Non-Binary Turbo-Coded OFDM-PLC System in the Presence of Impulsive Noise

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***Abstract*—The power-line communication (PLC) channel causes information-bearing signals to be affected by impulsive noise and the effects of the multipath fading. To mitigate these effects, we propose the employment of non-binary turbo codes, since non-binary error-correcting codes generally promise an enhanced performance in such harsh environments. In this paper, we investigate the performance of non-binary turbo-codes on PLC channels that exhibit frequency selectivity with additive Middleton Class A noise and compare with a comparable binary turbo-coded PLC system. In order to reduce the effect of multipath and impulsive noise, orthogonal frequency-division multiplexing (OFDM) with non-linear receivers (blanking and clipping) have been employed. The system is examined on extremely impulsive channels where the value of the impulsive index (A) is 0.01 and the noise ratio (Γ) is 0.01. The results show that non-binary turbo codes are very robust and achieve a large gain over binary turbo codes on PLC channels.**

***Index Terms*—non-binary turbo codes, power-line communication, impulsive noise, Middleton Class A noise, OFDM,**

I. INTRODUCTION

Power-line communication (PLC) utilizes the established electrical grid, but since power networks are not designed for communication services, there are many factors that make reliable communication over transmission lines challenging. These include: attenuation, impulsive noise and multipath frequency selectivity. Communication over the transmission line is most likely to be exposed to impulsive noise due to electromagnetic interference and this is commonly modeled by the Middleton Class A probability density function (PDF) [1]. Employing error-correction codes with the PLC system is an effective method to enhance bit-error rate (BER) performance. As seen in [2]–[5], the authors employed different coding schemes and compared the performance of coded PLC systems with uncoded PLC systems and showed a significant gain over uncoded PLC systems. However, these results did not consider the multipath present in the powerline channel. Typical PLC multipath frequency selective channels specifications are presented in [6] and [7].

In this paper, non-binary turbo codes are proposed [8] for improved error correction in the presence of impulsive noise and multipath. To the best of our knowledge, non-binary turbo codes have not been considered on PLC systems. Furthermore, it is well known that orthogonal frequency-division multiplexing (OFDM) is a powerful tool to combat the frequency selectivity and it is highly resistant to the effect of impulsive noise by spreading the noise signal energy simultaneously over sub-carriers [9]. In addition to the use of OFDM, non-linear blanking and clipping operations are also applied to mitigate the effects of impulsive noise samples with large amplitudes [10], [11].

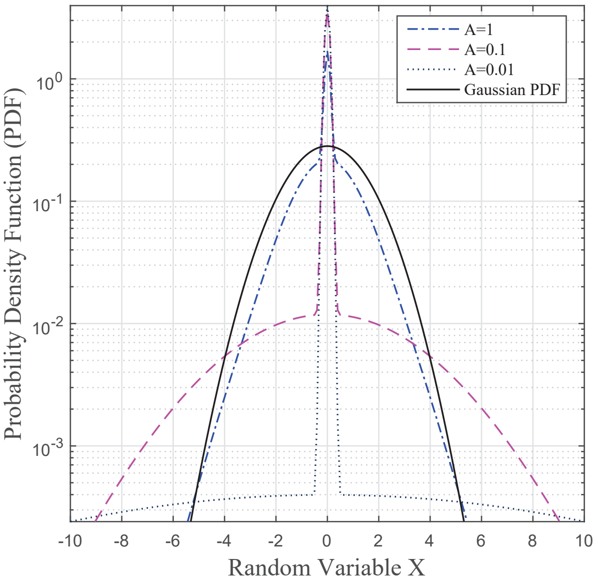

Fig. 1. Middleton Class A Probability Density Function

The BER performance of non-binary turbo codes defined in a Galois field GF(4) is investigated for an OFDM-PLC system on realistic power-line channels and compared with comparable binary turbo codes.

The paper is organized as follows: Section II-A introduces Middleton class A distributions. In Section III, the encoding and decoding of non-binary turbo codes are presented. In Section IV the system model components are explained in detail. In Section V, the simulation results are shown. Finally,

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section VI offers our conclusions.

