

## Joint Learning of Local Fingerprint and Content Modulation

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### ABSTRACT

This paper proposes learning a linear map with local content modulation for robust content fingerprinting. The goal is to estimate a data adapted linear map that provides bounded modulation distortion and features with targeted properties. A novel problem formulation is presented that jointly addresses the fingerprint learning and the content modulation. A solution by iterative alternating algorithm is proposed. The algorithm alternates between linear map update step and linear modulation estimate step. Global optimal solutions for the respective iterative steps are proposed, resulting in convergent algorithm with locally optimal solution.

A computer simulation using local image patches, extracted from publicly available data set is provided. The advantages under additive white Gaussian noise (AWGN), lossy JPEG compression and projective geometrical transform distortions are demonstrated.

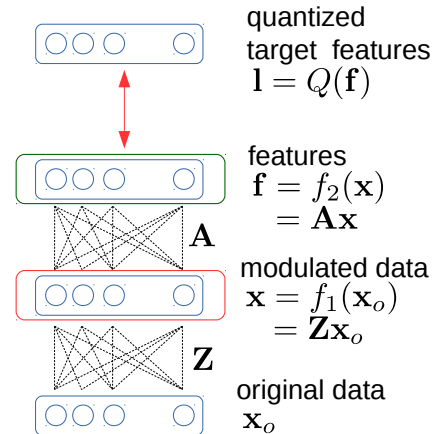
**Index Terms**— active content fingerprint, modulation, feature map learning, robustness

### 1. INTRODUCTION

Active Content Fingerprinting (aCFP) has emerged as a synergy between digital watermarking (DWM) and passive content fingerprinting (pCFP) [1]. This alternative approach covers a range of applications in the case when content modulation is appropriate, prior to content distribution/reproduction. Possible advantages may be seen in a number of applications, including content authentication, identification and recognition.

Recently, theoretically it was demonstrated that the identification capacity of aCFP [2] under additive white Gaussian channel distortions and  $\ell_2$ -norm embedding distortion is considerably higher to those of DWM and pCFP. Several scalar and vector modulation schemes for the aCFP have been proposed [3, 4] and have been tested on synthetic signals and collections of images. Despite of the attractive theoretical properties of aCFP, the practical implementation of aCFP modulation with an acceptable complexity, remains an open and challenging problem.

On the other hand in recent years, local, i.e., *patch-based*, compact, geometrically robust, binary descriptors such as SIFT [5], BRIEF [6], BRISK [7], ORB[8] and the family of



**Fig. 1.** Local active content fingerprint learning (aFIL) scheme. The linear modulation function is denoted as  $f_1(\cdot)$  and the linear feature extraction function is denoted as  $f_2(\cdot)$ . The resulting fingerprint is the quantized feature.

LBP [9] became a popular tool in image processing, computer vision and machine learning. These local descriptors are also considered as a form of local pCFP.

However, up to our best knowledge, there is small amount prior work on the modulation of local descriptors in the scope of aCFP or DWM. In [10] an aCFP with a linear modulation subject to convex constraint on the properties of the resulting local descriptors was proposed, together with an optimal solution when the feature map is invertible. The main open issues with the proposed optimal solution are related to the assumptions about the linear feature map. The authors in [11] addressed the general case from two distinct perspectives. Firstly, they propose a direct approximation of the linear feature map and secondly they present a novel problem formulation for the linear modulation and the constraints on the properties of the resulting local descriptor. In the former case, the used linear map is predefined and analytic, therefore the open issues are related to the properties of the used linear map that are crucial for the achievable modulation distortion and the resulting feature descriptor.

This paper proposes to learn a linear feature map with linear aCFP modulation to reduce the modulation distortion and explicitly regularize the features properties. The contributions of the paper are: (i) a novel problem formulation for joint

linear map learning and linear data modulation, (ii) an iterative alternating algorithm with optimal solutions in the corresponding iterating steps, (iii) a convergence result for the iterating sequence of the objective function values generated by the iterating steps of the proposed algorithm and (iv) validation by a computer simulation using publicly available data set of images under several image processing distortions, including AWGN, lossy JPEG compression and projective geometrical transform.

