A Novel Channel Estimation Scheme for Multicarrier Communications with the Type-I Even Discrete Cosine Transform

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***Abstract*—In this work, the problem of channel estimation in multicarrier communications with the Type-I even discrete cosine transform (DCT1e) is addressed. A novel scheme, based on using the DCT1e, both at the transmitter and the receiver, is introduced. The proposed approach does not require adding any redundancy or knowing the exact length of the channel's impulse response. By constructing a symmetric training sequence at the transmitter with enough leading and tail zeros, we show that an accurate estimation of the channel's impulse response can be attained. Simulations using the ITU-T pedestrian channel B illustrate the good behavior of the proposed scheme in terms of reconstruction signal to noise ratio.**

I. Introduction

Multicarrier modulation (MCM) has become the preferred technique in current state-of-the-art digital communication systems like mobile communications (3GPP LTE), wireless local/metropolitan area networks (WiFi/WiMax), digital subscriber lines (DSL), digital TV broadcasting (DVB-T) or power line communications (PLC). Most of these systems are based on the discrete Fourier transform (DFT), and the resulting MCM schemes are usually referred to collectively as orthogonal frequency division multiplexing (OFDM) [1], [2].

Recently, discrete trigonometric transforms (DTTs) have been considered as an alternative to the DFT for MCM systems due to their improved robustness with respect to some of the main weaknesses of OFDM systems. For instance, several works have established the good performance of DTT-based MCM schemes under carrier frequency offset [3]–[7]. Most of these systems are based on the Type-II discrete cosine transform (DCT2) or the Type-IV discrete cosine transform (DCT4), and require the estimation of the channel's impulse response, which is usually unknown and time-varying. To this aim, some training symbols (known both by the transmitter and the receiver) are often used [8].

In this work, we explore the use of the Type-I even Discrete Cosine Transform (DCT1e) to estimate the channel in MCM systems. The three key advantages of the DCT1e that we exploit in this work are the following:

- The inverse of the DCT1e is the same as the direct DCT1e transform, so we can use exactly the same block both at the transmitter and the receiver [9].
- The linear convolution of two vectors is transformed by the DCT1e into a pointwise product of their transforms, whenever *one of the two vectors* presents some symmetry conditions and has *enough* leading and tail zeros [9], [10].
- Signals which present whole-point symmetry (WS), i.e., even symmetry around the central element of the sequence, are transformed into vectors with a high number of zero coefficients in the DCT1e transform domain (and viceversa).

The first property allows us to use the same hardware for the transmitter and the receiver. The second property enables the estimation of non-symmetric channels without introducing any additional transform in the receiver (like the DFT used in other works) by constructing a symmetric training signal with enough leading and tail zeros. Note that the use of a symmetric training signal was already studied in [8], but the scheme described in this work considered only the DCT2 even (DCT2e) and the DCT4 even (DCT4e) transforms, and required using the DFT in the receiver to estimate the channel. Finally, the third property ensures that the signals obtained are sparse. This sparsity was exploited in [11] to develop a compressed channel sensing scheme based on the DCT1e, but this approach was restricted to symmetric channels. In the following, we introduce a simple and efficient approach to estimate arbitrary non-symmetric channels for MCM based on the DCT1e. Furthermore, the proposed scheme only requires knowing the maximum possible length of the channel's impulse response, not its exact length.

The work is organized as follows. Firstly, in Section II we recall the general MCM scheme and the resulting channel estimation problem. Then, the proposed channel estimation procedure is presented in Section III, and its behaviour is illustrated in Section IV through simulations. Finally, the main contributions of this work are summarized in Section V.