II. CHANNEL MODEL

A. Middleton Class A Distributions

Middleton class A distributions are commonly used to model the impulsive noise of power-line channels [1] and their PDF is defined as

$$p(X) = \sum_{m=0}^{\infty} \frac{e^{-A}A^m}{m!} . \frac{1}{\sqrt{2\pi\sigma_m^2}} \exp\left(-\frac{|X|^2}{2\sigma_m^2}\right), \qquad (1)$$

where the variance σ_m^2 is given as

$$\sigma_m^2 = \sigma_u^2 \left(\frac{\frac{m}{A}+\Gamma}{1+\Gamma}\right) \qquad (2)$$

and

$$\sigma_u^2 = \sigma_G^2 + \sigma_I^2, \qquad \Gamma = \frac{\sigma_G^2}{\sigma_I^2}. \qquad (3)$$

The parameters σ_G^2 and σ_I^2 are the variances of Gaussian noise and impulsive noise, respectively. Γ is the background to impulsive noise ratio parameter which indicates the strength of impulsive noise compared to Gaussian noise. A is the impulsive index which increases the impulsive behavior as it becomes smaller and conversely the noise become Gaussian when A is large. Figure 1 shows the PDF of Middleton Class A noise for various values of A when $\Gamma = 1$ and the Gaussian PDF is also displayed as a reference. In addition, the probability of an error for Middleton Class A noise when employing M-ary phase-shift keying (MPSK) is given in [9] as

$$P_e = (1-A)\frac{M-1}{M} Q\left(\sqrt{\frac{E_b}{\sigma_G^2}}\right) + A\frac{M-1}{M} Q\left(\sqrt{\frac{E_b}{\sigma_G^2(1+\frac{1}{A\Gamma})}}\right), \qquad (4)$$

where M is the order of the PSK modulation, and E_b is the bit energy.

B. The Multipath model for the Power-line Channel

The behaviour of a PLC multipath channel can be described by its frequency response as

$$H(f) = \sum_{l=0}^{L-1} g_l e^{-(a_0+a_1 f^k)dl} e^{2\pi f \frac{dl}{v_p}}, \qquad (5)$$

where L is the number of paths, g_l is the weighting factor, a_0 and a_1 are attenuation parameters, d_l is the path length, $k \in$ [0.5,1] is the respective attenuation of an echo and v_p is the phase velocity. This is calculated as

$$v_p = \frac{c_0}{\sqrt{\varepsilon_r \mu_r}}, \qquad (6)$$

where c_0 is the speed of light, ε_r is the dielectric constant and μ_r is the permeability of the metal. In this paper, we use 4 and 15 path channels as used in [7]. Fig 2 illustrates the magnitude of the transfer function and the frequency response for both channels. Obviously, the attenuation caused by the 15 paths channel is greater than the attenuation caused by the 4 paths channel, by a value of 30dB at 20MHz. At the receiver, after adding the Middleton Class A noise, the received signal will be processed by the non-linear blanking or clipping operation to reduce the effect of impulsive noise.

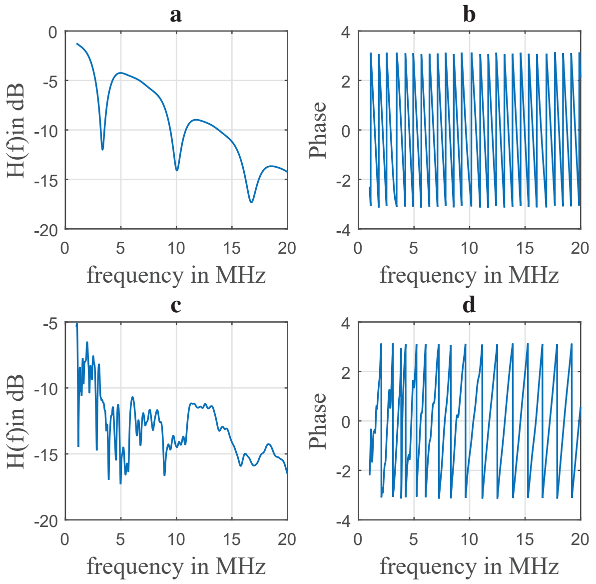

Fig. 2. Frequency and phase response of the realistic PLC multipath channels. a) Frequency response for 4 path PLC channel. b) Phase response for 4 path PLC channel. c) Frequency response for 15 path PLC. channel d) Phase response for 15 path PLC channel.