The paper is organized as follows. In Section 2 the problem is introduced, a short description of the local pCFP is given and the local aCFP modulation is presented. In Section 3 the main result is stated. Section 4 is devoted to computer simulation and Section 5 concludes the paper.

## 2. LEARNING LINEAR MAP AND MODULATION

This paper proposes joint linear feature learning and linear modulation scheme named as local active content Fingerprint Learning (aFIL). The aFIL scheme is shown in Figure 1. The modulation is prior to the content reproduction and the descriptor extraction includes feature mapping using a learned liner map and quantization.

The core idea behind the aCFP modulation [3] is based on the observation that the magnitude of the feature coefficients before the quantization influences the probability of the bit error in the descriptor bits. The descriptor bit flipping is more likely for low magnitude coefficients. Therefore, it is natural to modify the original content by an appropriate modulation and to increase these magnitudes subject to some distortion constraint. Obviously, the modulation faces a trade-off between two conflicting requirements of feature coefficient magnitude increase for the probability of bit error reduction and the modulation distortion. Fortunately, the low magnitude coefficient are concentrated near zero and are easily affected by a low distortion modulation.

Note that the local aFIL scheme is also applicable in the context of global image descriptors. Nevertheless, considering either global or local image description, here a novel approach is presented, alternatively to the scheme considered in [4], [10] and [11]. Most importantly we highlight that the liner map is learned from data with explicitly regularized features properties.

### 2.1. Local pCFP: no patch modulation

Given a patch  $\mathbf{x}_o$  in the most general case, the local patch-based features are extracted using a mapping function  $f_2 : \mathbb{R}^{N \times 1} \rightarrow \mathbb{R}^{M \times 1}$ , where  $M$  is the length of the descriptor. Consider a linear function  $\mathbf{f}_o = f_2(\mathbf{x}_o) = \mathbf{A}\mathbf{x}_o$ , where  $\mathbf{A} \in \mathbb{R}^{M \times N}$  is a linear map. Note that the map can be either pre-defined, data independent and analytic or learned, data dependent and content adaptive.

The mapping, followed by a quantization  $Q(\cdot)$  results in a quantized local descriptor denoted as  $\mathbf{b}_{x_o} = Q(\mathbf{f}_o)$ . The differences between the existing classes of local descriptors are determined by the defined mapping  $f_2(\cdot)$  and the type of the quantization  $Q(\cdot)$ .

### 2.2. Local aFIL: patch modulation and learning

The analysis here is focused on the optimal solution with learning linear modulation and feature extraction maps and scalar quantizers.

**Data adaptive linear modulation.** We consider linear aCFP modulation  $f_1 : \mathbb{R}^{N \times 1} \rightarrow \mathbb{R}^{N \times 1}$ ,  $\mathbf{x} = f_1(\mathbf{x}_o) = \mathbf{Z}\mathbf{x}_o$ , with  $\mathbf{Z} \in \mathbb{R}^{N \times N}$ .

**Data adaptive linear feature extraction.** The considered feature extraction is linear  $\mathbf{f}_o = f_2(\mathbf{x}_o) = \mathbf{A}\mathbf{x}_o$ , where  $\mathbf{A} \in \mathbb{R}^{M \times N}$ ,  $M < N$  and is learned from data.

