An Empirical Study on Gamma Shadow Fading Based Localization

Saliha Büyükçorak*†, Güneş Karabulut Kurt*, Abbas Yongaçoğlu‡
*Istanbul Technical University, Faculty of Electrical and Electronics Engineering,
34469, Maslak, Istanbul, Turkey, Email: {buyukcorak, gkurt}@itu.edu.tr
†Gebze Technical University, Faculty of Electronics Engineering,
41400, Gebze, Kocaeli, Turkey,Email: sbuyukcorak@gtu.edu.tr
‡School of Electrical Engineering and Computer Science, University of Ottawa,
Ottawa, Canada, Email: yongac@uottawa.ca

***Abstract*—In this paper, we propose a maximum likelihood estimator for received signal strength (RSS) based indoor localization systems by exploiting gamma shadow fading model. In order to investigate the validity of proposed method in a realistic environment, we develop a testbed based on Wi-Fi technology. Through experimental analyses, we first demonstrate the gamma distribution is a good fit to lognormal distribution, and both of them can sufficiently accurately characterize the empirical RSS observations. Then, we observe that gamma distribution is worth investigating for indoor localization compared to lognormal model because it provides superior accuracy. We further analyze the impacts of uncertainties of considered distributions' parameters on the localization performance via simulations.**

***Index Terms*—Positioning; received power level; gamma shadow fading; maximum likelihood estimator; Wi-Fi**

I. Introduction

Over the recent decades, localization techniques improved considerably due to vital requirements of numerous applications towards military, industrial, medical, household and personal uses. Global navigation satellite systems (GNNS) based positioning techniques are well-investigated for outdoor environments. However such systems are unfavorable for dense cities, tunnels and indoor environments since satellite signals suffer from severe loss passing through walls and partitions. To circumvent this shortcoming, several GNNS free localization techniques have been actively studied with the help of radio frequency (RF) technologies including Wi-Fi, WSN [1]–[5]. In current RF based techniques, localization are frequently executed by utilizing measurement of received signal strength (RSS), direction of arrival (DOA), angle of arrival (AOA), time (difference) of arrival (T(D)OA), roundtrip-time (RTT), and a fusion of them. There is an inherent tradeoff between the localization accuracy and the implementation complexity of these systems. When compared to others, RSS is an attractive low-cost solution because of requiring no specialized hardware and no synchronization.

The wireless channel is complicated and can be characterized by several effects which include macroscopic and microscopic fadings. In RSS based positioning systems, proper modeling of wireless channel is the crucial challenge. To the best of authors' knowledge, in related literature the widely accepted channel model is characterized by log-distance path loss and lognormal shadow fading (i.e macroscopic fading). Lognormal distribution for modeling the shadow fading is in compliance with the physical justifications of the channel [6]. Since its introduction, this model sustained its validity by many empirical studies. Subsequently, the authors of [7] empirically proposed Gamma distribution based shadow fading model due to provide more tractable performance analysis. Following this, in many works focused on data-bit level analyses, gamma mixture distribution is utilized to characterize the shadow fading in composite models which include macroscopic and microscopic fadings together [8], [9]. Various other shadow fading models such as inverse Gaussian have also been considered in such composite channels [10].

In addition to the above mentioned studies, the composite channel models for localization have also been investigated in a few other works. In [11], a maximum likelihood (ML) based localization method considers a composite model which consists of two parts: gamma distributed microscopic fading and lognormal distributed shadow fading. Guo *et. al* experimentally introduced new composite model utilizing exponential and Rayleigh distributions to consider microscopic effects as well as exploring the validity of the model with a particle filter in localization and tracking [12].

In this paper, inspired by [7], we propose a ML based localization scheme utilizing the gamma shadow fading model. In order to empirically investigate the effectiveness of the proposed scheme, we implement a real-life experiment setup aided by Wi-Fi technology for an indoor environment. By utilizing empirical RSS measurements, we statistically demonstrate that gamma distribution can be a good substitute for lognormal distribution. Furthermore, the validity of the proposed gamma shadow fading aided localization scheme is empirically verified. In addition to experimental analyses, we investigate the impacts of the uncertainties of propagation environment conditions on localization via simulations.

