Performance Improvement for Wideband Beamforming with White Noise Reduction Based on Sparse Arrays

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***Abstract*—A method is proposed for reducing the effect of white noise in wideband sparse arrays via a combination of a judiciously designed transformation followed by highpass filters. The reduced noise level leads to a higher signal to noise ratio for the system, which can have a significant effect on the performance of various beamforming methods. As a representative example, the reference signal based (RSB) and the Linearly Constrained Minimum Variance (LCMV) beamformers are employed here to demonstrate the improved beamforming performance, as confirmed by simulation results.**

I. Introduction

The area of wideband beamforming has been in the focus of research with various applications such as radar, sonar and wireless communications for many years [1], [2], [3]. The spacing between adjacent sensors for uniform linear arrays (ULAs) must be half the wavelength of the highest frequency of the desired signal for preventing the spatial aliasing problem. Considering arrays with a large aperture size, the cost of a large number of required sensors can be problematic. Sparse arrays are a good alternative [1], [4], [5], as they allow adjacent sensor spacings to be greater than half the wavelength of the corresponding desired signal with the highest frequency, while avoiding grating lobes, since the sensor locations have a non-uniform structure. In addition, an optimum beam response can be achieved, since sparse arrays provide more degrees of freedom with the same number of sensors.

Generally, the performance of all wideband beamforming algorithms for both ULAs and sparse arrays is affected by the amount of white noise, so a better performance can be achieved by reducing the noise level in the system. In most cases, noise in a wideband array is temporally and spatially white, therefore, the noise between the sensors is uncorrelated with each other. After processing designed for the signal part, there will be some level of noise left, which we can not do much about it.

In our previous work [6], a method was developed for reducing the effect of white noise in wideband ULAs via a combination of a judiciously designed transformation followed by highpass filters to improve the performance for wideband direction of arrival (DOA) estimation. In this paper, we extend that idea to sparse arrays and as a result, the transformation is re-designed using the least squares method to adjust the noise reduction method for the non-uniform sensor layout of sparse arrays.

To make sure the transformation is invertible, a prototype filter is first designed and then modulated to different subbands to cover the full normalised frequency band from $-\pi$ to π. The diagonal loading method is used to keep the condition number to a low level [7]. Similar to the ULA case, the overall signal to noise ratio (SNR) of the system can be improved by up to 3dB, which then leads to performance enhancement for beamforming as demonstrated using two well-known adaptive beamformers, namely the reference signal based (RSB) [8], [9], [10], [11], and the linearly constrained minimum variance (LCMV) beamformers [3], [12].

This paper is organised as follows. In Sec. II, the proposed white noise reduction method for sparse arrays is introduced. In Sec. III, the least squares approach for designing the transformation matrix is explained. Simulation results are presented

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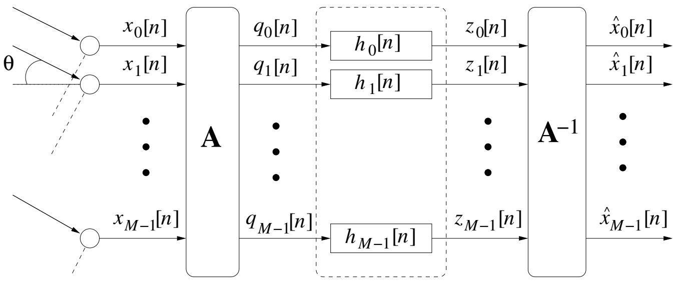

Fig. 1: The general structure of the proposed noise reduction approach for sparse arrays.

in Sec. IV, followed by conclusions in Sec. V.

II. The Proposed White Noise Reduction Method for Sparse Arrays

The general structure of the proposed white noise reduction approach for sparse arrays is presented in Fig. 1. M array signals $x_m[n]$, $m = 0, \ldots, M-1$, are received by the sensors, which are then transformed by an $M \times M$ transformation matrix **A**, in the next stage, its outputs $q_m[n]$, $m = 0, \ldots, M-1$, are processed by highpass filters with $h_m[n]$, $m = 0, \ldots, M-1$, as their impulse responses. The outputs of the highpass filters are denoted by $z_m[n]$, $m = 0, \ldots, M-1$. Finally $z_m[n]$, $m = 0, \ldots, M-1$, are block-transformed by $\mathbf{A}^{-1}$.

