Hybrid Digital-Analog Joint Source Channel Coding for Broadcast Multiresolution Communications

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***Abstract*—Multilayer coding represents an appealing solution for the broadcasting of common data when we want to ensure the receivers decode the signal with different distortion levels depending on the quality of the received signal. In addition, the combination of digital and analog techniques to encode the source data allows to solve the limitation of these approaches when they are applied separately. In this work, we consider a Hybrid Digital-Analog (HDA) multilayer system where the digital layers are employed to satisfy different Quality of Service (QoS) requirements and an analog layer refines the estimates of the source symbols. The resulting scheme provides good performance for different expansion factors and number of layers, and it also presents good scaling for all SNR values.**

I. Introduction

Communication systems designed according to the source-channel separation principle are in general suboptimal for broadcast channels. In this case, when the receivers experience a different quality of the received signal, the superimposition of several data layers transmitted with different levels of protection provides better performance [1]. In such multilayer schemes, source coding based on successive refinement is employed to iteratively generate new information layers that successively reduce the distortion and, at the same time, it is possible to reconstruct the source data from any layer for a given distortion target [2].

Different approaches have been considered to decode the source data at the receivers with different resolutions or distortion levels, such as hierarchical modulations and embedded channel codes [3]. Another strategy is multilayer Hybrid Digital-Analog (HDA) where coarse information is encoded using digital schemes to ensure all the receivers are able to decode at least that information, while the quantizer errors are directly transmitted to refine the source estimates in the best receivers. These schemes are particularly helpful in broadcast settings where users at different distances must be able to decode a common message [4].

Multilayer HDA schemes usually send the information corresponding to the analog part using linear encoding or uncoded transmission. Non-linear strategies can alternatively be considered to properly adjust the analog rate to the digital one. Analog Joint Source Channel Coding (JSCC) mappings based on space-filling parametric curves have been shown to achieve near optimal performance in different scenarios [5]–[7], specially for bandwidth compression. In the case of bandwidth expansion, other non-linear mappings such as spherical codes [8], non-differentiable analog mappings [9], orthogonal polynomials [10] or analog codes based on chaotic dynamical systems [11] have been proposed.

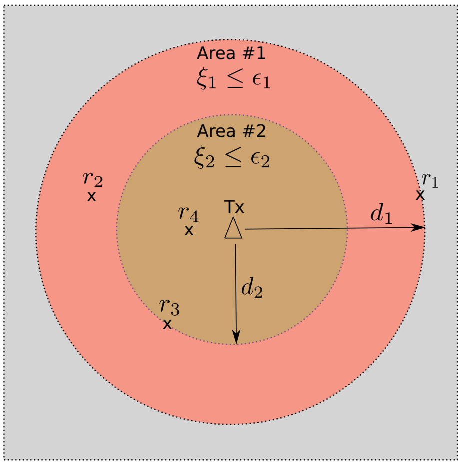

Fig. 1. Example of the broadcast scenario with different two coverage regions.

In this work, we focus on the broadcasting of common messages from one transmitter to several receivers at different distances. A multilayer HDA scheme is considered where the digital layers satisfy different QoS requirements and the last layer encodes the information using some adequate analog JSCC mapping. Compared to [4], the matched tandem codes used to transmit the quantization errors are replaced by analog JSCC mappings.

II. System Model

Let us consider the transmission of a common message from one transmitter to several receivers placed at different distances. We are interested in ensuring that the furthest receivers be able to decode at least part of the message, while the source information is decoded with better quality at locations closer to the transmitter.

Let us divide a coverage area into L circular broadcast regions with their corresponding QoS requirements, i.e.

$$\xi_{\text{rx}} \leq \epsilon_i \qquad \forall\ \text{rx} : d_{\text{rx}} \leq d_i, \qquad \forall\ i = 1, \ldots, L, \qquad (1)$$