II. Problem Statement: Channel Estimation in Multicarrier Modulation

Let us consider the MCM block diagram shown in Fig. 1. In the transmitter, a symbol $\mathbf{X} = [X_0, \dots, X_{N-1}]^T$ is constructed in the transformed domain, converted to the time domain sequence $\mathbf{x} = [x_0, \dots, x_{N-1}]^T = \mathbf{T}^{-1}\mathbf{X}$ through the

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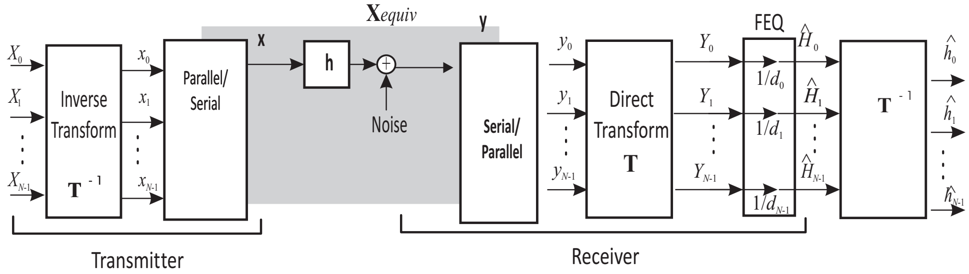

Fig. 1. General block diagram for channel estimation in a multicarrier modulation (MCM) system.

inverse transform $\mathbf{T}^{-1}$, and prepared for transmission through the channel by a parallel/serial conversion. We consider a channel of maximum length L, whose impulse response is $\mathbf{h} = [h_0, \ldots, h_{L-1}]^T$.[1] The received signal is then $\mathbf{y} = \mathbf{x} * \mathbf{h} + \mathbf{n}$, where $\mathbf{n} = [n_0, \ldots, n_{N-1}]^T$ is the additive white Gaussian noise (AWGN) vector and $*$ denotes the standard linear convolution operator.

In the receiver, our aim is recovering the information contained in $\mathbf{X}$. In order to fulfill this goal, we need to estimate $\mathbf{h}$ first. A common approach in the literature is transmitting a known training sequence, $\mathbf{x}$, and applying the procedure shown in Fig. 1 (see Section III for further details): serial/parallel conversion of the N relevant samples of the received signal, $\mathbf{y} = [y_0, \ldots, y_{N-1}]^T$; conversion to the transformed domain, $\mathbf{Y} = [Y_0, \ldots, Y_{N-1}]^T = \mathbf{T}\mathbf{y}$ using the direct transform $\mathbf{T}$; estimation of $\mathbf{H} = [H_0, \ldots, H_{N-1}]^T$ in the transformed domain using the one-tap filters d_k (for $k = 0, 1, \ldots, N-1$) computed from $\mathbf{X}$ and $\mathbf{Y}$; and transformation back to the time domain to obtain $\hat{\mathbf{h}}_{zp} = [\hat{h}_0, \ldots, \hat{h}_{N-1}]^T$, from which the relevant non-null coefficients of the channel's impulse response can be easily extracted.

This is the typical approach used in OFDM (where the DFT is the underlying transform), whereas [8] analyzed this scheme when $\mathbf{T}$ corresponds either to the DCT2e or the DCT4e. Here we consider the DCT1e, for which $\mathbf{T} = \mathbf{T}^{-1} = \mathbf{C}_{1e}$, with

$$[\mathbf{C}_{1e}]_{k,j} = a_j \cos\left(\frac{kj\pi}{N-1}\right),\ 0 \le k, j \le N-1,$$

where

$$a_j = \begin{cases} \frac{1}{\sqrt{2(N-1)}}, & \text{if} \quad j \in \{0, N-1\}, \\ \frac{2}{\sqrt{2(N-1)}}, & \quad\text{otherwise}. \end{cases}$$

This is the definition of $\mathbf{C}_{1e}$ given in [9], except for the normalization factor $\sqrt{2(N-1)}$, which has been introduced here in order to ensure the involution property: $\mathbf{C}_{1e}^{-1} = \mathbf{C}_{1e}$.

III. Channel estimation for DCT1e-based Multicarrier Modulation

We focus now on the channel estimation problem of Fig. 1, by using the DCT1e both at the transmitter and the receiver. Without loss of generality, we assume that the maximum length L of the channel filter is odd (otherwise, we simply introduce an extra null component), so $L = 2\nu + 1$, and we propose applying the following procedure:

1) Construct a training symbol with WS symmetry, whose first and last $L-1 = 2\nu$ components are null, and whose length is $N = 2M + 2L - 1 = 2M + 1 + 4\nu$,

$$\tilde{\mathbf{x}} = [\underbrace{0, \ldots, 0}_{2\nu}, \underbrace{x_M, \ldots, x_1, x_0, x_1, \ldots, x_M}_{2M+1}, \underbrace{0, \ldots, 0}_{2\nu}]^T,$$

either directly or through the inverse DCT1e of some appropriate sequence $\mathbf{X}$ in the transformed domain. For instance, a valid sequence is $X_k = (-1)^{k/2}$ if k is even, and $X_k = 0$ if k is odd.

