Denoising Galaxy Spectra with Coupled Dictionary Learning

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***Abstract*—The Euclid satellite aims to measure accurately the global properties of the Universe, with particular emphasis on the properties of the mysterious Dark Energy that is driving the acceleration of its expansion. One of its two main observational probes relies on accurate measurements of the radial distances of galaxies through the identification of important features in their individual light spectra that are *redshifted* due to their receding velocity. However, several challenges for robust automated spectroscopic redshift estimation remain unsolved, one of which is the characterization of the types of spectra present in the observed galaxy population. This paper proposes a denoising technique that exploits the mathematical frameworks of Sparse Representations and Coupled Dictionary Learning, and tests it on simulated Euclid-like noisy spectroscopic templates. The reconstructed spectral profiles are able to improve the accuracy, reliability and robustness of automated redshift estimation methods. The key contribution of this work is the design of a novel model which considers coupled feature spaces, composed of high- and low-quality spectral profiles, when applied to the spectroscopic data denoising problem. The coupled dictionary learning technique is formulated within the context of the Alternating Direction Method of Multipliers, optimizing each variable via closed-form expressions. Experimental results suggest that the proposed powerful coupled dictionary learning scheme reconstructs successfully spectral profiles from their corresponding noisy versions, even with extreme noise scenarios.**

I. Introduction

The Euclid satellite[1] is a space mission currently under development by the European Space Agency (ESA) [1]. It aims to improve our understanding of the evolution and of the major constituents of the Universe over the past 10 billion years. It will focus on two main cosmological probes: the spatial correlations of galaxy shape distortions from the weak gravitational lensing effect due to the total dark matter distribution along the photon paths, and the 3D correlation function of galaxy positions, which tracks the characteristic scale of primordial baryonic acoustic oscillations (BAO) imprinted in the galaxy distribution. This latter measurement, in particular, requires a large number of exquisite redshift measurements to allow a precise determination of the radial distances of galaxies. To that effect, Euclid will perform a spectroscopic survey of over 50 million galaxies with slitless spectroscopy in the near-infrared wavelength range.

Spectroscopic redshift estimation from Euclid data presents several complex challenges. Current redshift estimation techniques [2], [3] possess limited efficiency in the low signal-to-noise regime in which Euclid will observe most of the high-redshift galaxies. In particular, one of the main sources of systematic uncertainties is the definition of the spectral energy distributions (SEDs) templates required by the most redshift estimation methods. For cross-correlation techniques to work appropriately, the SED template library needs to be representative of the existing dataset, otherwise the chosen best-fit template and redshift might not be correct. Existing SED libraries, mostly defined from lower redshift surveys, may suffer from such representativeness problems in ways that are hard to quantify. Additionally, these SEDs have been observed with different instruments, under a wide range of observational conditions, that might not be extensible to unknown regimes. Therefore, a new instrument like Euclid's slitless spectrograph, probing a much less explored high-redshift regime, is vulnerable to the impact of previously uncharacterised selection effects in the SED libraries.

Enhancing galaxy spectra has been a subject of significant research for many years. Recently, the authors in [4], proposed a sparse-based denoising technique for hyperspectral astrophysical data, adhering to the specificities of the MUSE instrument[2]. Another spectroscopic enhancement technique was developed in [5] where the authors tackle the problem of joint signal restoration and parameter estimation of the MUSE instrument, using a sparsity regularization technique, based on the Iterative Coordinate Descent (ICD) principle. Additionally, the authors in [6] examine the problems of deconvolution and denoising of spectroscopic data.

In this work we focus on the enhancement of Euclid-like simulated spectroscopic data. Specifically, this paper employs the concept of *spectroscopic data denoising*, where noisy and clean training examples are used within a computational learning framework to enhance the degraded spectroscopic data. Us-

[1]http://sci.esa.int/euclid/

[2]https://www.eso.org/sci/facilities/develop/instruments/muse.html

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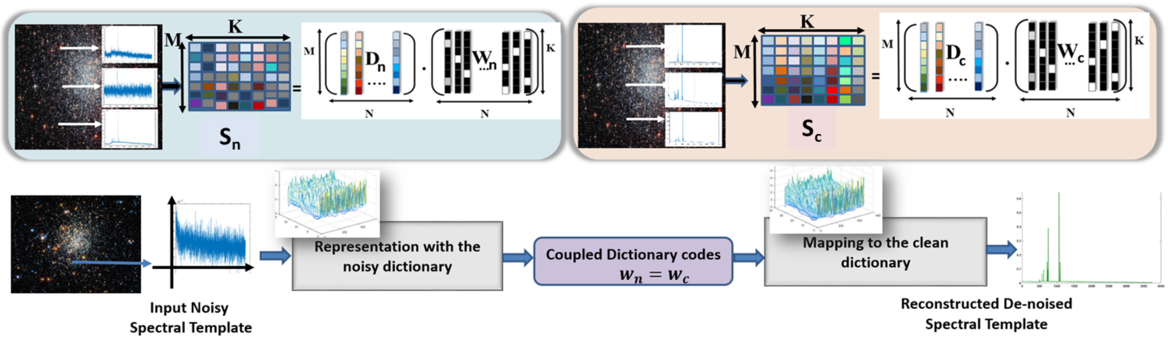

