(i)}, W_{k}^{(i)}\}_{i=1}^{N_s}$ using Eqs. (14)-(15).

S5. *Resampling*: In case of degeneracy, particles using the replacement approach [32].

This completes the computation of the likelihood function in idle scenarios (no transmission).

(ii) Update based on Point Measurements ($\gamma_k = 1$): In this case, the estimator receives the sensor measurement $\boldsymbol{z}_k$, therefore, the hybrid likelihood function $P(\boldsymbol{y}_k|\boldsymbol{x}_k)$ reduces to the sensor likelihood function $P(\boldsymbol{z}_k|\boldsymbol{x}_k)$. Consequently, the hybrid likelihood function is given by

$$P(\boldsymbol{y}_k|\boldsymbol{x}_k, \gamma_k = 1) = P(\boldsymbol{z}_k|\boldsymbol{x}_k) = \Phi\left(\frac{\boldsymbol{z}_k - \boldsymbol{H}_k\boldsymbol{x}_k}{\sqrt{R_k}}\right). \quad (22)$$

This complete the presentation of the proposed EBPF. Algorithm 1 outlines the steps of the EBPF implementation.

IV. SIMULATIONS

In this section, simulation experiments are developed to evaluate the performance of the proposed EBPF. Following the recent literature on event-based estimation [5], a target tracking problem is considered where observations from a sensor are used to sequentially estimate the state of the target denoted by $\boldsymbol{x}_k$ consisting of its position and speed. Target's dynamic is given by $\boldsymbol{x}_k = \begin{bmatrix} 0.8 & 1 \\ 0 & 0.95 \end{bmatrix} \boldsymbol{x}_{k-1} + \boldsymbol{w}_k$, where $\boldsymbol{w}_k \sim \mathcal{N}\left(\boldsymbol{0}, \boldsymbol{Q} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}\right)$. The sensor periodically measures the position and speed of the target based on the following observation model $\boldsymbol{z}_k = \begin{bmatrix} 0.7 & 0.6 \end{bmatrix} \boldsymbol{x}_k + v_k$. In this experiment, the observation noise variance is $\sigma_v^2 = 0.01$. The following results are computed over Monte-Carlo (MC) simulations of 1000 runs. The object's position and speed used in each simulation run changes randomly to provide a fair experimental benchmark. Furthermore, the following four estimators are implemented and compared for accuracy: (i) The full-rate estimation based on KF where the sensor communicates its observation to the remote estimator every iteration; (ii) The full-rate estimation based on particle filter; (iii) Open-loop and event-based KF, where SOD triggering is used, and; (iii) The proposed open-loop and event-based estimation algorithm developed in Section III, where the triggering decisions at the sensor level are made based on SOD mechanism and the fusion is performed by jointly incorporating set-valued and point-valued measurements based on the proposed EBPF.

Fig. 2 illustrates the estimated mean-square errors (MSE)