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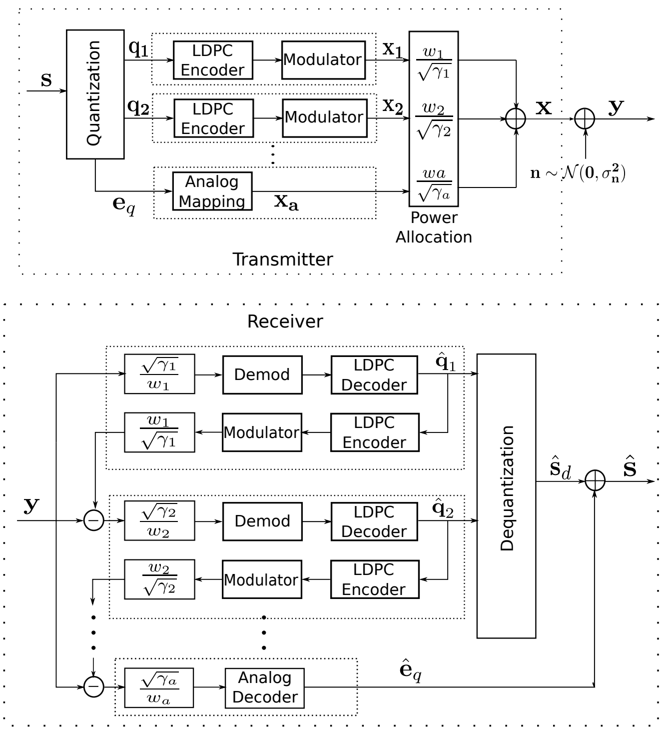

Fig. 2. Block-diagram of the proposed multilayer architecture.

where ϵ_i is the distortion level required for the i-th region, d_i is the distance from the transmitter to the furthest points in the i-th region, ξ_{rx} represents the actual distortion between the source information and the data decoded by the receiver rx, and d_{rx} is the distance between rx and the transmitter. Figure 1 shows an example for $L = 2$ with two circular broadcast regions around the transmitter, and their corresponding distortion requirements.

In this context, a multilayer transmitter with L digital layers is considered to satisfy the distortion requirements in (1). Figure 2 shows the block-diagram of the proposed multilayer system. As observed, a vector of analog source symbols $\mathbf{s}$ is first quantized and then encoded with a particular digital scheme at each layer $i = 1, \ldots, L$. The digital components and the allocated power must conveniently be chosen at each layer i to ensure that the signal distortion for the receivers in the i-th region is lower than ϵ_i. In Section III-A, we will provide the details for the design of the digital layers.

Once the transmit power is properly allocated to the L digital layers, the remaining power can be exploited to improve the signal quality of those receivers closer to the transmitter. In this case, two alternatives can be considered: 1) using one or several digital layers in a way similar to previous layers, or 2) using a purely analog layer where the quantization errors are directly mapped to analog channel symbols. As already commented, we choose the second approach since the utilization of an analog layer avoids the floor effect of the digital schemes, and the signal distortion continuously decreases as the distance to the transmitter lowers.

The symbol vectors corresponding to the $L + 1$ layers are superimposed into a single vector to be transmitted. Thus,

$$\mathbf{x} = \sum_{i=1}^{L} \sqrt{\frac{w_i}{\gamma_i}} \mathbf{x}_i + \sqrt{\frac{w_a}{\gamma_a}} \mathbf{x}_a, \tag{2}$$

where $\mathbf{x}_i$ and $\mathbf{x}_a$ represent the symbol vector of the i-th digital layer and the analog mapped symbols, respectively, and γ_i and γ_a are normalization factors to guarantee the power of the encoded symbols at each layer is equal to 1. Finally, w_i and w_a are the power allocated to the i-th digital layer and to the analog one, respectively.

The resulting vector $\mathbf{x}$ is assumed to be broadcast over an Additive White Gaussian Noise (AWGN) channel. Hence, the received signal can be expressed as $\mathbf{y} = \mathbf{x} + \mathbf{n}$, where $\mathbf{n} \sim \mathcal{N}_{\mathbb{C}}(0, \sigma_n^2)$ represents the AWGN. Without loss of generality, the variance of the channel noise is assumed to be $\sigma_n^2 = 1$. Hence, the Signal-to-Noise Ratio (SNR) at the receivers is directly measured as $\text{SNR}_{\text{rx}} = P_{\text{rx}}$, where P_{rx} is the power of the received signal. Assuming free space propagation, the path loss exponent is $n = 2$ and the received power is given by $P_{\text{rx}} = P_{\text{tx}} d_{\text{rx}}^{-2}$, with P_{tx} the transmit power.

At the receivers, an estimate of the symbols is computed from the received vector $\mathbf{y}$. The different layers are iteratively decoded using a Succesive Interference Cancellation (SIC) strategy. In this case, each receiver starts decoding the first layer. If successful, it subtracts the information corresponding to the first layer from the received signal and proceeds to decode the next layer. It repeats these operations until it is not able to decode a layer. If a receiver successfully decodes the L digital layers, it can exploit the analog data to refine its estimate of the source symbols. In general, the receivers in the l-th region are able to decode only the first l layers, while those which fall into the closest region can also improve their estimation with the analog data of the last layer as long as the received power is above a certain value. Otherwise, the analog estimates are disregarded.