**Binary quantization.** Let  $\mathbf{A}\mathbf{x}_o, \mathbf{A}\mathbf{x}_e, \mathbf{A}\mathbf{x}_n \in \mathbb{R}^M$ , where  $\mathbf{x}_n$  and  $\mathbf{x}_e$  are the modulation and the signal processing distortions, respectively. The quantization is defined as  $\mathbf{b}_y = Q(\mathbf{f})$ , where  $\mathbf{f} \in \mathbb{R}^M$ ,  $Q(a) = \text{sign}(a) = 1$ , if  $a \geq 0$  and 0, otherwise. The content fingerprint extracted from original content is  $\mathbf{b}_{x_o} = Q(\mathbf{A}\mathbf{x}_o)$ , from non-distorted modulated content is  $\mathbf{b}_x = Q(\mathbf{A}\mathbf{x}_o + \mathbf{A}\mathbf{x}_n)$  and from distorted modulated content is  $\mathbf{b}_y = Q(\mathbf{A}\mathbf{x}_o + \mathbf{A}\mathbf{x}_n + \mathbf{A}\mathbf{x}_e)$ .

## 3. LOCAL ACTIVE CONTENT FINGERPRINT LEARNING (AFIL)

Assume that a data set  $\mathbf{X}_o = [\mathbf{x}_{o,1}, \mathbf{x}_{o,2}, \dots, \mathbf{x}_{o,L}] \in \mathbb{R}^{N \times L}$  and a corresponding target feature matrix  $\mathbf{L} = [l_1, l_2, l_3, \dots, l_L] \in \{-1, 1\}^{M \times L}$  is given, with a total of  $L$  available data samples. The learning of a linear feature map for aCFP with linear modulation is addressed by considering the following problem formulation:

$$\begin{aligned} \min_{\mathbf{X}, \mathbf{A}} g(\mathbf{X}, \mathbf{A}) &= \min_{\mathbf{X}, \mathbf{A}} \Omega_1(\mathbf{X}_o, \mathbf{X}) + \Omega_2(\mathbf{A}) \\ &\text{subject to} \\ (\mathbf{A}\mathbf{X}) \odot \mathbf{L} &\geq_e \tau \mathbf{1}\mathbf{1}^T, \end{aligned} \quad (1)$$

where  $\odot$  denotes Hadamar product,  $\geq_e$  denotes element-wise inequality,  $\mathbf{X}$  are the *modulated data* and the matrix  $\mathbf{A}$  is the *linear map*. The terms  $\Omega_1(\mathbf{X}_o, \mathbf{X})$ ,  $\Omega_2(\mathbf{A})$  and the inequity  $(\mathbf{A}\mathbf{X}) \odot \mathbf{L} \geq_e \tau \mathbf{1}\mathbf{1}^T$  induce constraints on *modulation error*, the *properties of the linear map*  $\mathbf{A}$  and the *modulated features*  $\mathbf{A}\mathbf{x}_i$ , respectively.

The penalty  $\Omega_1(\mathbf{X}_o, \mathbf{X})$  is defined as  $\Omega_1(\mathbf{X}_o, \mathbf{X}) = \frac{\lambda_1}{2} \sum_{i=1}^L \|\mathbf{x}_{o,i} - \mathbf{x}_i\|_2^2 + \frac{\lambda_2}{2} \sum_{i=1}^L \|\mathbf{A}(\mathbf{x}_{o,i} - \mathbf{x}_i)\|_2^2$  and  $\Omega_2(\mathbf{A})$  is defined as  $\Omega_2(\mathbf{A}) = \frac{\lambda_3}{2} \|\mathbf{A}\|_F^2 - \lambda_4 \log |\det \mathbf{A}^T \mathbf{A}|$ , respectively, where  $\lambda_k$  are Lagrangian multipliers  $\forall k \in \{1, 2, 3, 4\}$ . The  $\|\mathbf{A}\|_F$  penalty helps regularize the scale ambiguity in the solution of (1). The  $\log |\det \mathbf{A}^T \mathbf{A}|$  and  $\|\mathbf{A}\|_F^2$  are functions of the singular values of  $\mathbf{A}$  and together help regularize the conditioning of  $\mathbf{A}$ .