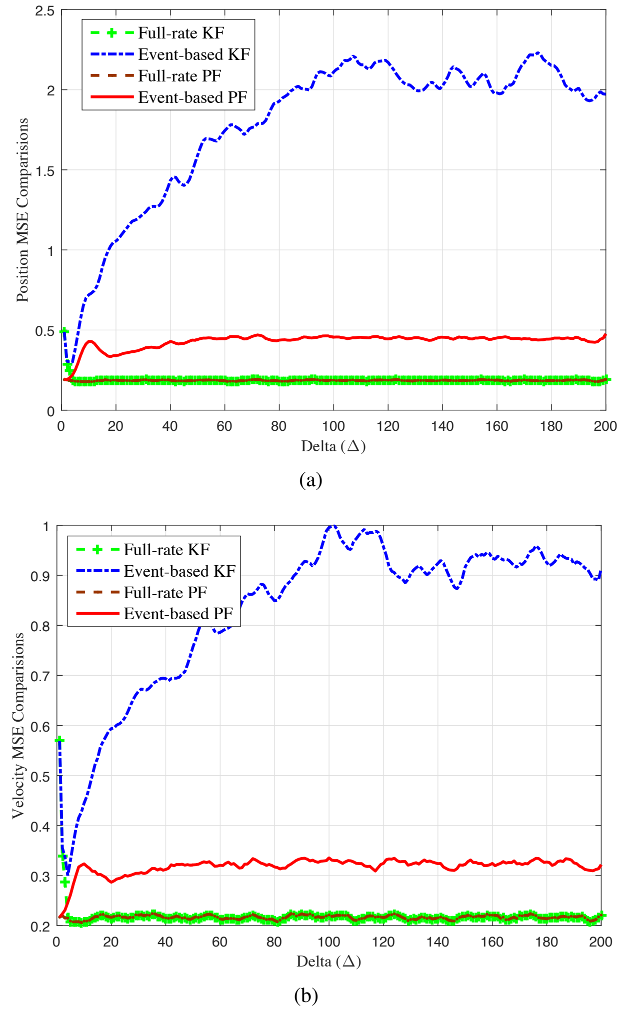

Fig. 2. The MSE comparison when $\Delta = 1.2$. (a) Position MSE. (b) Velocity MSE.

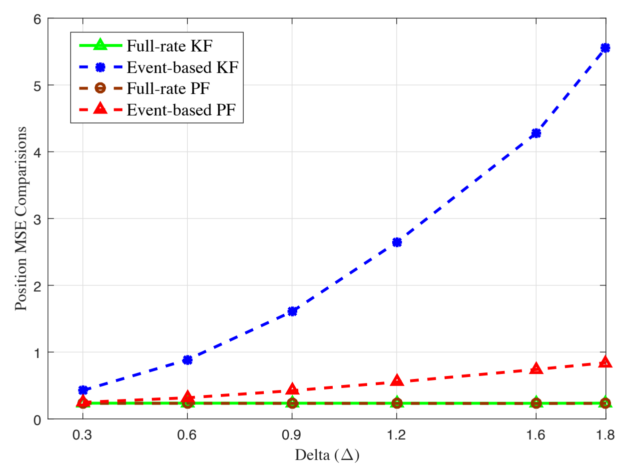

Fig. 3. Position MSE comparison over different values of Δ.

ISBN 978-0-9928626-7-1 © EURASIP 2017

obtained from the four implemented filters. In this experiment, the value of Δ is set equal to 1.2. In this low communication rate scenario, it is observed that the proposed EBPF algorithm provides acceptable results and closely follows its full-rate counterparts and shows significant improvements in comparison to its KF-based counterpart. Fig. 3, shows the position MSE plots over varying values of Δ which in turn results in varying values of the communication rate. It is observed that the proposed EBPF algorithm provides acceptable results in very low communication rates (high values of Δ) and closely follows its full-rate counterparts in high communication rates. Besides, when the communication rate increases (i.e., small values for Δ), the proposed event-based methodology approaches the full-rate estimator. Finally, it is observed that the proposed EBPF provides significantly superior results in comparison to its KF-based counterpart.

V. Conclusion

In this paper we proposed an event-based particle filter (EBPF) framework for distributed state estimation in systems with communication/power constraints at the sensor side. An event-based and open-loop estimation architecture (i.e., no feedback communication is incorporated from the FC to local sensors) is considered. Local sensor uses practical send-on-delta (SOD) event triggering mechanism resulting in availability of side information at the fusion Centre (FC) in the absence of an observation. Utilization of this side information results in estimation with joint set-valued and point-valued measurements which consequently translates in to a non-Gaussian state estimation problem. The proposed EBPF is a systematic and intuitively pleasing non-Gaussian estimation algorithm which jointly incorporates point and set-valued measurements within the particle filter framework by capitalizing on the fact that particle filters only require new measurements to evaluate the likelihood function during the weight update step. In presence of an observation (point-valued measurement), the likelihood function can exactly be evaluated for each particle. In the absence of an observation, the likelihood becomes the probability that the observation belongs to the triggering set which is derived in the paper to utilize set-valued measurements in the proposed EBPF framework. The simulation results depicts that the proposed EBPF outperforms its counterparts specifically in low communication rates, and also confirms the effectiveness of the proposed nonlinear estimation based on joint point and set-valued measurements.