The paper is organized as follows. In Section II, gamma shadow fading model and its relation to lognormal distribution are introduced. In Section III, gamma shadow fading aided localization procedure is formulated. The real-life experimental studies for both model verification and positioning are given in Section IV. The extended analysis results based on simulation

ISBN 978-0-9928626-7-1 © EURASIP 2017

are presented in Section V. In Section VI, the concluding remarks are summarized.

II. Gamma Shadow Fading Model

Depending upon the nature of wireless channels, received power level is mainly determined by path loss, macroscopic and microscopic effects. Macroscopic fading originates from the shadow fading effect by buildings, foliage and other objects, while microscopic fading results from multipath characteristics. Many available RSS based localization techniques eliminate the microscopic effects by averaging the measurement values in time, thus they need to consider only shadow fading. Lognormal shadow fading model is used in almost all positioning systems. In this paper, by adopting the gamma shadow fading model, we propose a localization procedure based on gamma shadow fading model.

For future convenience, we start with a brief explanation of the lognormal and gamma distributions and the relation between them. The shadow fading is represented by a random variable (RV). While the uppercase term X_S denotes this RV, the lowercase x_S denotes its specific measured value. Based on lognormal distribution, probability density function (pdf) of shadow fading is written as follows

$$f(x_S) \sim \mathcal{LN}(\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}x_S} \exp\left(\frac{-(\ln x_S - \mu)^2}{2\sigma^2}\right). \quad (1)$$

Here, the parameters of this distribution are calculated as $\mu = \mathbb{E}[\ln X_S]$ and $\sigma = \sqrt{\mathbb{E}[(\ln X_S)^2] - \mathbb{E}^2[\ln X_S]}$, where $\mathbb{E}[\cdot]$ represents the expectation operator. The first and second moments of RSS RV according to lognormal distribution are defined as, respectively

$$\mathbb{E}[X_S] = \exp(\mu + \sigma^2/2), \quad (2)$$

$$\mathbb{E}[X_S^2] = \exp(2\mu + 2\sigma^2). \quad (3)$$

When the shadow fading is modeled with the gamma distribution, the pdf of this RV is formulated as

$$f(x_S) \sim \mathcal{G}(\alpha, \beta) = \frac{1}{\beta^\alpha \Gamma(\alpha)} x_S^{\alpha-1} \exp\left(\frac{-x_S}{\beta}\right), \quad (4)$$

where α is the shape parameter, β is the scale parameter and $\Gamma(\cdot)$ is the gamma function. The first and second moments of this RV aided gamma distribution are written as

$$\mathbb{E}[x_S] = \alpha\beta, \quad (5)$$

$$\mathbb{E}[x_S^2] = \alpha(\alpha+1)\beta^2. \quad (6)$$

Hence, the shape and scale parameters are directly computed by using the above identities as follows:

$$\alpha = \frac{\mathbb{E}^2[x_S]}{\mathbb{E}[x_S^2] - \mathbb{E}^2[x_S]}, \quad (7)$$

$$\beta = \frac{\mathbb{E}[x_S^2] - \mathbb{E}^2[x_S]}{\mathbb{E}[x_S]} = \frac{\mathbb{E}[x_S]}{\alpha}. \quad (8)$$

With the purpose of determining the relation between the parameters of lognormal and gamma distributions (i.e. μ, σ, α and β), equations given in (2), (5) and (3), (6) are equal to each other respectively, and hence the following formulas are obtained

$$\alpha = (e^{\sigma^2} - 1)^{-1}, \quad (9)$$

$$\beta = \exp\left(\mu + \frac{\sigma^2}{2}\right)(e^{\sigma^2} - 1), \quad (10)$$

$$\mu = \ln\left(\frac{\alpha\beta}{\sqrt{(1+1/\alpha)}}\right), \quad (11)$$

$$\sigma^2 = \ln(1 + 1/\alpha). \quad (12)$$

III. Localization based on Gamma Shadow Fading

Next we explain the proposed localization scheme via maximum likelihood estimator exploiting the gamma shadow fading model defined above.

