Spectrum Reconstruction with Nonuniform Fast Fourier Transform for MIMO SAR Azimuth Nonuniform Sampling

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Abstract—Multiple input multiple output (MIMO) synthetic aperture radar (SAR) shows much potential compared with traditional SAR in many interesting applications. In MIMO SAR high resolution wide swath imaging, azimuth ambiguity is a problem when the system azimuth sampling is nonuniform. A spectrum reconstruction method based on periodic nonuniform sampling theory has been used for azimuth ambiguity suppression. However, the computation cost is very high for MIMO SAR with a lot of transmitters/receivers. In this paper, MIMO SAR spectrum reconstruction with nonuniform fast Fourier transform (NUFFT) is proposed. The simulation results show the effectivity of the proposed spectrum reconstruction method.

Keywords—Azimuth ambiguity; reconstruction algorithm; nonuniform fast Fourier transform(NUFFT); multiple input multiple output synthetic aperture radar (MIMO SAR)

I. INTRODUCTION

Multiple input multiple output (MIMO) synthetic aperture radar (SAR) has much potential in imaging and target detection[1-2]. Several along-track receiving apertures in MIMO SAR can receive the echoes from the same illuminated area at a low pulse repetition frequency (PRF), which can realize high resolution wide swath imaging. However, it needs the PRF and the displacement of apertures satisfies the so-called uniform PRF requirement. Any deviation from the PRF or the apertures' displacement will result in the problem of azimuth nonuniform sampling.

The filter bank reconstruction algorithm (FBA) is first proposed in [3]. However, it requires matrix inversion and cannot be used when the matrix is not square or some of the phase centers of different channels are overlapped.

Some adaptive methods for azimuth ambiguity suppression have been proposed, such as Capon[4], MSANR[5] and MMSE[6]. These methods are suitable for complex situation. But when the SNR is low, the algorithm performance will be deteriorated.

Yih-Chyun Jenq proposed a perfect spectrum reconstruction algorithm from nonuniformly sampled signals [7]. Based on this method, Z Zhu put forward a fast algorithm with several short Fourier transform[8]. However, this algorithm works efficiently only when the channel number is small.

Nonuniform fast Fourier transform (NUFFT) is a fast algorithm for nonuniform Fourier transform and it has been widely researched in many aspects[9-13]. This paper combines NUFFT with spectrum reconstruction algorithm for the problem of MIMO SAR azimuth nonuniform sampling.

II. SIGNAL MODEL OF MIMO SAR AZIMUTH NONUNIFORM SAMPLING

Suppose a MIMO SAR system with M transmitters and N receivers, transmitting waveforms are $\left\{s_1(t),...,s_m(t),...,s_M(t)\right\}$. All of the antennas are along-track linear array on the same radar platform or different platforms with the same moving parameters. $\{x_1^t, x_2^t, ..., x_M^t\}$ and $\{x_1^r, x_2^r, ..., x_N^r\}$ are the along-track position of each transmitter and receiver respectively. v is the radar platform velocity, and H is the radar flight altitude, c is the light velocity.

Suppose a target $\mathrm{P}(x_0, y_0)$ in the image area, η is the slow time, $R_m^t(\eta)$ is the range from the mth transmitter to the target and $R_n^r(\eta)$ is the range from the target to the nth receiver. The echo signal of the nth receiver is

$$s_n(\tau,\eta) = \sum_{m=1}^{M} s_m^t[\tau - \frac{1}{c}(R_m^t(\eta) + R_n^r(\eta))] \qquad (1)$$

After MIMO SAR signal separation, the echo signal of each channel can be expressed as

$$s_{mn}(\tau,\eta) = s_m^t[\tau - \frac{1}{c}(R_m^t(\eta) + R_n^r(\eta))] \qquad (2)$$

where $R_m^r(\eta) = \sqrt{(x_0 - x_m^t - v\eta)^2 + y_0^2 + H^2}$ and $R_n^r(\eta) = \sqrt{(x_0 - x_n^r - v\eta)^2 + y_0^2 + H^2}$.