Problem (1) is non-convex in the variables  $\mathbf{A}$  and  $\mathbf{X}$ . If the variable  $\mathbf{A}$  is fixed, (1) is convex, conversly if  $\mathbf{X}$  is fixed (1) is convex again. This paper proposes an alternating algorithm that has two steps and solves (1) by iteratively updating  $\mathbf{A}$  and  $\mathbf{X}$ . In **step 1** (*Linear modulation*) given the linear map  $\mathbf{A}$ , the modulated data  $\mathbf{X}$ , are estimated by a global optimal solution. In **step 2** (*Liner map estimate*) given  $\mathbf{X}$  the linear map  $\mathbf{A}$  is estimated by an global optimal solution.

### 3.1. Step 1 : Linear modulation

Given the linear map  $\mathbf{A}^{t-1}$ , note that (1) is separable for all  $\mathbf{x}_i^t$ , and per individual  $\mathbf{x}_i^t$  reduces to the following problem:

$$\begin{aligned} \min_{\mathbf{x}_i^t} & \frac{\lambda_1}{2} \|\mathbf{x}_{o,i} - \mathbf{x}_i^t\|_2^2 + \frac{\lambda_2}{2} \|\mathbf{A}^{t-1}(\mathbf{x}_{o,i} - \mathbf{x}_i^t)\|_2^2 \\ \text{subject to} & \\ & (\mathbf{A}^{t-1}\mathbf{x}_i^t) \odot \mathbf{l}_i \geq_e \tau \mathbf{1}, \end{aligned} \quad (2)$$

where  $\mathbf{x}_i^t$  is the modulated data that has to be estimated and the modulation distortion is related to the error  $\|\mathbf{x}_{o,i} - \mathbf{x}_i^t\|_2^2$ . Introduce an auxiliary variable as  $\mathbf{v} \in \mathfrak{R}^M$  and define an element-wise indicator function  $\mathbb{I}(v(m)) = +\infty$ , if  $v(m) > 0$  and  $\mathbb{I}(v(m)) = 0$ , if  $v(m) = 0$ , then (2) equivalently is:

$$\begin{aligned} \min_{\mathbf{x}_i^t, \mathbf{v}} & \frac{\lambda_1}{2} \|\mathbf{x}_{o,i} - \mathbf{x}_i^t\|_2^2 + \frac{\lambda_2}{2} \|\mathbf{A}^{t-1}(\mathbf{x}_{o,i} - \mathbf{x}_i^t)\|_2^2 + \\ & \sum_m \mathbb{I}(v(m)) \\ \text{subject to} & \\ & (\mathbf{A}^{t-1}\mathbf{x}_i^t) \odot \mathbf{l}_i - \tau \mathbf{1} - \mathbf{v} =_e \mathbf{0}. \end{aligned} \quad (3)$$

The augmented Lagrangian of (3) is  $\mathcal{L}(\mathbf{x}_i, \mathbf{v}, \mathbf{s}) = \frac{\lambda_1}{2} \|\mathbf{x}_{o,i} - \mathbf{x}_i^t\|_2^2 + \frac{\lambda_2}{2} \|\mathbf{A}^{t-1}(\mathbf{x}_{o,i} - \mathbf{x}_i^t)\|_2^2 + \sum_m \mathbb{I}(v(m)) + \mathbf{s}^T ((\mathbf{A}^{t-1}\mathbf{x}_i^t) \odot \mathbf{l}_i - \tau \mathbf{1} + \mathbf{v}) + \frac{\rho}{2} \|(\mathbf{A}^{t-1}\mathbf{x}_i^t) \odot \mathbf{l}_i - \tau \mathbf{1} + \mathbf{v}\|_2^2$ . Denote for clarity and simplicity  $\mathbf{x}_o = \mathbf{x}_{o,i}$ ,  $\mathbf{x} = \mathbf{x}_i^t$ ,  $\mathbf{l} = \mathbf{l}_i$  and  $\mathbf{A} = \mathbf{A}^{t-1}$ . The *Alternating Direction Method of Multipliers* (ADMM) [12] is used for (3) and the problem is solved by iterating with the following 3 steps.