A. RSS Measurement Model

Suppose that there are N Wi-Fi access points (APs) and a single client to be localized. Without loss of generality, we consider a 2D indoor monitored region, thus the known location of ith AP is denoted by $\theta_i = (x_i, y_i)$, and the target location of client is denoted by $\theta = (x, y)$. Let average received power level be represented by P_R which is a RV due to shadow fading. A specific measured value of the variable is represented by p_R. For simplicity, assume that radio signals are emitted by omnidirectional antennas and all APs can communicate with the client, i.e $p_{R,i} > \gamma_{th}$ where γ_{th} is the receiver sensitivity for a signal to be received.

Under the log-distance path loss model and shadow fading variable, the RSS measurement observed from the wireless channel between the client and each AP in decibel[1] can be expressed as [13]:

$$\ln(P_{R,i}) = \ln(\kappa P_T) - n_p \ln(d_i) + X_{S,i}, \;\; i = 1, \ldots, N, \quad (13)$$

where κ is a constant associated with the system properties, P_T is the transmit power, n_p is the path loss exponent (PLE) and $d_i = \sqrt{(x-x_i)^2 + (y-y_i)^2}$ is the distance between the client and the ith AP. The shadow fading RV is represented by $X_{S,i}$ which is characterized with gamma distribution in this section.

B. Maximum Likelihood (ML) Estimator

With the purpose of integrating the gamma shadow fading model to localization systems, we develop an ML based localization procedure given below. An ML estimator is frequently utilized because it provides asymptotically efficient and high accuracy estimation. In order to formulate a localization procedure as an ML estimation problem, initially, the likelihood function of RSS measurement obtained from the channel

[1] RSS in decibel is prevalently written with regard to $\log_{10}(\cdot)$, but for mathematical convenience, we here utilize the natural logarithm.

between the client and ith AP can be written. Based on gamma shadow fading model given in (4), the likelihood function can be expressed as

$$f(p_{R,i};\theta) = \frac{1}{\beta(\theta)^{\alpha}\Gamma(\alpha)} p_{R,i}^{\alpha-1} \exp\Big(-\frac{p_{R,i}}{\beta(\theta)}\Big), \tag{14}$$

where $\theta = (x, y)$ represents the target parameter (i.e. unknown location of client) and $p_{R,i}$ represents the observation of average received power level. Observe from the parameters of gamma distribution given in (9) and (10) that only the scale parameter β is dependent on θ because of including the mean of shadow fading RV. Thus we can rewrite it as follows

$$\beta(\theta) = \frac{1}{\alpha} \exp\big(\ln(\kappa P_T) - n_p \ln d_i + \sigma^2/2\big). \tag{15}$$

By taking logarithm of the above identity, log-likelihood function of RSS RV is obtained in the following form

$$\mathcal{L}(p_{R,i};\theta) = -\alpha \ln\big(\beta(\theta)\big) + \ln\big(\Gamma(\alpha)\big) + (\alpha-1)\ln(p_{R,i}) - \frac{p_{R,i}}{\beta(\theta)}. \tag{16}$$

Let us define by $\mathcal{P}$ the observation vector of the mentioned RSS variables obtained from APs, (i.e. $\mathcal{P} = (p_{R,1}, \ldots, p_{R,N})$), then the joint likelihood function of $\mathcal{P}$ can be expressed as

$$f(\mathcal{P};\theta) = \prod_{i=1}^{N} f(p_{R,i};\theta). \tag{17}$$

Also, the joint log-likelihood function is written in the form of summation of each log-likelihood expression provided in (16), i.e. $\mathcal{L}(\mathcal{P};\theta) = \sum_{i=1}^{N} \mathcal{L}(p_{R,i};\theta)$.