There are two components for the received array signal $x_m[n]$ at the m-th sensor: the signal part $s_m[n]$ and the white noise part $\bar{n}_m[n]$. Therefore,

$$x_m[n] = s_m[n] + \bar{n}_m[n]. \tag{1}$$

The total signal vector $\mathbf{x}[n]$ is

$$\mathbf{x}[n] = \mathbf{s}[n] + \bar{\mathbf{n}}[n], \tag{2}$$

where

$$\mathbf{x}[n] = [x_0[n], x_1[n], \cdots, x_{M-1}[n]]^T,$$

$$\mathbf{s}[n] = [s_0[n], s_1[n], \cdots, s_{M-1}[n]]^T,$$

$$\bar{\mathbf{n}}[n] = [\bar{n}_0[n], \bar{n}_1[n], \cdots, \bar{n}_{M-1}[n]]^T.$$

By applying the transformation matrix **A** to the received signal vector $\mathbf{x}[n]$, the output signal vector $\mathbf{q}[n]$ is processed as

$$\mathbf{q}[n] = \mathbf{A}\mathbf{x}[n], \tag{3}$$

where $\mathbf{q}[n] = [q_0[n], \cdots, q_{M-1}[n]]^T$.

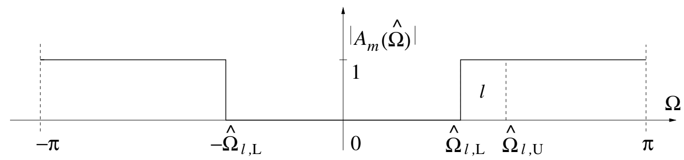

Fig. 2: The ideal frequency response of a sample row vector of **A**, and its corresponding highpass filter.

The element of **A** at the m-th row and l-th column is denoted by $a_{m,l}$, i.e., $[\mathbf{A}]_{m,l} = a_{m,l}$. Each row vector of **A** acts as a beamformer, and the output $q_m[n]$ is

$$q_m[n] = \sum_{l=0}^{M-1} a_{m,l} x_l[n]. \tag{4}$$

The beam response $R_m(\Omega, \theta)$ of this simple beamformer where Ω and θ are the normalized frequency and the DOA angle respectively, is given by

$$R_m(\Omega, \theta) = \sum_{l=0}^{M-1} a_{m,l} e^{-j \frac{d_l}{cT_s}(\Omega \sin\theta)} = A_m(\Omega \sin\theta), \tag{5}$$

where d_l is the spacing between the zero-th sensor and the l-th sensor (where d_0=0), $\Omega = \omega T_s$, c is the wave propagation speed, T_s is the sampling period, $j = \sqrt{-1}$ and ω is the angular frequency of the signals.

With $\hat{\Omega} = \Omega \sin\theta$, we have

$$A_m(\hat{\Omega}) = \sum_{l=0}^{M-1} a_{m,l} e^{-j \frac{d_l}{cT_s} \hat{\Omega}}, \tag{6}$$

where $A_m(\hat{\Omega})$ is the frequency response of the m-th row vector of the transformation matrix **A**, considering each row vector as the impulse response of a finite impulse response (FIR) filter. Similar to [13], the frequency responses $A_m(\hat{\Omega})$, $m = 0, 1, \cdots, M-1$, are set to have bandpass characteristics, each with bandwidth of $2\pi/M$. The row vectors of **A** all effectively cover $[-\pi; \pi]$ which is the entire frequency band.