III. NON-BINARY TURBO CODE

A. Non-Binary Turbo Encoder

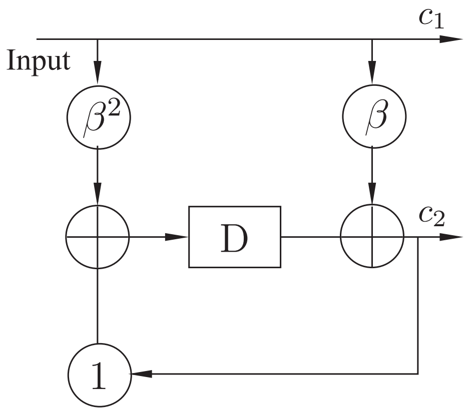

Fig. 3. $\beta^2\beta/1$ Non-Binary Convolutional Encoder.

The non-binary turbo code is a set of parallel concatenated non-binary convolutional encoders separated by a pseudorandom interleaver $\prod$. A non-binary convolutional encoder comprises a set of memory elements and multipliers defined in a finite field $GF(q)$ with q elements $\{0, 1, \beta, \beta^2, ..., \beta^{q-2}\}$, where β is a primitive element. Figure 3 shows the $\beta^2\beta/1$ non-binary convolutional encoder defined in GF(4) with code rate $R = \frac{1}{2}$ [12]. There are two feed-forward multipliers β^2 and β, one feedback multiplier 1. For more information on non-binary convolutional encoder, please refer to [8] and [12].

B. Non-binary Turbo Decoding over Impulsive Noise

There are various optimal and sub-optimal algorithms used to decode binary turbo codes on the additive white Gaussian noise (AWGN) channel [13]. In this paper, the Max-Log-MAP algorithm is employed to decode non-binary turbo codes on impulsive Middleton Class A noise channels. Basically, this algorithm finds the maximum probability input symbol by estimating the probability of each trellis edge that corresponds to one of the q inputs. The probability of the state transitions from the state s_0 at time $t-1$ to state s_1 at time t is given as

$$P(s_0, s_1, \boldsymbol{y}) = P(s_0, \boldsymbol{y}_t^-)P(s_1, \boldsymbol{y}_t|s_0)P(\boldsymbol{y}_t^+|s_1), \tag{7}$$

where $\boldsymbol{y}_t$, $\boldsymbol{y}_t^-$, and $\boldsymbol{y}_t^+$ are the received symbols at time (t), the received symbols before time (t), and the received symbols after time (t), respectively [14]. If we take the logarithm of (7), we obtain

$$\begin{aligned} m_t(s_0, s_1) &= \ln P(s_0, \boldsymbol{y}_t^-) + \ln P(s_1, \boldsymbol{y}_t|s_0) + \ln P(\boldsymbol{y}_t^+|s_1) \\ &= \alpha_{t-1}(s_0) + \gamma_t(s_0, s_1) + \delta_t(s_1) \end{aligned} \tag{8}$$

where γ_t is the probability transition metric between s_0 and s_1, α_t is the forward recursion, β_t is the backward recursion. In the Max-Log-MAP algorithm, α_t and β_t can be calculated as

$$\alpha_t(s_0) = \max_i \{\alpha_{t-1}(s_i) + \gamma_t(s_i, s_0)\}, \tag{9}$$

$$\delta_t(s_1) = \max_i \{\delta_{t+1}(s_i) + \gamma_{t+1}(s_1, s_i)\}. \tag{10}$$