$$\begin{aligned} \mathbf{x}^k &= \underset{\mathbf{x}^k}{\operatorname{argmin}} \frac{\rho}{2} \|(\mathbf{A}\mathbf{x}^k) \odot \mathbf{l} - \tau \mathbf{1} - \mathbf{v}^{k-1} + \mathbf{s}^{k-1}\|_2^2 + \\ & \frac{\lambda_1}{2} \|\mathbf{x}_o - \mathbf{x}^k\|_2^2 + \frac{\lambda_2}{2} \|\mathbf{A}(\mathbf{x}_o - \mathbf{x}^k)\|_2^2 \\ \mathbf{v}^k &= \max((\mathbf{A}\mathbf{x}^k) \odot \mathbf{l} - \tau \mathbf{1} + \mathbf{s}^{k-1}, \mathbf{0}) \\ \mathbf{s}^k &= \mathbf{s}^{k-1} + (\mathbf{A}\mathbf{x}^k) \odot \mathbf{l} - \tau \mathbf{1} - \mathbf{v}^k. \end{aligned} \quad (4)$$

Note that the problem related to  $\mathbf{x}^k$  has a closed form solution as:

$$\begin{aligned} \mathbf{x}^k &= \mathbf{B}^\dagger \mathbf{B}^T (\rho \mathbf{A}^T ((\tau \mathbf{1} - \mathbf{v}^{k-1} + \mathbf{s}^{k-1}) \odot \mathbf{l}) + \\ & (\lambda_1 \mathbf{I} + \lambda_2 \mathbf{A}^T \mathbf{A}) \mathbf{x}_o), \end{aligned} \quad (5)$$

where  $\mathbf{B} = (\rho + \lambda_2) \mathbf{A}^T \mathbf{A} + \lambda_1$ . The matrices  $\mathbf{B}$  and the pseudo-inverse  $\mathbf{B}^\dagger = (\mathbf{B}\mathbf{B}^T)^{-1} \mathbf{B}^T$  are computed only once and reused in the solutions for all  $\mathbf{x}^k$  and for all  $\mathbf{x}_i^t, i \in \mathcal{I}$ .

Since (3) is equality constrained convex optimization problem over the non-negative orthant  $\mathfrak{R}_+^M$  the procedure with the ADMM (4) gives the optimal solution to (3).

### 3.2. Step 2 : Linear map estimation

Let the original data  $\mathbf{X}_o$ , modulated data  $\mathbf{X}$  and the linear map  $\mathbf{A}^{t-1}$  be given, then (1) reduces to the following problem:

$$\begin{aligned} \min_{\mathbf{A}^t} & \frac{\lambda_2}{2} \|\mathbf{A}^t(\mathbf{X}_o - \mathbf{X}^t)\|_2^2 + \frac{\lambda_3}{2} \|\mathbf{A}^t\|_F^2 - \\ & \lambda_4 \log |\det((\mathbf{A}^t)^T \mathbf{A}^t)| \\ \text{subject to} & \\ & (\mathbf{A}^t \mathbf{X}^t) \odot \mathbf{L} \geq_e \tau \mathbf{1} \mathbf{1}^T. \end{aligned} \quad (6)$$

Again by introducing an auxiliary variable  $\mathbf{W} \in \mathfrak{R}^{M \times L}$ , problem (6) equivalently has the following form:

$$\begin{aligned} \min_{\mathbf{A}^t, \mathbf{W}} & \frac{\lambda_2}{2} \|\mathbf{A}^t(\mathbf{X}_o - \mathbf{X}^t)\|_2^2 + \frac{\lambda_3}{2} \|\mathbf{A}^t\|_F^2 - \\ & \lambda_4 \log |\det((\mathbf{A}^t)^T \mathbf{A}^t)| + \sum_{m,l} \mathbb{I}(W(m,l)) \\ \text{subject to} & \\ & (\mathbf{A}^t \mathbf{X}^t) \odot \mathbf{L} - \tau \mathbf{1} \mathbf{1}^T - \mathbf{W} =_e \mathbf{0} \mathbf{0}^T. \end{aligned} \quad (7)$$