Finally, the distortion between the source and estimated symbols is measured according to the Mean Square Error (MSE) criterion as

$$\xi_{\text{rx}} = \|\mathbf{s} - \hat{\mathbf{s}}_{\text{rx}}\|^2, \tag{3}$$

where $\hat{\mathbf{s}}_{\text{rx}}$ is the vector estimated for the receiver rx.

III. Multilayer Scheme

In the ensuing subsections, we describe the transmit layers and the SIC decoder corresponding to the digital part of the system, and the mapping and the decoder employed in the analog scheme. Finally, the power allocated to each layer is determined depending on the number of layers, the quality requirements and the available transmit power.

A. Digital Layers

As observed in Figure 2, the digital part of the multilayer transmitter consists of a quantizer and L layers where a channel encoder is concatenated to a digital modulator. The discrete-time continuous-amplitude source symbols are quantized to obtain a discrete representation of the original analog message. The quantization is based on source coding with successive refinement [2]. Thus, the quantizer provides L bit sequences such that the first sequence corresponds to a coarse representation of the input. The next sequences can be employed to refine this representation iteratively. For simplicity, in this work we focus on scalar quantization, but other type of quantizers can be considered instead.

Let $b_1, b_2, \ldots b_L$ be the number of bits at the quantizer output for each layer. The multilayer quantizer can be interpreted as a collection of L scalar quantizers $Q_l^{H(l)}(\cdot)$ with $H(l) = 2^{\sum_{i=1}^{l} b_i}$ the number of quantization levels for the l-th layer. Each quantizer splits the source space into $H(l)$ equal length intervals (or bins), $T_r = (t_{r-1}, t_r], r = 1, \ldots, H(l)$, each one represented by its middle point, v_r. Notice that the bins for the $l+1$-th layer are determined from the bins of the previous layer by splitting each bin into several smaller bins depending on b_{l+1}.

Given a source vector $\mathbf{s} = [s_1, s_2, \ldots, s_N]$, the quantized values for the l-th layer are hence obtained as

$$Q_l^{H(l)}(s_i) = v_r \quad i = 1, \ldots, N, \tag{4}$$

when s_i falls into the r-th bin, i.e., $t_{r-1} < s_i \le t_r$. Next, the binary representation of the source vector is determined by encoding the quantized values to the corresponding b_l-bit words. It is important to assign properly the words to the quantizer bins to guarantee the successive refinement when the received signal is decoded.

The quantization error for the l-th layer is computed as the distortion between the original and quantized symbols, measured according to the MSE metric, i.e.

$$D_l = \mathbb{E}\left[\left|\mathbf{s} - Q_l^{H(l)}(\mathbf{s})\right|^2\right] = \sum_{i=1}^{H(l)} \int_{t_r-1}^{t_r} |s - v_r|^2 p(s)\, ds, \tag{5}$$

where $p(s)$ is the source pdf. Notice that when the encoder-modulator tandem is properly designed to ensure that no errors occur during the transmission, the distortion after decoding the received signal is given by the quantization error in (5). In this situation, the distortion observed by a receiver rx that is able to decode the first l layers is $\xi_{\text{rx}} = D_l$ and, hence, the number of quantization bits for each layer l can be determined from the distortion requirements of each area to satisfy that $D_l \le \epsilon_l$.

Next, the bits of each layer are individually encoded using a channel encoder and a digital modulator. Assuming the l-th channel encoder has rate $R_c(l)$ and the l-th modulator uses $M(l)$ levels, the number of source symbols transmitted per channel use is given by

$$R_l = \frac{R_c(l)\log_2(M)}{b_l}. \tag{6}$$

The parameters of the channel encoders and modulators are chosen in such way that R_l is the same for all the digital layers, and it should be selected to ensure that the distortion requirements are feasible. In general, we focus on scenarios with $R(l) \le 1$, where digital communications clearly outperform analog JSCC techniques. As explained earlier, the observed symbols are decoded at the receivers using the SIC strategy.

B. Analog Layer

As introduced in Section II, an analog layer is incorporated into the multilayer scheme to improve the signal quality of the receivers in the nearest region. In this layer, the quantization errors are mapped using a particular analog JSCC scheme.