The bandpass filters, which are used as row vectors of **A**, have highpass filtering behaviour, considering the whole range of θ for the received signal. As an example, the frequency response of the l-th row vector is given by

$$\left| A_l(\hat{\Omega}) \right| = \begin{cases} 1, & \text{for } \hat{\Omega} \in [\hat{\Omega}_{l,L}; \hat{\Omega}_{l,U}] \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

Since $\hat{\Omega} = \Omega \sin\theta$, for $\hat{\Omega}_{l,L} > 0$, the frequency range of the output of the l-th row vector is $|\Omega| \geq \hat{\Omega}_{l,L}$, with the lower bound equal to $\hat{\Omega}_{l,L}$. So, the frequency components of the received signal with $\Omega \in [-\hat{\Omega}_{l,L}; \hat{\Omega}_{l,L}]$, do not pass through

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the l-th row vector, regardless of the DOA angle θ. Similarly, when $\hat{\Omega}_{l,L} < \hat{\Omega}_{l,U} < 0$, the lower bound is equal to $|\hat{\Omega}_{l,U}|$ and the frequency components with $\Omega \in [-|\hat{\Omega}_{l,U}|; |\hat{\Omega}_{l,U}|]$ do not pass through the corresponding row vector.

So, the l-th row vector has a highpass filtering effect on the spectrum of the directional signal part of its output $q_l[n]$ as shown in Fig. 2. As mentioned before, the noise component at the array sensors is spatially white, therefore, the spectrum of the noise at the output of the row vectors is constant, and covers the entire spectrum. Assume that the row vectors of $\mathbf{A}$ are normalized to unity norm, then the total power of the noise before and after the transformation $\mathbf{A}$ would be the same.

As illustrated in Fig. 1, every highass filter $h_l[n], l = 0, \cdots, M-1$, processes its corresponding input $q_l[n], l = 0, \cdots, M-1$. The highpass filters $h_l[n], l = 0, \cdots, M-1$, have the same highpass frequency responses as their corresponding row vectors of the transformation $\mathbf{A}$ and are designed to cover the entire bandwidth of the directional signal. So ideally, the directional signal part should not experience any distortion after being processed by the highpass filters and the highpass filters will not remove any part of the directional signal. In contrast, the highpass filters remove parts of the white noise with frequency components matching their stopband. $\mathbf{z}[n] = [z_0[n], \cdots, z_{M-1}[n]]^T$ denotes the output of the aforementioned highpass filters.

By processing $\mathbf{z}[n]$ with the inverse transformation $\mathbf{A}^{-1}$, the original array signal will be recovered without distortion in the ideal case, while the noise power will be reduced, leading to an improved overall SNR. Following the same analysis in the ULA case as discussed in [6], when $\mathbf{A}$ is unitary, we can draw the same conclusion that up to 3dB total SNR improvement can be obtained by the proposed method. However, in practice, the SNR improvement will be less than 3dB due to limited number of sensors and difficulty in designing a unitary transformation matrix with the required bandpass responses.

The transformation matrix $\mathbf{A}$ is required to be unitary to make sure that the row vectors of both $\mathbf{A}$ and $\mathbf{A}^{-1}$ have unity norm and preserve the signal power after transforming the signal and also after transforming it back. If $\mathbf{A}$ is not unitary, then the noise might be amplified by some significant amount during the process even if some of it has been removed, this subsequently leads to a reduced output SNR. Moreover, a unitary matrix automatically ensures that $\mathbf{A}$ is of full rank.

III. Least Squares Based Design for the Transformation Matrix

As an example for a unitary matrix with a satisfactory bandpass response, we could consider the discrete Fourier transform (DFT) matrix as in the ULA case of [6]. However, it is not really applicable here since the sparse array does not have a uniform spacing and the resultant beams by each row vector of such a transformation matrix will be significantly distorted.

Therefore, we have to adopt a different approach for the design of the transformation matrix for sparse arrays and introduce a least squares based design method here. The idea is to use an ideal unitary beam response such as those of a DFT matrix as the reference response for the least squares method to design a sparse prototype filter $\mathbf{p}$ (where $[\mathbf{p}]_l = p_l, l = 0, \cdots, M-1$). Then, we modulate it into different subbands in a uniform way to form the required transformation matrix.