In the sequel, ML estimation exploits the maximization of the aforementioned joint log-likelihood function. Thus ML based localization problem based on the gamma shadow fading model can be expressed as

$$\begin{aligned}\theta^* = (x^*, y^*) &= \arg\max_{\theta} f(\mathcal{M};\theta) = \arg\max_{\theta} \mathcal{L}(\mathcal{M};\theta) \\ &= \arg\min_{\theta} \sum_{i=1}^{N} \alpha \ln(\beta(\theta)) + p_{R,i}/\beta(\theta). \end{aligned}\tag{18}$$

ML based solution can be analytically obtained by equating the first-order derivative of this function to zero. Due to the nature of localization problem, a closed-form solution for ML expression given in (18) cannot be obtained. However, an optimal solution can be achieved with the help of an iterative solver. In this paper, we use grid search method, also called as multiresolution projection in the literature. In this straightforward method, the monitored region is divided into search points called as grid blocks, then an exhaustive search made over these points to find the optimal solution.

IV. Experimental Results

In this section, we describe our real-life testbed which was implemented for two purposes: to empirically examine whether gamma distribution is a good substitute to lognormal distribution and to test the proposed gamma shadow fading aided ML based localization procedure. As depicted in Fig. 1, the testbed covers an office building, a typical indoor environment including hallways, tables, chairs, with the floor size of 13 m $\times$ 12 m. The system infrastructure is based on Wi-Fi technology in order to take advantage of the off-the-shelf Wi-Fi APs to estimate the positions of Wi-Fi enabled devices, such as laptops, smartphones. It is seen from this figure that our example system consists of a client and three Wi-Fi APs. The client acts as a target device located at $\theta =$ [4.77 m 4.56 m], while the Wi-Fi APs use as reference devices placed at $\theta_1 =$ [12.09 m 3.46 m], $\theta_2 =$ [0.64 m 0.64 m], and $\theta_3 =$ [1.84 m 1.62 m]. For RSS data collection as involving variations in time and propagation environment, the described system is operated from 22^{00} to 10^{00}. By utilizing standard Wi-Fi AP, we collect 2000 RSS observations for each device.

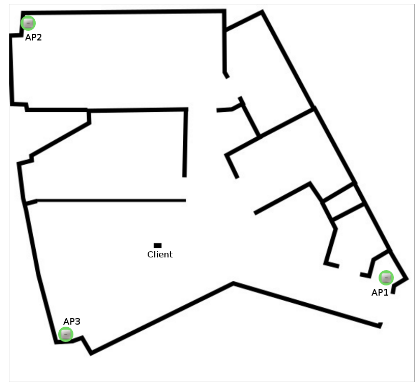


Fig. 1. An illustration of our testbed layout. In a typical indoor office environment, the testbed consists of one client, and three WiFi APs.

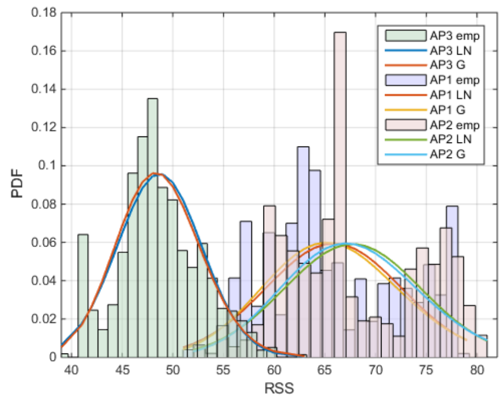
A. Empirical Verification of Models

In order to investigate whether gamma and lognormal statistical distributions are proper to characterize arbitrary RSS RVs, we execute histogram and cumulative distribution function (CDF) of RSS observation sets obtained from aforementioned measurement scenario. For RSS values observed from the three APs, the statistical histograms and lognormal, gamma pdf estimates are illustrated in Fig. 2(a). As can be seen from this figure, gamma and lognormal distributions exhibit a similar behavior and neither one is superior. In the sequel, the empirical CDFs and their estimations are provided in Fig. 2(b). It is seen from this figure that gamma distribution is a good fit for lognormal distribution especially for AP3 this model is well-suited to empirical RSS observations. Consequently, from these results, we verify the mentioned two shadow fading models are interchangeable for representing actual RSS measurements.