The PLC channel exhibits severe impulsive nature, OFDM can spread the noise over all sub-carriers during the FFT process. Hence the noise after OFDM demodulation can be regarded as Gaussian but with a difference variance and the channel log-likelihood ratio (LLR) can be calculated as

$$\begin{aligned} L^z(c = z|\boldsymbol{y}) &= \ln \frac{P^z(c = z|\boldsymbol{y})}{P^z(c = 0|\boldsymbol{y})} \\ &= \ln \frac{P^z(\boldsymbol{y}|c = z)}{P^z(\boldsymbol{y}|c = 0)} + \ln \frac{P^z(c = z)}{P^z(c = 0)} \\ &= L^z(\boldsymbol{y}|c = z) + L^z(c) \end{aligned} \tag{11}$$

where $L^z(c)$ is the a priori LLR and $\boldsymbol{y}$ is the received sequence. For $z \in GF(2^p)$, each element z contains p bits and $\boldsymbol{y}$ also consists of p bits. $L^z(\boldsymbol{y}|c = z)$ is calculated as

$$L^z(\boldsymbol{y}|c = z) = \ln \frac{P^z(y_1, ..., y_p|c = z)}{P^z(y_1, ..., y_p|c = 0)} = \sum_{l:c_l=1} \frac{2y_l}{\sigma^2},$$

In our case, σ^2 is the variance of the Middleton Class A noise and as shown in [15], it can be approximated as

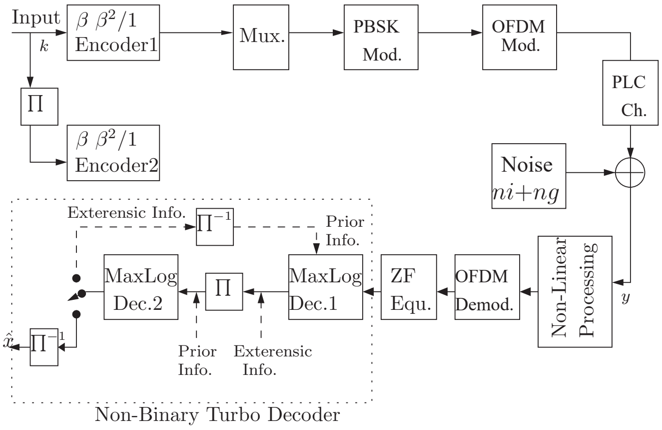

Fig. 4. Coded OFDM-PLC system.

$$\sigma^2 = \sigma_G^2 \left(1 + \frac{1}{\Gamma}\right). \tag{12}$$

Hence, γ_t can be calculated as

$$\gamma_t(s_0, s_1) = L^z(c) + \ln P^z(\boldsymbol{y}|\boldsymbol{x}), \tag{13}$$

where $\boldsymbol{x}$ is the vector of modulated symbols. Finally, the output LLR of the decoded message symbols are given as

$$\begin{aligned} L^z(c = z|\boldsymbol{y}) = &\max_{s^i - s^j \in s^z} \{\alpha_{t-1}(s_0) + \gamma_t(s_0, s_1) + \delta_t(s_1)\} \\ &- \max_{s^i - s^j \in s^0} \{\alpha_{t-1}(s_0) + \gamma_t(s_0, s_1) + \delta_t(s_1)\}, \end{aligned} \tag{14}$$

where s^z represents the set of all state transitions corresponding to $c \neq 0$ and s^0 is the set of all state transitions corresponding to $c = 0$. This output LLR will be used as the extrinsic information for the other component decoder.

IV. CODED OFDM-PLC SYSTEM WITH NON-LINEAR PROCESSING

Fig 4 shows the system model that used in this paper. The input is a set of non-binary symbols k, where $k \in$ GF(4). First, k message symbols are encoded by a non-binary turbo encoder and then modulated by using binary phase shift keying modulation (BPSK). This is then passed to the OFDM block. OFDM is a powerful solution to mitigate the effect of strong impulsive noise in multipath environment. The OFDM modulator applies an inverse fast Fourier transform (IFFT) to generate a complex baseband OFDM signal as

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{\frac{j2\pi kt}{T_s}}, \qquad 0 < t < T_s, \tag{15}$$

where X_k is the data after the mapping process, N is the number of sub-carriers and T_s is the active symbol interval. Due to the effect of multipath, the receiver will receive many copies of the original signal with different delays. At the receiver, after adding the Middleton Class A noise, the received signal will be processed by the blanking or clipping