2) Remove the first and last $\nu = (L-1)/2$ null components of $\tilde{\mathbf{x}}$, to obtain the transmitted vector,

$$\mathbf{x} = [\underbrace{0, \ldots, 0}_{\nu}, \underbrace{x_M, \ldots, x_1, x_0, x_1, \ldots, x_M}_{2M+1}, \underbrace{0, \ldots, 0}_{\nu}]^T,$$

of length $2M + L = 2M + 2\nu + 1$, which also has WS symmetry and its first and last ν components are null.

3) After transmitting $\mathbf{x}$ through the channel, the received data, $\mathbf{y} = \mathbf{x} * \mathbf{h} + \mathbf{n}$, has length $2M + 2L - 1 = N$.

4) Following the approach of [8], we can express the convolution matricially as

$$\mathbf{y} = \widetilde{\mathbf{X}} \cdot \mathbf{h} + \mathbf{n}, \tag{1}$$

where $\widetilde{\mathbf{X}}$ is a lower triangular Toeplitz matrix whose first column contains the information about the known symbol padded with zeros: $[\mathbf{x}, 0, ..., 0]^T$. In our case, this matrix (of size $N \times L$) is given by

$$\widetilde{\mathbf{X}} = \begin{bmatrix} & \mathbf{O}_{\nu \times L} & & \\ x_M & 0 & \ddots & 0 \\ \vdots & x_M & \ddots & \vdots \\ x_1 & \ddots & \ddots & 0 \\ x_0 & \ddots & \ddots & x_M \\ \vdots & \ddots & \ddots & \vdots \\ x_M & \ddots & \ddots & x_0 \\ 0 & x_M & \ddots & x_1 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & x_M \\ & \mathbf{O}_{\nu \times L} & & \end{bmatrix}. \tag{2}$$

[1] If the length of the channel's impulse response is $L' < L$, then $h_{L'} = h_{L'+1} = \cdots = h_{L-1} = 0$, which does not alter the proposed scheme.

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5) Now, let us notice that the convolution can also be expressed as

$$\mathbf{x} * \mathbf{h} = \widetilde{\mathbf{X}} \cdot \mathbf{h} = \mathbf{X}_{equiv} \cdot \begin{bmatrix} \mathbf{0}_{M+\nu} \\ \mathbf{h} \\ \mathbf{0}_{M+\nu} \end{bmatrix},$$

where $\mathbf{X}_{equiv}$ is any $N \times N$ square matrix which is obtained as an extension of the central matrix $\widetilde{\mathbf{X}}$, by appending $M+\nu$ arbitrary columns on the left and $M+\nu$ arbitrary columns on the right. Our key idea is that we can build $\mathbf{X}_{equiv}$ so that it can be perfectly diagonalized via the DCT1e: in effect, notice that we can write $\mathbf{X}_{equiv} = \mathbf{X}_T + \mathbf{X}_H$ as a sum of a Toeplitz matrix $\mathbf{X}_T$ and a Hankel-type matrix $\mathbf{X}_H$, defined as:

$$\mathbf{X}_T = \begin{bmatrix} x_0 & x_1 & \cdots & x_M & 0 & \cdots & 0 \\ x_1 & \ddots & \ddots & & & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & & & 0 \\ x_M & & \ddots & \ddots & & & x_M \\ 0 & & & & & & \vdots \\ \vdots & \ddots & & & & & x_1 \\ 0 & \cdots & 0 & x_M & \cdots & x_1 & x_0 \end{bmatrix},$$

$$\mathbf{X}_H = \begin{bmatrix} 0 & x_1 & \cdots & x_M & \cdots & 0 & 0 \\ \vdots & \vdots & \iddots & 0 & \iddots & \vdots & 0 \\ 0 & x_M & \iddots & & \iddots & 0 & 0 \\ 0 & 0 & \iddots & & \iddots & x_M & 0 \\ \vdots & \vdots & \iddots & & \iddots & \vdots & \\ 0 & 0 & \cdots & x_M & \cdots & x_1 & 0 \end{bmatrix}.$$