We can get that

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$$\begin{aligned} &R_m^t(\eta) + R_n^r(\eta) \\ &\approx 2R_0 + \frac{(x_0 - x_m^t - v\eta)^2}{2R_0} + \frac{(x_0 - x_n^r - v\eta)^2}{2R_0} \\ &\approx 2R_0 + \frac{(x_0 - x_{mn} - v\eta)^2}{R_0} \end{aligned} \tag{3}$$

where $R_0 = \sqrt{y_0^2 + H^2}$ is the minimum slant range, $x_{mn} = (x_m^t + x_n^r)/2$ is the virtual phase center. $\Delta R_{mn} \approx \frac{d_{mn}^2}{4R_0}$ (where $d_{mn} = \left|x_m^t - x_n^r\right|$) is the range error, which should be corrected for virtual phase center.

The signal after range cell migration correction and range compensation can be written as

$$\begin{aligned} s_{rc,mn}(\tau,\eta) = A\mathrm{sinc}[B_r(\tau - 2R_0/c)]\cdot \\ w_a(\eta)\exp(-j2\pi(R_m^t(\eta) + R_n^r(\eta))/\lambda) \end{aligned} \tag{4}$$

where, B_r is the signal bandwidth, A is the amplitude, $w_a(\cdot)$ is the azimuth function, λ is the wavelength.

The azimuth signal of the range cell with the target can be expressed as

$$s_{a,mn}(\eta) = \exp(-j2\pi(2R_0^2 + \frac{(x_0 - x_{mn} - v\eta)^2}{R_0})/\lambda) \tag{5}$$

Reject the overlapping positions in $\{x_{mn}\}$, we can get L channels with the phase centers $\{x_0, x_2, ..., x_L\}$ in ascending order. Ignore the constant phase, the signal of (5) can be rewritten as

$$s(t_{n_a,l}) = \exp(-j2\pi\frac{(x_0 - vt_{n_a,l})^2}{\lambda R_0}) \tag{6}$$

$$t_{n_a,l} = n_a T + \frac{x_k}{v} = (n_a L + l)T' + \gamma_l T' \tag{7}$$

where $n_a = 0,1,...,N_a - 1, l = 0,1,...,L-1$, N_a is the system sampling number during synthetic aperture time, $\gamma_l = \frac{x_l}{vT'} - l$, $T' = T/L$, $T = 1/PRF$, and PRF is the system pulse repeat frequency.

Rearrange the signal of (6), $t_{n_a,l}$ can be transform to t_i, $i = 0,1,...,LN_a - 1$, the signal can be rewritten as

$$s(t_i) = \exp(-j2\pi\frac{(x_0 - vt_i)^2}{\lambda R_0}) \tag{8}$$

If γ_l in (7) is not zero, that is to say azimuth sampling of MIMO SAR is nonuniform, which leads azimuth ambiguity in the imaging result.

III. SPECTRUM RECONSTRUCTION ALGORITHM

A. The basic reconstruction Method

From [7] we can see, nonuniform sampling signal (6) can be perfect reconstructed by the following method.

Define the nonuniform discrete Fourier transform (NUDFT) of signal (8) as

$$S_d(k) = \sum_{i=0}^{LN_a-1} s(t_i)e^{-j\frac{2\pi k}{LN_a}\frac{t_i}{T'}} \tag{9}$$

Suppose the discrete Fourier transform of uniformly spaced samples is $S_c(\cdot)$, $\mathbf{S}_c$ and $\mathbf{S}_d$ are the vector composed of $S_c(k)$ and $S_d(k)$, respectively. We can get $\mathbf{S}_c$ by

$$\mathbf{S}_c = \mathbf{A}^{-1}\mathbf{S}_d \tag{10}$$

where $\mathbf{S}_c = [S_c(k)S_c(k+N_a)...S_c(k+(L-1)N_a)]^{\mathrm{T}}$, $\mathbf{S}_d = [S_d(k)S_d(k+N_a)...S_d(k+(L-1)N_a)]^{\mathrm{T}}$, $-\frac{LN_a}{2} \le k \le N_a - \frac{LN_a}{2}$.