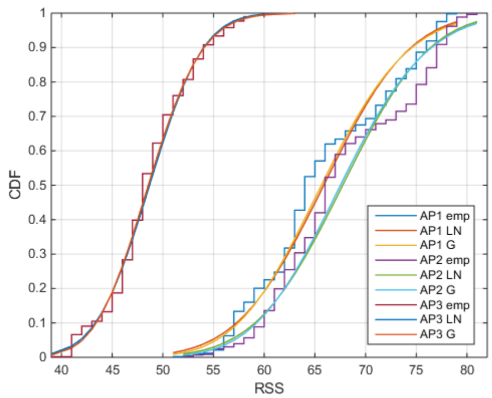
B. Empirical Verification of Localization

With the purpose of verifying the proposed ML based localization scheme compliance with the practical scenarios, we investigate the performance in terms of real-life RSS measurements comparing lognormal and gamma distributions.

ISBN 978-0-9928626-7-1 © EURASIP 2017



(a)



(b)

Fig. 2. (a) Empirical histograms and pdf estimates of gamma, lognormal distribution for AP1, AP2, AP3. (b) Empirical CDFs and their estimates.

By using the grid search method as iterative solver to carry out the ML expressions for gamma and lognormal distributions provided in (18) and Ref. [5], respectively, the obtained result for arbitrary RSS observation of each AP is given in Fig. 3. In this analysis, the floor of monitored region is divided into grid blocks, where the length of a block is set as $\lambda = 1$ m. Furthermore, the PLE is estimated as $n_p = 3.1$ by utilizing MMSE method. It is shown in the figure that gamma distribution based ML solution provides consistent superior localization accuracy. As a leading remark, we infer from this result that gamma distribution for localization problem can be noteworthy according to lognormal distribution.

V. Numerical Results

In order to find out the effect of imperfect estimation of indoor wireless channel parameters (i.e. PLE, shadow fading mean and variance) to localization performance, we employ extensive analyses via simulations. For this, let us define an error term denoted by ξ, the uncertainties on actual parameters for lognormal and gamma distributions are formulated as respectively

$$(\mu_\xi, \sigma_\xi) = (\mu, \sigma) + \xi_{\mathcal{LN}}, \qquad (19)$$
$$(\alpha_\xi, \beta_\xi) = (\alpha, \beta) + \xi_{\mathcal{G}}.$$

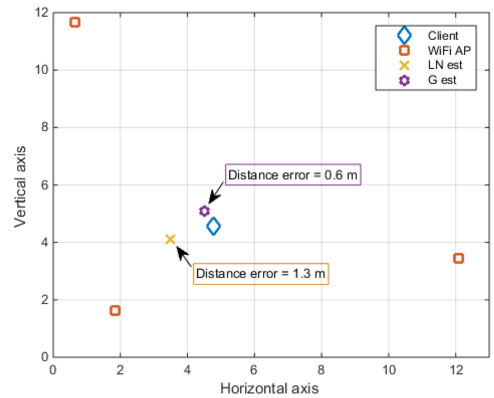


Fig. 3. Position estimation of gamma and lognormal model in terms of experimental RSS measurements.

This term is an RV that is modeled with Gaussian distribution with a zero mean and σ_ξ^2 variance, i.e. $\xi \sim \mathcal{N}(0, \sigma_\xi^2)$.

All simulations conduct on $Q = 1000$ Monte Carlo (MC) runs, thus the localization performance is visualized in terms of the root mean square error (RMSE) which is defined as

$$\sqrt{\frac{1}{Q}\sum_{q=1}^{Q}\left[(x_q - x_q^*)^2 + (y_q - y_q^*)^2\right]}, \qquad (20)$$

where (x, y) denote the true location of the client and (x_q^*, y_q^*) denote its estimation in the qth MC run. For fair comparison of considered distributions, the synthetic RSS measurements are generated according to propagation model given in (13) by considering both gamma and lognormal shadow fading. The synthetic RSS data set based on lognormal is named as $\mathcal{D}_{\mathcal{LN}}$, while based on gamma is named as $\mathcal{D}_{\mathcal{G}}$.