Problem (7) is approached similary as in the previous subsection, therefore, first denote  $\mathbf{A} = \mathbf{A}^t$  and  $\mathbf{X} = \mathbf{X}^t$  then the augmented Lagrangian to (7) is evaluated as  $\mathcal{L}(\mathbf{A}, \mathbf{W}, \mathbf{Q}) = \frac{\lambda_2}{2} \|\mathbf{A}(\mathbf{X}_o - \mathbf{X})\|_2^2 + \sum_{m,l} \mathbb{I}(W(m,l)) + \frac{\lambda_3}{2} \|\mathbf{A}\|_F^2 - \lambda_4 \log |\det(\mathbf{A}^T \mathbf{A}^t)| + \mathbf{S}^T ((\mathbf{A}\mathbf{X}) \odot \mathbf{L} - \tau \mathbf{1} \mathbf{1}^T - \mathbf{W}) + \frac{\rho}{2} \|(\mathbf{A}\mathbf{X}) \odot \mathbf{L} - \tau \mathbf{1} \mathbf{1}^T - \mathbf{W}\|_F^2$ .

Denote  $\mathbf{G} = \frac{\lambda_3}{2} \mathbf{I} + \frac{\lambda_2}{2} (\mathbf{X}_o - \mathbf{X})(\mathbf{X}_o - \mathbf{X})^T + \frac{\rho}{2} \mathbf{X} \odot \mathbf{L} \mathbf{L}^T \odot \mathbf{X}^T$  and  $\mathbf{Z}^{k-1} = \tau \mathbf{1} \mathbf{1}^T - \mathbf{W}^{k-1} + \mathbf{S}^{k-1}$  then the ADMM steps for the solution of (7) are the following:

$$\begin{aligned} \mathbf{A}^k &= \underset{\mathbf{A}^k}{\operatorname{argmin}} \operatorname{Tr}\{\mathbf{A}^k \mathbf{G} (\mathbf{A}^k)^T - \mathbf{A}^k \mathbf{X} (\mathbf{Z}^{k-1})^T\} - \\ & \lambda_4 \log |\det((\mathbf{A}^k)^T \mathbf{A}^k)| \\ \mathbf{W}^k &= \max((\mathbf{A}^k \mathbf{X}) \odot \mathbf{L} - \tau \mathbf{1} \mathbf{1}^T + \mathbf{S}^{k-1}, \mathbf{0}) \\ \mathbf{S}^k &= \mathbf{S}^{k-1} + (\mathbf{A}^k \mathbf{X}) \odot \mathbf{L} - \tau \mathbf{1} \mathbf{1}^T - \mathbf{W}^k. \end{aligned} \quad (8)$$

Note that for the problem related to  $\mathbf{A}^k$  a closed form solution exists as:

$$\mathbf{A}^k = \frac{1}{2} \mathbf{U} \left( \Sigma_{LXZ} + (\Sigma_{LXZ} + \lambda_4 \mathbf{I})^{1/2} \right) \mathbf{V} \mathbf{L}^{-1}, \quad (9)$$

where  $\mathbf{U} \Sigma_{LXZ} \mathbf{V}^T$  is the singular value decomposition of  $\mathbf{L}^{-1} \mathbf{X} (\mathbf{Z}^{k-1})^T$ ,  $\mathbf{L} \mathbf{L}^T$  is Cholesky factorization of  $\mathbf{G}$ . Due

to space limitations we refer to the complete proof given in [13].