And $\mathbf{A}$ is a Toeplitz matrix

$$\mathbf{A} = \begin{bmatrix} A(0) & A(-1) & \cdots & A(-L+1) \\ A(1) & A(0) & \cdots & A(-L+2) \\ \vdots & \vdots & \ddots & \vdots \\ A(L-1) & A(L-2) & \cdots & A(0) \end{bmatrix} \tag{11}$$

where $A(k) = \frac{1}{LT'}\sum_{l=0}^{L-1} e^{-jk\gamma_l\frac{2\pi}{L}}e^{-jkl\frac{2\pi}{L}}$.

The matrix $\mathbf{A}$ can be obtained from the position of the phase centers of all the MIMO SAR channels. And then, we can see that the spectrum can be reconstructed from the NUDFT of the azimuth nonuniform sampling signal.

B. Fast reconstruction algorithm with NUFFT

Consider the calculation of (9). Suppose the output $S_d(k)$ is a LN_a-element sequence, it requires $(LN_a)^2$ complex multiplications and $LN_a(LN_a - 1)$ complex additions by direct NUDFT calculation, which is definitely computationally expensive especially when L is large for MIMO SAR. Here, we use NUFFT to accelerate the calculation.

$$X_k = \sum_{n=0}^{N-1} x_n \exp(-j2\pi z_n k/K) \tag{12}$$
$$k = -K/2,...,K/2-1$$

where z_n is the sampling position. Use Gaussian kernel [9] $\hat{\phi}(x) = e^{-bx^2}$, $\phi(\xi) = \begin{cases} e^{-\xi^2/4b}, |\xi| \le \alpha \\ 0 \quad , |\xi| > \alpha \end{cases}$, where b and α are coefficients of Gaussian kernel.

$$e^{-jx\xi} = \frac{(2\pi)^{-1/2}}{\phi(\xi)} \sum_m \hat{\phi}(x-m) e^{-jm\xi} \tag{13}$$

The NUFFT of non-equispaced data can be expressed as

$$\begin{aligned} X_k &= \frac{(2\pi)^{-1/2}}{\phi(2\pi k / cK)} \\ &\cdot \sum_{n=0}^{N-1} \sum_m x_n \hat{\phi}(cz_n - m) \exp(-j2\pi mk / cK) \\ k &= -K/2, \dots, K/2-1 \end{aligned} \tag{14}$$

where c is the oversampling factor.

Flowchart for the fast MIMO SAR reconstruction algorithm is shown in fig.1.

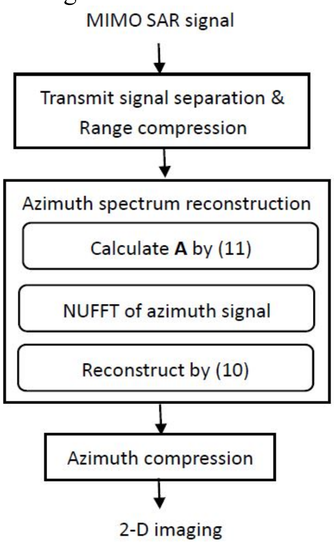

Fig. 1. Flowchart for the fast MIMO SAR reconstruction algorithm

IV. EXPERIMENTAL RESULTS

Simulation MIMO SAR system with two transmitters (Tx1 and Tx2) and three receivers (Rx1, Rx2 and Rx3). The antenna layout is shown in fig.2, and the position data of antennas is shown in Table I.

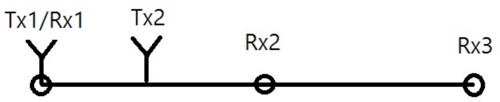

Fig. 2. The layout of MIMO SAR antennas for simulation

TABLE I. ANTENNAS POSITION DEVIATION FOR SIMULATION

Antennas	**Position**	**deviation** Δ_n
Tx1/Rx1	$\Delta_1 \cdot L_r$	*0.01*
Tx2	$(1+\Delta_2) \cdot L_r$	*-0.15*
Rx2	$(2+\Delta_3) \cdot L_r$	*-0.1*
Rx3	$(4+\Delta_4) \cdot L_r$	*0.2*

The main parameters of simulation system are given in Table II.