The least squares filter design method has been well studied in the past [14], [15]. Given the desired beam pattern $P_d(\hat{\Omega})$ and considering $\mathbf{d}(\hat{\Omega})$ as the steering vector of the sparse array with

$$\mathbf{d}(\hat{\Omega}) = \left[1, \quad e^{-j\frac{d_1}{cT_s}\hat{\Omega}}, \quad \cdots, \quad e^{-j\frac{d_{M-1}}{cT_s}\hat{\Omega}}\right]^T, \tag{8}$$

the problem can be solved by minimizing the sum of the squares of the error between the designed response $P(\hat{\Omega})$ and $P_d(\hat{\Omega})$ over the desired frequency range, i.e.,

$$\min_{\mathbf{p}} \sum_{\hat{\Omega}_{pb}} |P(\hat{\Omega}) - P_d(\hat{\Omega})|^2. \tag{9}$$

The standard least squares solution is achieved by minimizing the above cost function with respect to the coefficients vector $\mathbf{p}$, so

$$\mathbf{p}_{opt} = \mathbf{G}_{ls}^{-1}\mathbf{g}_{ls}, \tag{10}$$

where

$$\mathbf{G}_{ls} = \sum_{\hat{\Omega}_{pb}} \mathbf{d}(\hat{\Omega})\mathbf{d}^H(\hat{\Omega}),$$

$$\mathbf{g}_{ls} = \sum_{\hat{\Omega}_{pb}} (\mathbf{d}_R(\hat{\Omega})P_{d,R}(\hat{\Omega}) + \mathbf{d}_I(\hat{\Omega})P_{d,I}(\hat{\Omega})),$$

with $\{\cdot\}^H$ denoting the Hermitian transpose, $\mathbf{d}_R(\hat{\Omega})$ and $P_{d,R}(\hat{\Omega})$ are the real parts of $\mathbf{d}(\hat{\Omega})$ and $P_d(\hat{\Omega})$, $\mathbf{d}_I(\hat{\Omega})$ and $P_{d,I}(\hat{\Omega})$ are their imaginary parts and $\hat{\Omega}_{pb}$ is the passband of the prototype filter $\mathbf{p}$.

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Then, we modulate $\mathbf{p}$ to cover the whole normalized frequency band [16],

$$A_{m,l} = e^{-j\frac{2\pi}{M}m\frac{d_l}{cT_s}} p_l, \quad (11)$$

where $m = 0, \cdots, M-1$, $l = 0, \cdots, M-1$.

At this point, if the condition number of the resultant transformation matrix is high, we can reduce it using the diagonal loading method [7],

$$\mathbf{A}_L = \mathbf{A} + \alpha\mathbf{I}\ , \quad (12)$$

where $\mathbf{I}$ denotes an $M \times M$ identity matrix and α is a constant representing a small loading coefficient.

Note that the transformation matrix obtained by the above procedure will not be unitary in general and how to design a unitary matrix with the required bandpass filtering effect is still an open problem for our future study. However, we will see in our simulations that the transformation matrix obtained by the above procedure works well and provides a satisfactory performance improvement.

IV. Simulation Results

In this section, simulation results are provided and compared to verify the effectiveness of the proposed noise reduction preprocessing method for sparse arrays. They are based on a sparse array example provided in [17] and the sensor locations are listed in Table I, where λ is the wavelength corresponding to the normalized frequency of $\Omega = \pi$. It has 15 sensors ($M = 15$) and the desired signal arrives from the broadside ($\theta_d = 0$). The transformation matrix $\mathbf{A}$ is a 15×15 matrix obtained by the design procedure described in Sec. III, and its frequency response is shown in Fig. 3.

n	d_n/λ	n	d_n/λ	n	d_n/λ
1	0	6	4.09	11	6.72
2	0.81	7	4.24	12	7.58
3	1.62	8	5.00	13	8.38
4	2.42	9	5.81	14	9.19
5	3.28	10	5.96	15	10

TABLE I: Sensor locations for the wideband sparse array example.

The received signals are processed by the designed sparse transformation and after that, they are passed through the highpass filters. For highpass filters, 50-tap linear-phase FIR filters with a common delay of 25 samples are employed.