Given a source vector $\mathbf{s} = [s_1, s_2, \ldots, s_N]$, the elements of the error vector $\mathbf{e}_q = [e_{q_1}, e_{q_2}, \ldots, e_{q_N}]$ are given by

$$e_{q_i} = s_i - Q^H(s_i) \quad i = 1, \ldots, N, \tag{7}$$

where $H = 2^{\sum_{i=1}^{L} b_i}$ is the total number of bins considering the L digital layers.

In this work, we consider spherical codes based on the exponentially chirped modulation to map the quantization errors. These codes have been shown to provide good performance and good scaling with the SNR for bandwidth expansion [8]. Using this type of spherical codes, the encoded data corresponding to the analog layer is obtained as

$$\mathbf{x}_a = \Delta\Big[\cos(2\pi\mathbf{e_q}), \sin(2\pi\mathbf{e_q}), \cos(2\pi\alpha\mathbf{e_q}), \sin(2\pi\alpha\mathbf{e_q}), \ldots,\ \cos(2\pi\alpha^{\rho/2-1}\mathbf{e_q}), \sin(2\pi\alpha^{\rho/2-1}\mathbf{e_q})\Big], \tag{8}$$

where α is the parameter that determines the mapping shape, Δ is the normalization factor to ensure the power of the mapping output is equal to 1, and ρ represents the expansion factor. Since the quantization errors are real-valued, while the digital symbols and the channel response are complex-valued, the expansion factor of the analog mapping is $\rho = 2R_l$. Note that although ρ is a non integer value, we can achieve any expansion factor by using an adequate analog $K{:}S$ scheme, where part of the information is encoded with an $1{:}S$ spherical expansion code, while the remaining is sent uncoded. After the quantization errors are coded, the resulting symbols are interleaved to generate the corresponding complex-valued symbols and then superimposed to the data of the digital layers.

At the receiver, if the L digital layers are successfully decoded, an estimate of the quantization errors can be computed by using the Minimum Mean Square Error (MMSE) estimator. Let $\hat{\mathbf{x}}_a$ be the vector of analog symbols after decoding the L digital layers. Then, the MMSE estimates for the quantization errors are calculated from

$$\hat{\mathbf{e}}_q = \mathbb{E}[\mathbf{e}|\hat{\mathbf{x}}_a] = \frac{1}{p(\hat{\mathbf{x}}_a)} \int_{-\infty}^{+\infty} \mathbf{e}\ p(\hat{\mathbf{x}}_a|\mathbf{e}) p(\mathbf{e})\ d\mathbf{e}. \tag{9}$$

The estimates are obtained by solving the integral numerically.

C. Power Allocation

An important issue is the allocation of the available transmit power among the $L+1$ layers. As commented, each digital layer is employed to satisfy the quality requirements for a given region. Thus, the first layer should ensure that the receivers placed at a distance d_1 from the transmitter achieve the distortion target ϵ_1, and the same for the other layers. In general, this is accomplished when the power allocated to the l-th layer guarantees an error free transmission of the quantized bits –or with negligible error probability– to those receivers whose distance to the transmitter is d_l. Hence, we

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design an iterative allocation scheme such that the digital encoder-modulator tandem is able to provide a low error probability at each layer in the worst case, i.e., considering the largest distance in the l-th region, d_l.

In that point, the received power is $P_{\text{rx}}^l = P_{\text{tx}}/d_l^2$ and the actual SNR for the l-th layer is

$$\eta_l = \frac{w_l P_{\text{rx}}^l}{1 + \sum_{i=l+1}^{L+1} w_i P_{\text{rx}}^l}, \quad (10)$$

where w_l is the power for the l-th layer and $\sum_{i=l+1}^{L+1} w_i$ represents the interference caused by the next layers.

Assuming that the encoder-modulator duple in the l-th layer achieves a sufficiently low error probability (e.g., $p_e \leq 10^{-5}$) for SNRs larger than a given threshold η_l^{th}, the minimum power which must be allocated to the l-th layer is determined from (10) by equating η_l to η_l^{th}. From the transmit power constraint $\sum_{l=1}^{L+1} w_l = P_{tx}$, the term $\sum_{i=l+1}^{L+1} w_i$ can be rewritten as

$$\sum_{i=l+1}^{L+1} w_i = P_{\text{tx}} - \left(\sum_{i=1}^{l-1} w_i + w_l \right) \quad (11)$$

and, therefore, (10) is transformed into

$$\eta_l^{\text{th}} = \frac{w_l P_{\text{rx}}^l}{1 + P_{\text{rx}}^l \left(P_{\text{tx}} - \left(\sum_{i=1}^{l-1} w_i + w_l \right) \right)}. \quad (12)$$