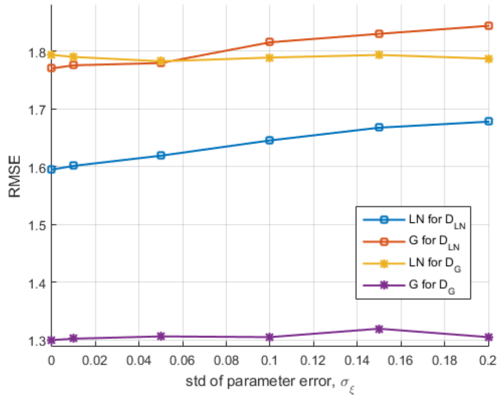
The effect of imperfections in parameters of both shadow fading models on localization are investigated for several σ_ξ values, the obtained results are provided in Fig. 4. As shown in Fig. 4(a), large values of σ_ξ slightly degrades the performance of these models. When compared with the mentioned datasets, it is seen that $\mathcal{D}_{\mathcal{G}}$ can achieve more accurate positioning than $\mathcal{D}_{\mathcal{LN}}$. In Fig. 4(b), we also plot the CDFs of the localization errors for both datasets at two distinct σ_ξ values.

Furthermore, we analyzed the error in estimation of PLE. The results obtained for RMSE and CDFs are illustrated in Figs. 5(a) and 5(b), respectively. It is seen from these figures that as the errors in the PLE estimates increases the performance slightly worsens. From all the simulation results, we conclude that the impact of information uncertainties about the channel characteristics on localization accuracy is marginal and the gamma shadow fading model is suitable and potentially can lead to analytical expressions.

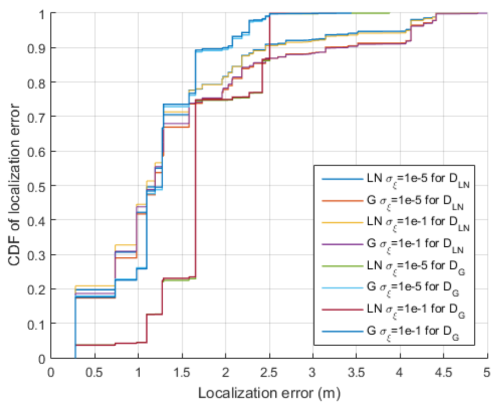
VI. Concluding Remarks

We introduce a localization method by utilizing the ML estimator based on gamma shadow fading model which may characterize the channel conditions. In order to validate the proposed model on practical scenarios, we develop an RSS measurement testbed exploiting the standard Wi-Fi hardware

ISBN 978-0-9928626-7-1

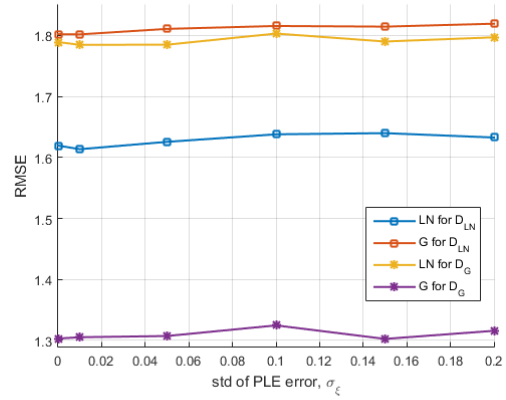


(a)

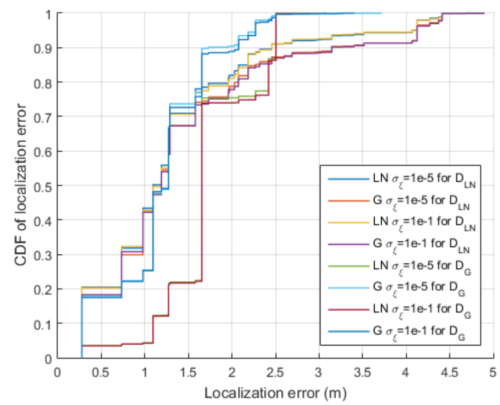


(b)

Fig. 4. The localization performance investigation for the effect of imperfections in shadow fading models' parameters (a) RMSE (b) CDF. $\mathcal{D}_{\mathcal{LN}}$ represents the synthetic RSS values aided by lognormal distribution, while $\mathcal{D}_{\mathcal{G}}$ represents aided by gamma distribution.