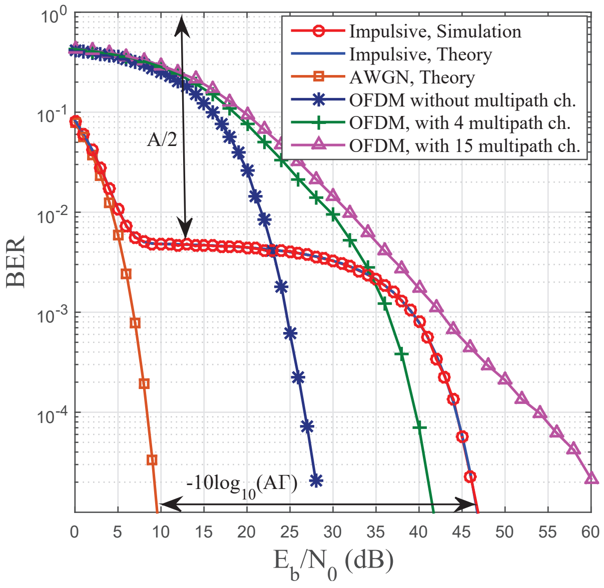

Fig. 5. BER performance of uncoded AWGN, Middeleton Class A, and Uncoded OFDM-PLC on different realistic PLC channels, versus SNR (dB).

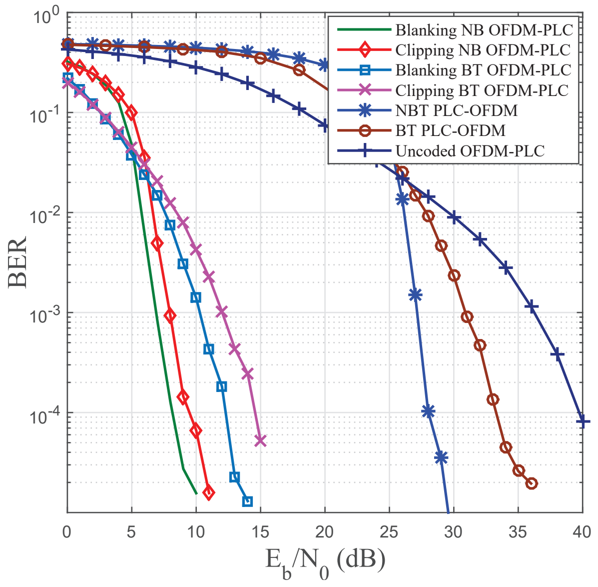

Fig. 6. BER performance of uncoded OFDM-PLC and coded (BT and NBT) OFDM-PLC versus SNR (dB) on 4 path frequency selective channel.

operation to reduce the effect of impulsive noise. Blanking is a non-linear process that is used to reduce the impulsive noise effect on the received signal y and this block is also shown in Fig. 4. After blanking, the received signal is given as

$$r_i = \begin{cases} y_i, & |y_i| < T_B \\ 0, & Otherwise \end{cases} ,0 \leq i \leqslant K-1, \qquad (16)$$

where T_B is the blanking threshold and y_i is the received signal, given by $y_i = x_i + n_i$ and n_i is the Middleton Class A noise. Clipping is another non-linear process which limits the received signal and the output is given as:

$$r_i = \begin{cases} y_i, & |y_i| < T_C \\ T_C e^{jarg(y_i)}, & Otherwise \end{cases} ,0 \leq i \leqslant K-1\ , \qquad (17)$$

where T_C is the clipping threshold value. These non-linear operations are applied before the OFDM demodulator on the receiver side. Then the OFDM demodulator is performed by a fast Fourier transform (FFT). After OFDM demodulation the signal will pass through a zero forcing (ZF) detector to compensate for the channel distortion, defined by [16]

$$w(k) = \frac{H^*(k)}{|H(k)^2|}, \qquad (18)$$

where $H(k)$ is the channel frequency response. Finally, a MAX-Log-Map non-binary turbo decoder is applied as explained in III-B.