Since (7) is convex problem, the iterative sequence by the solutions of the proposed ADMM method (8) converges to the optimal solution of (7).

### 3.3. Algorithm analysis: local convergence

This section presents the result on the local convergence of the proposed alternating algorithm.

**Lemma 1:** *Given initial  $\{\mathbf{X}_o, \mathbf{L}\}$ , the sequence  $\{\mathbf{A}^t, \mathbf{X}^t\}$  generated by the proposed algorithm is monotone decreasing i.e.,  $g(\mathbf{A}^t, \mathbf{X}^t) \leq g(\mathbf{A}^{t-1}, \mathbf{X}^t) \leq g(\mathbf{A}^{t-1}, \mathbf{X}^{t-1})$ , the function  $g(\mathbf{A}, \mathbf{X})$  is lower bounded therefore the alternating algorithm converges to an finite value denoted as  $g^*$*

*Proof:* Given  $\mathbf{A}^{t-1}$ ,  $g(\mathbf{A}^{t-1}, \mathbf{X}^t)$  is convex and the global optimal solution of  $g(\mathbf{A}^{t-1}, \mathbf{X}^t)$  is given by the iterative solution (4) therefore  $g(\mathbf{A}^{t-1}, \mathbf{X}^t) \leq g(\mathbf{A}^{t-1}, \mathbf{X}^{t-1})$ . Given  $\mathbf{X}^t$ , again  $g(\mathbf{A}^t, \mathbf{X}^t)$  is convex and the global optimal solution of  $g(\mathbf{A}^t, \mathbf{X}^t)$  is given by the iterative solution (8), combining the both results we have that  $g(\mathbf{A}^t, \mathbf{X}^t) \leq g(\mathbf{A}^{t-1}, \mathbf{X}^t) \leq g(\mathbf{A}^{t-1}, \mathbf{X}^{t-1})$ , implying that the sequence  $\{g(\mathbf{A}^t, \mathbf{X}^t)\}$  is monotone decreasing sequence. The result that the function  $g(\mathbf{A}^t, \mathbf{X}^t)$  is lower bounded is given in [13]. Since any lower bounded, monotone decreasing sequence is a convergent sequence then the proposed alternating algorithm converges to a local optimal value denoted as  $g^*$   $\square$

## 4. COMPUTER SIMULATIONS

A computer simulation is performed to demonstrate the advantages of the local aFIL scheme over aCFP and pCFP, under several signal processing distortions, including AWGN, lossy JPEG compression and projective geometrical transform. The goal is to achieve the same performance as the aCFP method proposed in [11], however with considerably lower distortion.

The UCID [14] image database was used to extract local image patches. The ORB detector [8] was run on all images, and  $\sqrt{N} \times \sqrt{N}$  pixel patches, with  $\sqrt{N} = 31$  were extracted around each detected feature point. The features were sorted by scale-space, 30 patches were extracted from individual image.

We consider three scenarios: **pCFP**, **aCFP** [11] and **aFIL** for the computer simulation. In order to make fair comparison between pCFP, aCFP and aFIL we use one predefined matrix  $\mathbf{A}$  for the pCFP and aCFP scenario. For the aFIL we use the same matrix  $\mathbf{A}$  to initialize the proposed algorithm and define the target labels as  $\mathbf{L} = \text{sign}(\mathbf{A}\mathbf{X}_o)$ . Half of the total 1000 image patches are used for aFIL learning with target feature  $\mathbf{L}$ .