References

[1] R.E. Kalman, "A New Approach to Linear Filtering and Prediction Problems," *J. Basic. Eng.*, vol. 82, no. 1, pp. 35-45, 1960.

[2] S.K. Khaitan and J.D. McCalley, "Design Techniques and Applications of Cyberphysical Systems: A Survey," *IEEE Systems Journal*, vol. 9, no. 2, pp. 350-365, June 2015.

[3] J.P. Hespanha, P. Naghshtabrizi, Y. Xu, "A Survey of Recent Results in Networked Control Systems," *Proc. IEEE*, vol. 95, pp. 138-162,2007.

[4] L. Canzian and M. van der Schaar, "Timely Event Detection by Networked Learners," *IEEE Trans. Signal Process.*, vol. 63, no. 5, pp. 1282-1296, March, 2015.

[5] D. Han, K. You, L. Xie, J. Wu, L. Shi, "Optimal Parameter Estimation Under Controlled Communication Over Sensor Networks," *IEEE Trans. Signal Process.*, vol. 63, no. 24, pp. 6473-6485, Dec. 2015.

[6] K. You and L. Xie, "Kalman Filtering with Scheduled Measurements," *IEEE Trans. Signal Process.*, vol. 61, no. 6, pp. 1520-1530, March 2013.

[7] D. Shi, T. Chen, and L. Shi, "An Event-Triggered Approach to State Estimation with Multiple point- and set-valued Measurements," *Automatica*, vol. 50, no. 6, pp. 16411648, 2014.

[8] D. Shi, T. Chen and L. Shi, "On Set-Valued Kalman Filtering and Its Application to Event-Based State Estimation," *IEEE Transactions on Automatic Control*, vol. 60, no. 5, pp. 1275-1290, May 2015.

[9] J. Sijs, M. Lazar, "Event based State Estimation with Time Synchronous Updates," *IEEE Trans. Autom. Control*, vol. 57, pp. 2650-2655, 2012.

[10] M.G. Cea, G.C. Goodwin, "Event based Sampling in Nonlinear Filtering", *Control Eng. Practice*, vol. 20, no. 10, pp. 963-971, 2012.

[11] N. Sadeghzadeh-Nokhodberiz, M. Davoodi and N. Meskin, "Stochastic Event-Triggered Particle Filtering for State Estimation," *Int. Conf. Event-based Control, Comm., & Sig. Process.*, Krakow, 2016, pp. 1-4.

[12] M.A. Sid and S. Chitraganti, "Nonlinear Event-based State Estimation using Particle Filtering Approach," *Int. Conf. Model., Ident. & Control*, Algiers, 2016, pp. 874-879.

[13] C. Yang and L. Shi, "Deterministic Sensor Data Scheduling under Limited Communication Resource," *IEEE Trans. Signal Process.*, vol. 59, no. 10, pp. 5050-5056, 2011.

[14] L. Shi, H. Zhang, "Scheduling Two Gauss-Markov Systems: An Optimal Solution for Remote State Estimation under Bandwidth Constraint," *IEEE Trans. Signal Proc.*, vol. 60, no. 4, pp. 2038-2042, 2012.

[15] J. Wu, Q. Jia, K. Johansson, L. Shi, "Event-based Sensor Data Scheduling: Trade-off Between Communication Rate and Estimation Quality," *IEEE Trans. Autom. Control*, vol. 58, no. 4, pp. 1041-1045, 2013.