V. SIMULATION RESULTS

In this section we present simulation results for a non-binary turbo (NBT) coded PLC-OFDM system, binary turbo (BT) coded PLC-OFDM system and uncoded PLC-OFDM system. BPSK modulation has been used for all simulated and theoretical implementations. The system is examined on very impulsive channels, where the impulsive index A=0.01 and the impulsiveness is greater than the Gaussian noise by 100 times (i.e. Γ=0.01). The 4 and 15 paths PLC channels are used to model the realistic measurement given in [6]. To make a fair comparison, simulation results for a comparable BT code comprising $(1, 7/5)_8$ recursive systematic convolutional codes and message length $k = 2048$ bits are compared with the $(\beta^2\beta/1)$ NBT code and message length of 1024 symbols. Both codes have a code rate of $\frac{1}{3}$ and can be realized by a 4-state trellis diagram. The maximum iterations for both decoders is set to 5. A channel bandwidth 5KHz - 20MHz is considered and cyclic prefix of 256.

Fig. 5 shows theoretical and simulated BER vs. SNR(dB) for the AWGN channel, the Middleton class A channel with and without OFDM and BER for uncoded PLC system over 4 and 15 multipath frequency selective channels. We can observe the hurdles that degrade the performance of the communications over PLC when compared with the conventional AWGN channel. It also illustrates the benefits of using OFDM on impulsive channel since the BER performance is enhanced by 15 dB at the low BER region. In addition, Fig. 5 displays the impact of different multipath models of PLC channels on the system performance.

Fig. 6 compares the BERs for NBT coded, BT coded and uncoded PLC-OFDM system on the 4-path frequency selective channel with and without the blanking and clipping techniques.

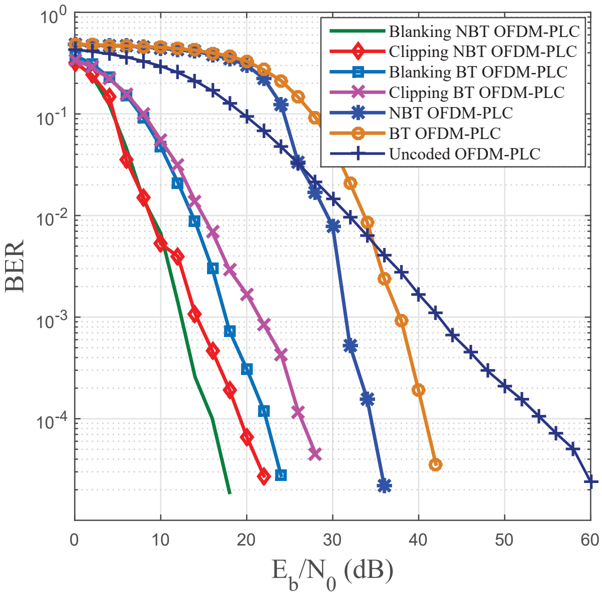

Fig. 7. BER performance of uncoded OFDM-PLC and coded (BT and NBT) OFDM-PLC versus SNR (dB) on 15 path frequency selective channel.

Although the performance of BT PLC-OFDM system with blanking has already shown a 25 dB gain over the uncoded system, the NBT coded PLC-OFDM system offers a 30dB and 5 dB further gain, over uncoded and coded BT PLC-OFDM system respectively. We also notice that the blanking is a more effective process to eliminate the effect of impulsive noise compared with the clipping operation.

Fig. 7 demonstrates the BERs of NBT and BT codes, similar to previous case but with a 15-path channel model. It is shown that the NBT code again shows a superior performance compared with the BT system with a 6dB gain for all situations at a BER of 10^{-4}. It should be noticed that the blanking and clipping techniques are still showing further improvement to the BER performance for both NBT and BT OFDM-PLC systems. Finally, employing NBT codes on PLC systems can achieve significant coding gain over using uncoded PLC systems, by 24dB.

VI. CONCLUSION

In this paper, an investigation into the performance of non-binary turbo codes on power-line channels, in term of BER has been presented. A non-Binary turbo-coded OFDM-PLC system employing two non-linear receivers, blanking and clipping, has been proposed. The system has been examined on realistic multipath frequency selective PLC channels with extremely impulsive Middleton class A noise, with A and $\Gamma = 0.01$. Finally, a comparison with a comparable binary turbo-coded PLC system in the same environment has been evaluated and simulation results have shown that non-binary turbo code offers a superior performance on powerline channels.

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