TABLE II. SYSTEM PARAMETERS FOR SIMULATION

Parameters	**Value**
Center frequency / GHz	5.39
Chirp width / us	5
Pulse bandwidth / MHz	150
PRF /Hz	25
Flight altitude / km	20
Look angle /degree	40
Number of transmitters M	2
Number of receivers N	3
Radar platform velocity /m·s^{-1}	250
Sub-aperture length L_r */m*	1.667
SNR/dB	10
Number of equivalent channels L	6
Number of system sampling N_a	256

A. Spectrum Reconstruction results

Take a range cell data with one point target for simulation. The MIMO SAR antennas' position deviations lead to the azimuth ambiguity as shown in fig.3(left) and fig.4(left). From fig.3(right) and fig.4(right) we can see that using the proposed method the azimuth signal can be well reconstructed and the ambiguity in the focused azimuth imaging can be reduced.

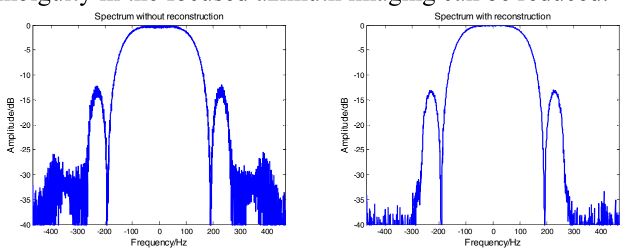

Fig. 3. The signal spectrum without reconstruction (left) and with reconstruction (right)

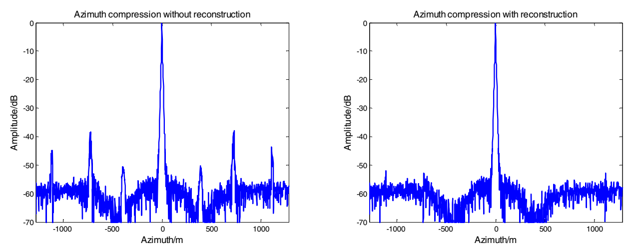

Fig. 4. Azimuth compression result without reconstruction (left) and with reconstruction (right)

B. Two-dimensional imaging results

Take five point targets for example in MIMO SAR 2-D imaging simulation. Suppose the targets are the same and the positions are (x_0, y_0), (x_0+300, y_0+200), (x_0+300, y_0-200), (x_0-300, y_0+200), and (x_0-300, y_0-200), here meter is used as the unit.

From the two-dimensional imaging results shown in fig.5, we can see that the azimuth ambiguity is disappeared with reconstruction.

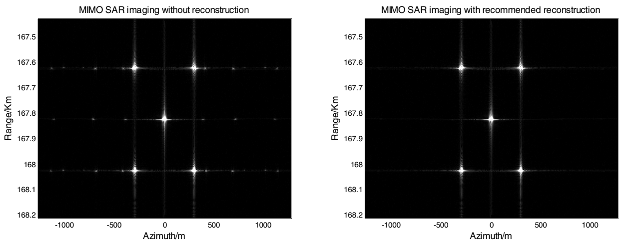

Fig. 5. Imaging result without reconstruction (left) and with reconstruction (right)

Compared with the basic reconstruction method using NUDFT, the proposed fast reconstruction method using NUFFT reduces the computational complexity significantly from $O(L^2 N_a^{\ 2})$ to $O(LN_a \log_2(LN_a))$. For the 2-D imaging simulation, the computer processing time is shown in Table III.

TABLE III. COMPUTER PROCESSING TIME OF RECONSTRUCTION METHOD USING NUDFT AND NUFFT

Method	**Time/s**
The basic reconstruction using NUDFT	1707.4
The fast reconstruction using NUFFT	1.285

V. CONCLUSION

This paper presents azimuth reconstruction for MIMO SAR based on NUFFT method. The MIMO SAR nonuniform azimuth sampling signal modeling and the basic reconstruction method is deduced. Then, a fast reconstruction method with NUFFT is proposed. Simulation results of azimuth compression and two-dimensional imaging are depicted to verify the proposed reconstruction method. In addition, the computer processing time of simulation shows that the proposed method is much more efficient than the basic method. The spectrum reconstruction performance of NUFFT with different kernels will be researched in the future work.

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