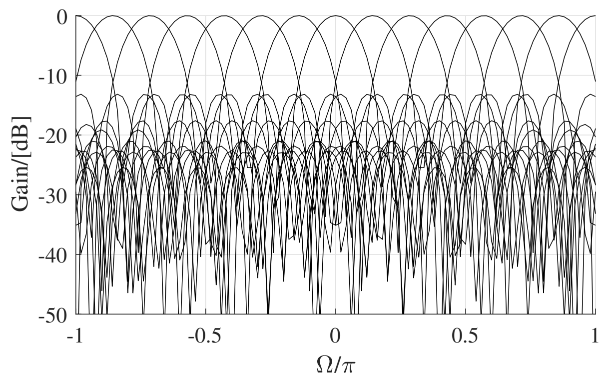

Fig. 3: The frequency response of the row vectors of the 15×15 sparse transformation $\mathbf{A}$.

Then, the signals are transformed back by inverse of the transformation matrix $\mathbf{A}^{-1}$.

Now we examine the effect of the proposed method on the performance of both the RSB beamformer and the LCMV beamformer. A desired bandlimited wideband signal with a bandwidth of $[0.3\pi, \pi]$ is received by the aforementioned sparse array from the broadside. Seven interfering signals are applied to the system, each with a -10dB input SIR and their DOAs are $\theta_i = 10°, 20°, 30°, 40°, 50°, 60°$ and $70°$, respectively. A tapped delay-line (TDL) with length of $J = 100$ is used for these beamformers [3].

The results are shown in Fig. 4, and we can see that a higher output SINR is achieved by the proposed noise reduction method for both beamformers, especially when the input SNR is greater than 0dB and generally the improvement becomes larger when input SNR increases.

However, one issue which cannot be clearly explained is that for an input SNR smaller than 0dB, there is not much improvement. In theory, we should always have a good improvement for all input SNR ranges. We checked the designed transformation matrix, and found that it has a relatively large condition number, which could be the reason for such a lower than expected performance for low SNR. As we mentioned at the end of Sec. III, further research is needed for designing a unitary transformation matrix with the desired frequency responses.

V. Conclusions

A method for mitigating the effect of white noise without affecting the directional signal in wideband sparse arrays has been introduced. With the proposed method, similar to the

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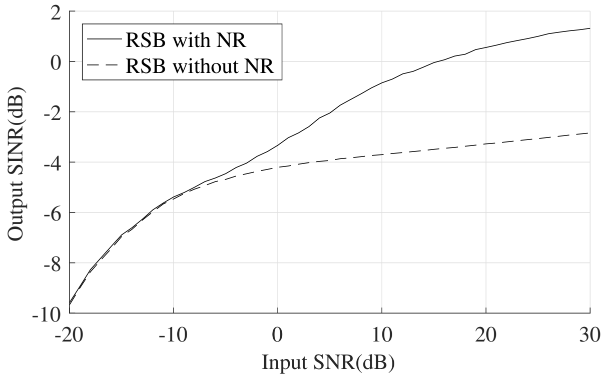
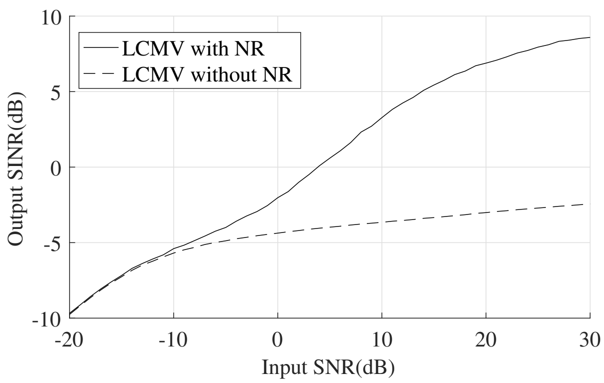


(a) RSB beamformer.



(b) LCMV beamformer.

Fig. 4: SINR performance of both beamformers with and without the proposed noise reduction (NR) method for the sparse array specified in Table I.

uniform linear array case, a maximum 3dB improvement is achieved for the total signal power to noise ratio (TSNR), which leads to possible performance enhancement in many sparse array signal processing applications. As an example, the effect of the method on adaptive beamforming was studied. The simulation results which were achieved by both the RSB and the LCMV beamforming methods, showed a clear improvement in performance, with respect to the output SINR for a large range of input SNR values.

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