Therefore, the result given in [10, p. 2634] guarantees that $\mathbf{X}_{equiv}$ is diagonalized by the DCT1e:

$$\mathbf{C}_{1e} \cdot \mathbf{X}_{equiv} \cdot \mathbf{C}_{1e}^{-1} = \mathbf{D}. \tag{3}$$

Moreover, [10] guarantees that the diagonal entries of matrix $\mathbf{D}$ (i.e., the eigenvalues of $\mathbf{X}_{equiv}$) are themselves the DCT1e of the $N-$length right-half vector $\mathbf{x}_{zp}^r = [x_0, \ldots, x_M, 0, \ldots, 0]^T$.

6) Let us denote as $\mathbf{Y} = \mathbf{C}_{1e} \cdot \mathbf{y}$ the received vector in the transformed domain, as $\mathbf{H} = \mathbf{C}_{1e} \cdot [\mathbf{0}_{M+\nu}\ \mathbf{h}\ \mathbf{0}_{M+\nu}]^T$ the transformed vector associated to the zero padded channel's impulse response, and as $\mathbf{N} = \mathbf{C}_{1e} \cdot \mathbf{n}$ the transformed noise vector. Then, using (3) we can express the received signal in the transformed domain as

$$\mathbf{Y} = \mathbf{D} \cdot \mathbf{H} + \mathbf{N},$$

where the diagonal entries of $\mathbf{D}$ are the DCT1e of a vector related to the training signal:

$$[\mathbf{D}]_{k,k} = d_k = \left[\mathbf{C}_{1e} \cdot \mathbf{x}_{zp}^r\right]_k.$$

These one-tap filters can be pre-computed and stored in memory for the training signal of choice. Finally, we obtain an estimation of $\mathbf{H}$ as

$$\widehat{\mathbf{H}}_k = \mathbf{Y}_k / d_k, \quad k = 0, ..., N-1, \tag{4}$$

and compute $\mathbf{C}_{1e}^{-1} \cdot \widehat{\mathbf{H}} = \widehat{\mathbf{h}}$, which provides a perfect estimation of $[\mathbf{0}_{M+\nu}\ \mathbf{h}\ \mathbf{0}_{M+\nu}]$ in the absence of noise.

Thus, we have been able to find a simple and efficient solution to the channel estimation problem for an MCM system based on the DCT1e. The whole procedure is summarized below.

SUMMARY OF THE PROCEDURE:

1) Choose a WS training signal of length $N-2\nu$ with ν zeros both at the beginning and the end, $\mathbf{x} = [0, \ldots, 0, x_M, \ldots, x_1, x_0, x_1, \ldots, x_M, 0, \ldots, 0]^T$, so that the $N-$length symbol at the first block of the transmitter is $\mathbf{X} = \mathbf{C}_{1e} \cdot [\mathbf{0}_{1\times\nu}, \mathbf{x}, \mathbf{0}_{1\times\nu}]^T$.
2) Compute the DCT1e of the right-half vector of $\mathbf{x}$ padded by zeros on the right, up to length N: $\mathbf{d} = \mathbf{C}_{1e} \cdot [x_0, \ldots, x_M, 0, \ldots, 0]^T$.
3) Transmit $\mathbf{x}$ through the channel, and get the $N-$length vector $\mathbf{y} = \mathbf{x} * \mathbf{h} + \mathbf{n}$ at the receiver.
4) Apply the DCT1e block: $\mathbf{Y} = \mathbf{C}_{1e} \cdot \mathbf{y}$.
5) For any $k = 0, \ldots, N-1$, compute $\widehat{\mathbf{H}}_k = \mathbf{Y}_k / d_k$ using the 1-tap per subcarrier coefficient d_k stored in Step 2.
6) Finally, obtain $\mathbf{C}_{1e}^{-1} \cdot \widehat{\mathbf{H}}$, which is the desired estimation of the zero padded channel filter $[0, \ldots, 0, \mathbf{h}, 0, \ldots, 0]$.