Since the above equation only depends on the power assigned to previous layers, we can determine the power allocation for each layer iteratively as

$$w_l = \frac{\eta_l^{\text{th}} \left(1 + P_{\text{rx}}^l \left(P_{\text{tx}} - \sum_{i=1}^{l-1} w_i \right) \right)}{P_{\text{rx}}^l \left(1 + \eta_l^{\text{th}} \right)} \qquad l = 1, \ldots, L. \quad (13)$$

Once the power is allocated among the first L layers, the remaining power is assigned to the last layer to improve the decoding of the closest receivers, i.e., $w_{L+1} = P_{\text{tx}} - \sum_{i=1}^{L} w_i$.

In the case of channel encoders based on iterative decoding and the exchange of log-likelihood ratio (LLR) messages, the SNR threshold from which the error bit probability drastically decays can be determined using EXtrinsic Information Transfer (EXIT) Chart [12] or Density Evolution (DE) [13].

IV. Results

We next present the results of several computer experiments to illustrate the performance of the proposed HDA multilayer scheme. The source symbols are generated from a zero-mean unit-variance Gaussian distribution and quantized using the multilayer quantizer in Section III-A. The L digital layers use the Low Density Parity Check (LDPC) codes of the IEEE 802.16 (WiMAX) standard, and PSK or QAM modulations. The number of quantizer bits, the code rates and the number of modulation levels are chosen to satisfy the distortion requirements and to ensure R_l is equal for the L digital layers. The quantization errors are encoded using the analog mappings described in Section III-B with a proper expansion factor.

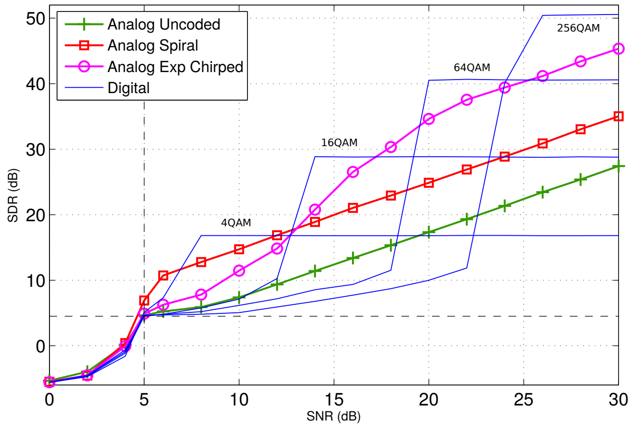

Fig. 3. Performance comparison between the HDA scheme with different mappings and the digital approaches with 2 layers and $R_1 = R_2 = 1/2$.

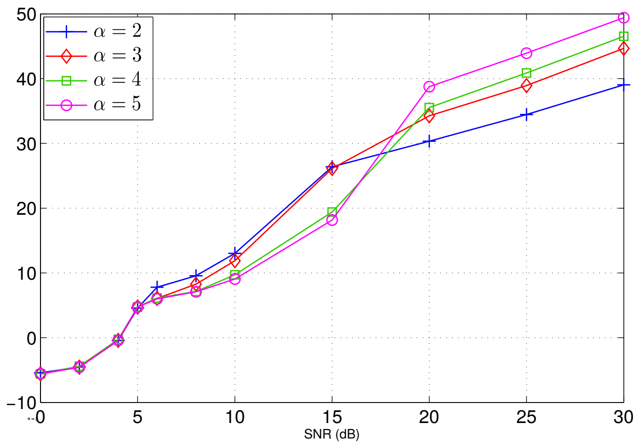

Fig. 4. Performance of analog chirped mappings for different values of α.

Since the source information is analog, the signal quality is measured in terms of the Signal-to-Distortion Rate (SDR)

$$\text{SDR}_{\text{rx}} = 10 \log_{10}(1/\xi_{\text{rx}}), \quad (14)$$

where ξ_{rx} represents the MSE between the source and decoded symbols given by (3). In this work, system performance is measured in terms of the SDR with respect to the SNR at reception, which depends of the distance to the transmitter.