**Three measured quantities** are used for evaluation:

1) *the modulation distortion*, 2) *the probability of bit error* and 3) *the modulation level*. The first is defined as

$mL$		$p_e$				
		pCFP	aCFP	aFIL		
$DWR$			10	60	10	60
			33.8	4.7	<b>36.1</b>	<b>6.9</b>
AWGN	0dB	.224	.217	.064	<b>.216</b>	<b>.063</b>
	5dB	.150	.142	<b>.022</b>	<b>.14</b>	<b>.022</b>
	10dB	.095	<b>.084</b>	.005	.085	<b>.004</b>
	20dB	.034	<b>.018</b>	0	.019	<b>0</b>
QF	0	.082	<b>.074</b>	.025	.075	<b>.024</b>
	5	.082	<b>.040</b>	<b>.015</b>	.041	<b>.015</b>
	10	.028	<b>.015</b>	.012	<b>.015</b>	<b>.011</b>
Proj., QF=5		.058	.049	.048	<b>.049</b>	<b>.047</b>

**Table 1.** The  $DWR$  and  $p_e$  using varying aCFP modulation under varying AWGN noise, JPEG quality factor and Projective transformation with QF=5.

$DWR = 10 \log_{10} \left( \frac{255^2}{\Delta} \right)$ ,  $\Delta = \frac{1}{N} \|\mathbf{x} - \mathbf{x}_o\|_2^2$ . The second one is defined by the average probability of bit error  $p_e = \frac{1}{L} \sum_{i=1}^L \mathbb{1}\{b_x(i) \neq b_y(i)\}$  with  $L = 256$  bits, where  $\mathbb{1}\{\cdot\}$  is indicator function that returns 1 if the argument is true and 0, otherwise. The third measure is the modulation level  $mL$ , expressed in percentage  $mL = \frac{K}{L} 100$ ,  $1 \leq K \leq L$ . This measure represents the fraction of coefficients  $\mathbf{x}_o$  that are modified. At single modulation level, the modulation threshold  $\tau$  is defined as  $\tau = \max_{1 \leq i \leq K} |s(i)|$ , where  $\mathbf{s}_o$ ,  $|s_o(i)| \leq |s_o(j)|$ ,  $1 \leq i \leq j \leq L$  is the sorted vector for the absolute values of  $\mathbf{A}\mathbf{x}_o$ .

**AWGN:** The results from a single patch was obtained as average of 100 AWGN realizations. Four different noise levels were used, defined in PSNR=  $10 \log_{10} \frac{255^2}{\sigma^2}$  are 0dB, 5dB, 10dB and 20dB. Two modulation levels (mL) were used 10 and 60.

**Lossy JPEG compression:** Three JPEG quality factors (QF) 0, 5 and 10 were used. The modulation levels (mL) that were used are 10 and 30.

**Projective transform with lossy JPEG compression:** A projective transformation  $\mathbf{P} \in \mathbb{R}^{3 \times 3}$  with parameter matrix:

$$\mathbf{P} = \begin{bmatrix} 1.0763 & 0.0325 & 0 \\ 0.0119 & 1.09 & 0 \\ -24.32 & -70.37 & 1 \end{bmatrix},$$

was used, followed by a lossy JPEG compression with QF=5. The modulation levels (mL) that were used are 10 and 60.

Table 1 summarizes the average results for 500 image patches that are used for testing.

The results show that the pair of highest  $DWR$  and lowest  $p_e$  is achieved for the aFIL scenario under all types of noise.

## 5. CONCLUSION

This paper presented local patch based active content Fingerprint Learning (aFIL) with objective to estimate data adaptive linear map that provides small active Content FingerPrint (aCFP) modulation distortion and features with targeted properties. Novel problem formulation was presented that jointly addresses the fingerprint learning and the content modulation. A solution by iterative alternating algorithm was proposed. A global optimal solutions for the respective iterative steps were proposed, resulting in convergent algorithm with locally optimal solution.

The computer simulation using local image patches, extracted from publicly available data set was provided. The results demonstrated that the proposed algorithm achieves small  $p_e$  under different and severe signal processing distortions. More importantly the introduced modulation distortion is smaller by using data adapted linear feature map compared to the modulation distortion by linear feature map without data adaptation.

A study on the overall performance for larger image collection, together with extensions considering other priors without target features is left for our future work.

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