IV. Numerical Results

In this section, we analyze the behaviour of the proposed channel estimation scheme by testing it on one of the standardized ITU-R M.1225 channels [12]. First of all, a training signal is constructed in the DCT1e domain by setting $X_k = (-1)^r$ if $k = 2r$ (for $r = 0, 1, \ldots, (N-1)/2$) and $X_k = 0$ otherwise. The inverse DCT1e of $\mathbf{X}$ is performed, the first and last $\nu = (L-1)/2$ zeros (out of the $L-1$ leading and tail zeros) are removed, and the resulting length $N-(L-1)$ time-domain signal, x_m, is transmitted.[2] After passing this signal through the channel, with impulse response h_m for $0 \leq m \leq L-1$, we obtain a length N time-domain signal, $z_m = [\mathbf{x} * \mathbf{h}]_m$. Then, zero-mean additive white Gaussian noise (AWGN) with variance σ_n^2 is added, resulting in a received signal

$$y_m = [\mathbf{x} * \mathbf{h}]_m + n_m = \sum_{r=0}^{N-1} h_r x_{m-r} + n_m,$$

where $n_m \sim \mathcal{N}(0, \sigma_n^2)$ is the AWGN, with $\mathcal{N}(\mu, \sigma^2)$ denoting a univariate Gaussian density with mean μ and variance

[2]Note that all this process can be avoided simply by pre-computing $\mathbf{x}$ and storing it.

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σ^2. The length N DCT1e of y_m (Y_k) is then computed and used to estimate the DCT1e of the channel's impulse response ($\widehat{H}_k$) as described in Section III. Finally, the length N inverse DCT1e of $\widehat{H}_k$ is obtained and the relevant central samples are extracted to obtain the estimate of the channel's impulse response, $\hat{h}_m$. The performance measure used is the reconstruction signal to noise ratio (SNR),

$$\widehat{\mathrm{SNR}}(\mathrm{dB}) = 10 \log_{10} \frac{P_h}{P_e},$$

where $P_h = \frac{1}{L}\sum_{m=0}^{L-1} |h_m|^2$, $P_e = \frac{1}{L}\sum_{m=0}^{L-1} |\hat{h}_m - h_m|^2$ and L is the maximum channel length. Both the transmitted SNR and the length of the DCT are allowed to change in order to see their effect in the channel's impulse response estimation.

As mentioned above, we consider one of the channels standardized by ITU-R for the evaluation of radio transmission technologies for IMT 2000 [12]. More precisely, we address the estimation of the ITU-T M.1225 pedestrian channel B for an increasing length of the DCT1e (from $N = 127$ to $N = 2047$). In the simulations, $N_c = 100$ randomly generated channels were tested and $N_s = 1000$ simulations were performed for each SNR ranging from -10 dB to 30 dB and each value of N. The channels were generated using Matlab's `stdchan` function using a carrier frequency $f_c = 2$ GHz and a sampling period $T_s = 100$ ns. With this sampling period, the channel's impulse response becomes

$$\begin{aligned} h_m = & A_0\delta_m + A_2\delta_{m-2} + A_8\delta_{m-8} + A_{12}\delta_{m-12} \\ & + A_{23}\delta_{m-23} + A_{37}\delta_{m-37}, \end{aligned}$$

where each of the A_i are independent Rayleigh distributed random variables and δ_{m-r} denotes Kronecker's delta, i.e., $\delta_{m-r} = 1$ if $m = r$ and $\delta_{m-r} = 0$ otherwise. Note that the length of the channel's impulse response is actually $L' = 38$, but we set $L = 41$ for the simulations in order to show the robustness of the proposed approach.[3]

In all cases the reconstruction SNR increases linearly when the transmitted signal power to noise ratio increases, as can be seen in Fig. 2. Indeed, the following simple relationship can be established:

$$\widehat{\mathrm{SNR}}(\mathrm{dB}) = \mathrm{SNR}(\mathrm{dB}) + \Delta\mathrm{SNR}(\mathrm{dB}), \qquad (5)$$

where $\mathrm{SNR}(\mathrm{dB}) = 10 \log_{10} \frac{P_x}{\sigma_w^2}$ with $P_x = \frac{1}{N}\sum_{m=0}^{N-1} |x_m|^2$, and $\Delta\mathrm{SNR}(\mathrm{dB})$ is given in Table I. Note that the reconstruction performance increases as more subcarriers are used, i.e., the larger the value of N the larger the reconstruction gain $\Delta\mathrm{SNR}(\mathrm{dB})$.