In the first experiment, we consider a broadcast transmission where receivers at distance $d \leq 18$ m must decode the information with $\text{SDR}_{\text{rx}} \geq 4.5$ dB. If the transmit power is $P_{\text{tx}} = 30$ dB, that distance corresponds to a $\text{SNR}_{\text{rx}} \approx 5$ dB. Since we have a single requirement, the transmitter uses one digital layer and one analog layer to refine the symbol estimates. In particular, we chose as digital parameters $b_1 = 2$, $R_c^1 = 1/2$ and $M_1 = 4$ (i.e., $R_1 = 0.5$). The analog mapping expansion factor is hence 1:4 and the parameter α is set to 3.

Figure 3 plots the SDRs obtained for the proposed scheme when the SNR at reception ranges from 0 dB to 30 dB. This performance is also compared to several schemes that use other well-known strategies in the analog layer. Red and green curves correspond to the use of the 1:2 Archimedean spiral

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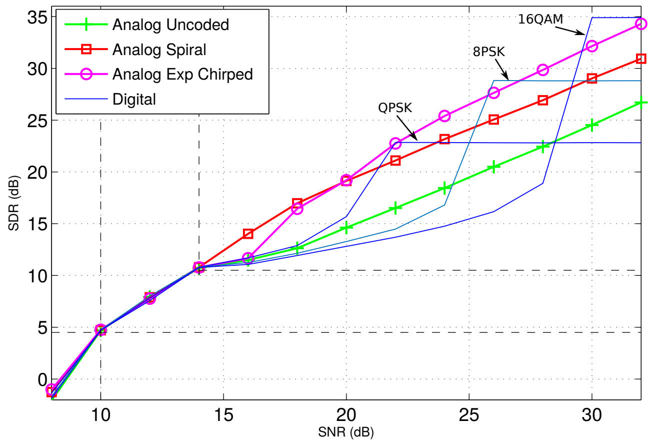

Fig. 5. Comparison of the the proposed HDA scheme to several digital approaches when considering three layers and an expansion factor $R_l = 1$.

to map the quantization errors and the uncoded transmission of those, respectively. Blue curves represent the performance when the refinement layer is also digital with a rate $1/2$ LDPC code and different QAM modulation schemes depending on the number of bits in the quantization step.

On one hand, the SDR values obtained for $\text{SNR}_{\text{rx}} > 5$ dB are larger than the threshold value, $\text{SDR}_{\text{rx}} \geq 4.5$ dB. On the other hand, the analog mapping based on the chirped modulation provides better performance than the other analog schemes, although it performs worse than the digital schemes. However, this comparison is not fair in practice because it is not possible to implement several digital schemes in a single layer and, consequently, we should choose one of them. Hence, if we restrict to a particular digital scheme, only some receivers will show better performance, while all users in the inner region exhibit good behaviour with the analog scheme.

Figure 4 compares the performance of the proposed multilayer scheme with the exponentially chirped mappings when considering different values of α. As observed, increasing α improves the system performance in the high SNR region (closest receivers) but penalizes the furthest receivers. The opposite behaviour is observed when α lowers. It is hence important to establish a trade-off by selecting α values which provide good performance for the whole range of SNRs.

In the second experiment, the requirements are $\text{SDR}_{\text{rx}}^1 \geq 4.5$ dB and $\text{SDR}_{\text{rx}}^2 \geq 10$ dB for $\text{SNR}_{\text{rx}}^1 > 10$ dB and $\text{SNR}_{\text{rx}}^2 > 14$ dB, respectively. The multilayer scheme hence comprises two digital layers and the analog one. Since the requirements are less restrictive in this case, the digital components are chosen such that $R_l = 1$, i.e., one source symbol per channel use is now transmitted. Figure 5 shows the performance curves obtained with the same schemes as in the previous experiment for the refinement layer: uncoded transmission, 1:2 Archimedean spiral, chirped modulation mappings and purely digital. Similar behaviour is observed for the different schemes. However, the multilayer scheme with chirped mappings closely approaches the fully digital multilayer schemes. This is because digital transmissions are more suitable for bandwidth expansion where the data is protected against the channel distortion by adding redundancy. As the expansion factor lowers, the improvement of digital transmissions vanishes until they even achieve inferior performance than analog communications for bandwidth compression. Similar results are expected for larger number of layers since the gap between purely digital systems and the proposed HDA scheme is more related to the rate R_l.

V. Conclusion

A HDA multilayer scheme for the transmission of common data from one transmitter to several receivers considering different QoS requirements has been proposed. Such scheme shows good behaviour and scaling for all the range of SNRs, as well as for different expansion factors and number of layers, with respect to previous analog and digital strategies. The system stands out for its flexibility since both the digital layers and the analog mappings can be configured depending on the considered scenario and the distortion constraints.

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