[16] D. Shi, T. Chen, L. Shi, "Event-triggered Maximum Likelihood State Estimation," *Automatica*, vol. 50, no. 1, pp. 247-254, 2014.

[17] S. Trimpe and R. D'Andrea, "Event-Based State Estimation with Variance-Based Triggering," *IEEE Trans. Automatic Control*, vol. 59, no. 12, pp. 3266,-281, Dec. 2014.

[18] X. Yonggang and J.P. Hespanha, "Estimation under Uncontrolled and Controlled Communications in Networked Control Systems," *IEEE Conf. on Decision and Control*, pp. 842-847, 2005.

[19] D. Han, Y. Mo, J. Wu, S. Weerakkody, B. Sinopoli, L. Shi, "Stochastic Event-Triggered Sensor Schedule for Remote State Estimation," *IEEE Trans. Automatic Control*, vol. 60, no. 10, pp. 2661-2675, Oct. 2015.

[20] J.W. Marck, J. Sijs, "Relevant Sampling Applied to Event-Based State-Estimation," *Int. Conf. Sensor Technol. & Appl.*, pp. 618-624, 2010.

[21] K. J. Astrom and B. M. Bernhardsson, "Comparison of Riemann and Lebesgue Sampling for First Order Stochastic Systems," *IEEE Conf. on Decision and Control*, vol. 2, pp. 2011-2016, 2002.

[22] M. Miskowicz, "Send-on-delta Concept: An Event-based Data Reporting Strategy," *Sensors*, vol. 6, no. 1, pp. 49-63, 2006.

[23] V.H. Nguyen, Y.S. Suh, "Improving Estimation Performance in Networked Control Systems Applying the Send-on-Delta Transmission Method," *Sensors*, vol. 7, no. 10, pp. 2128-2138, 2007.

[24] J. Sijs, B. Noack, and U. Hanebeck, "Event-based State Estimation with Negative Information," *16th International Conference on Information Fusion*, July 2013, pp. 2192-2199.

[25] A. Mohammadi and A. Asif, "Distributed Particle Fillter Implementation with Intermittent/Irregular Consensus Convergence," *IEEE Trans. Signal Process.*, vol. 61, pp. 2572-2587, 2013.

[26] A. Mohammadi, A. Asif, "Distributed Consensus + Innovation Particle Filtering for Bearing/Range Tracking With Communication Constraints," *IEEE Trans. Signal Process.*, vol. 63, no. 3, pp. 620-635,2015.

[27] A. Mohammadi, A. Asif, "Consensus-based Distributed Dynamic Sensor Selection in Decentralised Sensor Networks using the Posterior Cramer-Rao Lower Bound," *Signal Processing*, vol. 108, 2015, pp. 558-575.

[28] X. Zhong, A. Mohammadi, A.B. Premkumar, and A. Asif, "A distributed particle filtering approach for multiple acoustic source tracking using an acoustic vector sensor network," *Signal Processing*, vol. 108, March 2015, pp. 589-603.

[29] Y.H. Ruan, P. Willett, and A. Marrs, "Fusion of quantized measurements via particle filtering," *Proceedings of the 2003 Aerospace Conference*, pp. 1967-1978, 2003.

[30] R. Karlsson and F. Gustafsson, "Particle Filtering for Quantized Sensor Information," *European Signal Processing Conference*, 2005.

[31] A. Mohammadi, A. Asif, X. Zhong and A. B. Premkumar, "Decentralized Bayesian Estimation with Quantized Observations: Theoretical Performance Bounds," *IEEE International Conference on Distributed Computing in Sensor Systems*, Cambridge, MA, 2013, pp. 149-156.

[32] T. Li, M. Bolic and P. M. Djuric, "Resampling Methods for Particle Filtering: Classification, implementation, and strategies," *IEEE Signal Processing Magazine*, vol. 32, no. 3, pp. 70-86, May 2015.

ISBN 978-0-9928626-7-1 © EURASIP 2017