Finally, Fig. 3 shows two examples of the reconstructed channel (in the time and frequency domains, respectively) for $N = 1023$ and two different SNRs. Note that the channel's reconstruction is already very good for SNR = 0 dB (Figs. 3(a) and 3(b)) and it becomes almost perfect for SNR = 10 dB (Figs. 3(c) and 3(d)). Note also that the impulse and frequency responses of the channels used in the two examples are different, since they have been generated randomly, as described before.

[3]Indeed, we have tested several values of $L > L'$ without noticing significant differences in the results.

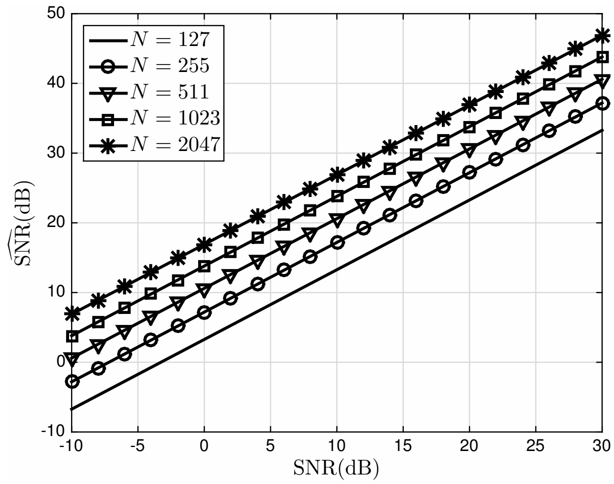

Fig. 2. Channel reconstruction SNR, $\widehat{\mathrm{SNR}}(\mathrm{dB})$, as a function of the signal power to noise ratio, $\mathrm{SNR}(\mathrm{dB})$, for different values of N. In all cases, $N_c = 100$ random channels have been tested and $N_s = 1000$ simulations have been performed for each N and SNR.

N	127	255	511	1023	2047
$N-(L-1)$	87	215	471	983	2007
$2M+1$	47	175	431	943	1967
$\Delta\mathrm{SNR}(\mathrm{dB})$	3.27	7.19	10.60	13.80	16.90

TABLE I
SNR GAIN IN THE ESTIMATION OF THE ITU-T M.1225 PEDESTRIAN CHANNEL B. $N = 2M + 2L - 1$: NUMBER OF SUBCARRIERS OF THE MCM SCHEME; $N - (L-1)$: LENGTH OF THE TRANSMITTED SEQUENCE; $2M + 1$: LENGTH OF THE NON-NULL CENTRAL PART OF THE TIME-DOMAIN TRAINING SEQUENCE.

V. CONCLUSIONS

In this work, we have presented a general procedure for the estimation of an arbitray channel's impulse response when the Type-I even Discrete Cosine Transform (DCT1e) is used within a multicarrier modulation system. The proposed approach is based on the construction of a symmetric training signal with enough leading and tail zeros. Unlike previous methods, which require the use of the DFT in the receiver, the scheme proposed here requires only a length N DCT1e both in the transmitter and the receiver, thus leading to a very efficient hardware implementation. Furthermore, no assumptions are required about the channel except for knowing the maximum possible length of its impulse response. Numerical simulations show that the proposed algorithm is able to provide very accurate channel reconstruction in noisy environments for ITU-T M.1225 pedestrian channel B.