(a)



(b)

Fig. 5. The localization performance investigation for the error in PLE estimation (a) RMSE (b) CDF.

for indoor environment. Through the experimental analyses, we show that gamma distribution can be a good substitute for the classical lognormal distribution in compliance with empirical RSS observations. In addition, it is highlighted that gamma distribution can provide better localization performance than lognormal distribution. Finally, via simulation that uses the same scenario with the real-life experiment, we observe that the imperfect information and errors about the wireless channel characteristics only slightly deteriorates the performance.

References

[1] J. Niu, B. Wang, L. Shu, T. Q. Duong, and Y. Chen, "ZIL: An energy-efficient indoor localization system using zigbee radio to detect WiFi fingerprints," *IEEE Journal on Selected Areas in Commun.*, vol. 33, no. 7, pp. 1431–1442, July 2015.

[2] I. Sabek, M. Youssef, and A. V. Vasilakos, "ACE: An accurate and efficient multi-entity device-free WLAN localization system," *IEEE Trans. on Mobile Comput.*, vol. 14, no. 2, pp. 261–273, Feb 2015.

[3] P. Abouzar, D. G. Michelson, and M. Hamdi, "RSSI-based distributed self-localization for wireless sensor networks used in precision agriculture," *IEEE Trans. on Wirel. Commun.*, vol. 15, no. 10, pp. 6638–6650, Oct 2016.

[4] W. Y. Chiu, B. S. Chen, and C. Y. Yang, "Robust relative location estimation in wireless sensor networks with inexact position problems," *IEEE Trans. on Mobile Comput.*, vol. 11, no. 6, pp. 935–946, June 2012.

[5] N. Patwari, A. O. Hero, M. Perkins, N. S. Correal, and R. J. O'Dea, "Relative location estimation in wireless sensor networks," *IEEE Trans. on Signal Processing*, vol. 51, no. 8, pp. 2137–2148, Aug 2003.

[6] J. Egli, "Radio propagation above 40 MC over irregular terrain," *Proc. of the IRE*, vol. 45, no. 10, pp. 1383–1391, 1957.

[7] A. Abdi and M. Kaveh, "On the utility of Gamma PDF in modeling shadow fading (slow fading)," in *IEEE 49th Veh. Tech. Conf.*, vol. 3, 1999, pp. 2308–2312 vol.3.

[8] S. Atapattu, C. Tellambura, and H. Jiang, "A mixture Gamma distribution to model the SNR of wireless channels," *IEEE Trans. on Wirel. Commun.*, vol. 10, no. 12, pp. 4193–4203, 2011.

[9] J. Jung, S. R. Lee, H. Park, S. Lee, and I. Lee, "Capacity and error probability analysis of diversity reception schemes over generalized-K fading channels using a mixture gamma distribution," *IEEE Trans. on Wirel. Commun.*, vol. 13, no. 9, pp. 4721–4730, Sept 2014.

[10] Karmeshu and R. Agrawal, "On efficacy of Rayleigh-inverse Gaussian distribution over K-distribution for wireless fading channels." *IEEE Wirel. Commun. and Mobile Comput.*, vol. 7, no. 1, pp. 1–7, 2007.

[11] A. Dogandzic and P. P. Amran, "Signal-strength based localization in wireless fading channels," in *Conference Record of the Thirty-Eighth Asilomar Conference on Signals, Systems and Computers, 2004.*, vol. 2, Nov 2004, pp. 2160–2164 Vol.2.

[12] Y. Guo, K. Huang, N. Jiang, X. Guo, Y. Li, and G. Wang, "An Exponential-Rayleigh model for RSS-based device-free localization and tracking," *IEEE Trans. on Mobile Comput.*, vol. 14, no. 3, pp. 484–494, March 2015.

[13] T. Rappaport, *Wireless Communications: Principles and Practice*, 2nd ed. Upper Saddle River, NJ, USA: Prentice Hall PTR, 2001.

ISBN 978-0-9928626-7-1 © EURASIP 2017