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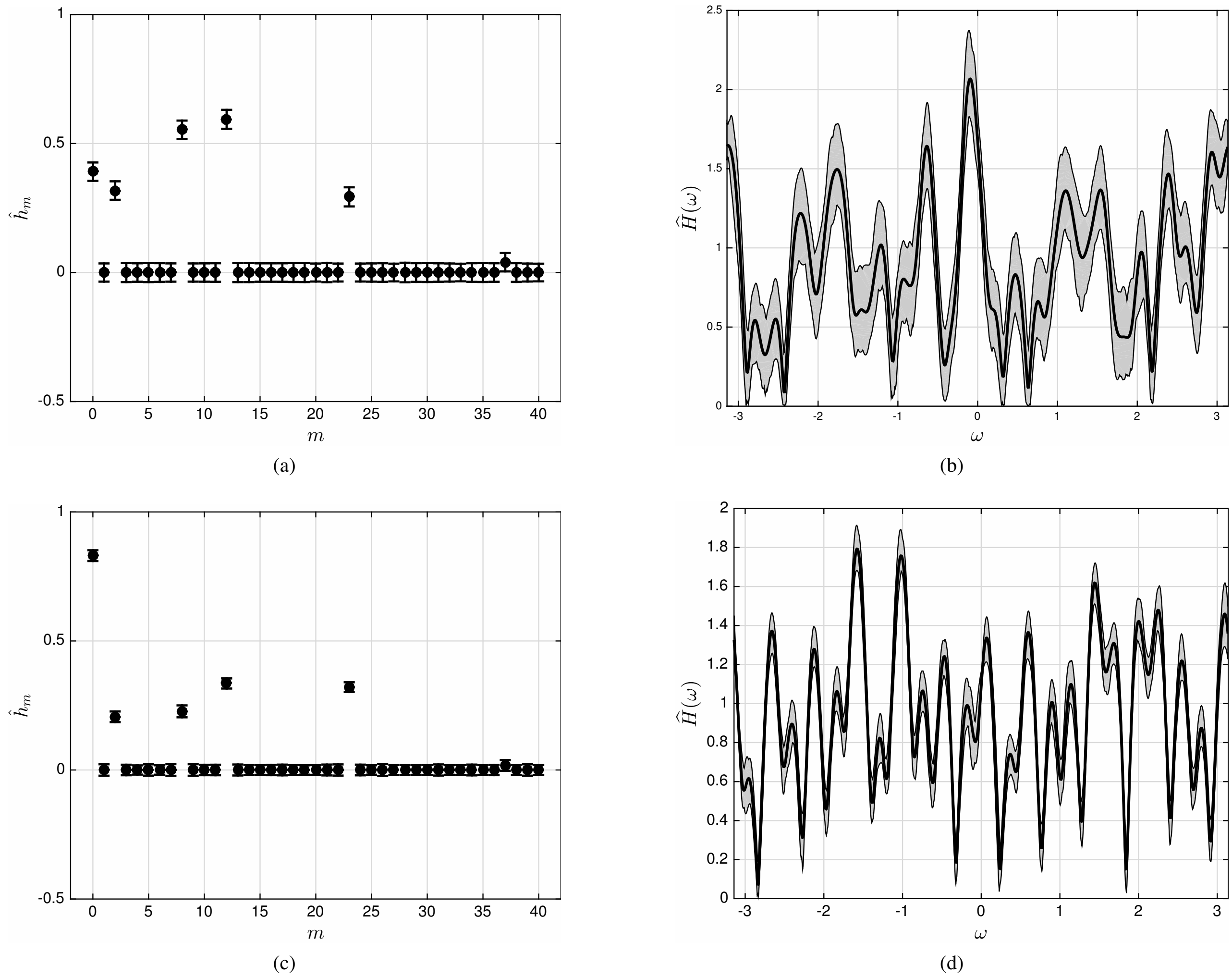

Fig. 3. (a): Estimated channel's impulse response (true value and two times the standard deviation) for SNR = 0 dB. (b): Estimated channel's frequency response (true value and range between maximum and minimum estimated values) for SNR = 0 dB. (c): Estimated channel's impulse response (true value and two times the standard deviation) for SNR = 10 dB. (d): Estimated channel's frequency response (true value and range between maximum and minimum estimated values) for SNR = 10 dB. In all cases, $N = 1023$ and $N_s = 100$.

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