Fig. 1: *Proposed system's block diagram: The system takes as input an acquired noisy spectral profile and produces an estimate of its denoised version. During the training phase, multiple "clean" and noisy spectral templates are utilized. Given these signal pairs, a coupled dictionary learning scheme is employed for learning two dictionaries, corresponding to the two different noisy scenarios. During the testing phase, noisy spectral profiles are mapped to the noisy dictionary and the identified sparse coding coefficients are subsequently combined with the denoised dictionary for producing the denoised estimates.*

ing a dual dataset, consisting of clean and noisy matched signal samples, we are able to learn the spectral energy distributions directly from the data, and subsequently use them for redshift estimation. In order to validate the reconstruction performance, we test our method with a set of fully clean simulated Euclid-like spectra, matched to corrupted versions with different noise levels. The proposed algorithm capitalizes on the *Sparse Representations* framework [7] and extents it by introducing a *Coupled Dictionary Learning* process for reconstructing the clean spectra data from their equivalent noisy versions. Furthermore, we solve the galaxy spectra denoising problem within the highly efficient Alternating Direction Method of Multipliers optimization framework [8], [9]. Figure 1 presents the proposed system's block diagram.

The rest of this paper is structured as follows. Section II provides the proposed spectroscopic-data denoising technique considered in this work, whereas Section III describes the associated coupled dictionary learning formulation. Section IV reports the experimental results, while conclusions and extensions of this work are presented in Section V.

II. Denoising of Spectroscopic Data

Our approach synthesizes a denoised spectral profile from its corresponding acquired noisy version by capitalizing on the *Sparse Representations* (SR) learning framework [7]. According to the SR framework, various spectral profiles can be represented as sparse linear combinations of a few elements from learned over-complete dictionaries. Traditional approaches consider a set of noisy and denoised signal pairs and assume that these signals are generated by the same statistical process under different noise conditions, and as such, they share approximately the same sparse coding, with respect to their corresponding noisy $\mathbf{D}_n \in \mathbb{R}^{M\times N}$, and denoised $\mathbf{D}_c \in \mathbb{R}^{M\times N}$ dictionaries. Each input noisy spectral template $\mathbf{s}_n \in \mathbb{R}^M$ can thus be expressed as a sparse linear combination, encoded in $\mathbf{w} \in \mathbb{R}^N$, of elements from a dictionary matrix, $\mathbf{D}_n \in \mathbb{R}^{M\times N}$, composed of training noisy spectral profiles, according to: $\mathbf{s}_n = \mathbf{D}_n\mathbf{w}$.

Although the ℓ_0-norm is theoretically the best regularizer for promoting sparsity, it leads to an intractable optimization. This problem is alleviated by replacing the ℓ_0-norm by its convex surrogate ℓ_1-norm, leading to robust solutions. The optimization problem is therefore formulated as:

$$\mathbf{w}^\star = \arg\min_{\mathbf{w}} ||\mathbf{s}_n - \mathbf{D}_n\mathbf{w}||_F^2 + \rho||\mathbf{w}||_1, \tag{1}$$

where ρ stands for a regularization parameter, that balances the fidelity of the solution. To obtain the denoised spectral profile, the optimal sparse code $\mathbf{w}^\star$ from (1), is directly projected onto the clean dictionary $\mathbf{D}_c \in \mathbb{R}^{M\times N}$, to synthesize the denoised spectral profile, according to $\mathbf{s}_c = \mathbf{D}_c\mathbf{w}^\star$. The two main challenges pertaining to the estimation of the denoised spectral profiles are related to the sufficient sparsity measure for the sparse coding vector $\mathbf{w}$ and the proper construction of the dictionary matrices, to efficiently sparsify the input signals. The following section describes explicitly the coupled dictionary procedure developed in this work.

III. Coupled Dictionary Learning

Coupled Dictionary Learning relies on generating a pair of dictionaries which jointly encode the noisy, $\mathbf{S}_n$ and the clean $\mathbf{S}_c$ feature spaces, where the signals have sparse representations in terms of the trained dictionaries [10]. The main task is to find a coupled dictionary pair $\mathbf{D}_n$ and $\mathbf{D}_c$ for the spaces $\mathbf{S}_n$ and $\mathbf{S}_c$, respectively [7]. Formally, one such pair, can be estimated by solving the following set of sparse decompositions:

$$\begin{aligned} &\underset{\mathbf{D}_c,\mathbf{D}_n,\mathbf{W}_c,\mathbf{W}_n}{\operatorname{argmin}} \quad ||\mathbf{S}_c - \mathbf{D}_c\mathbf{W}_c||_F^2 + ||\mathbf{S_n} - \mathbf{D}_n\mathbf{W}_n||_F^2 + \\ &\lambda_c||\mathbf{W}_c||_1 + \lambda_n\ ||\mathbf{W}_n||_1,\ \text{subject to } \mathbf{W}_c = \mathbf{W}_n, \\ &||\mathbf{D}_c(:,i)||_2 \leq 1,\ \ ||\mathbf{D}_n(:,i)||_2 \leq 1 \end{aligned} \tag{2}$$

where $\mathbf{W}_n$ and $\mathbf{W}_c$ correspond to the sparse coefficient matrices for the noisy and clean feature spaces respectively, while λ_n and λ_c denote the sparsity regularization parameters. A straightforward approach is to concatenate the coupled

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feature spaces and utilize a common sparse representation $\mathbf{W}$, able to reconstruct both $\mathbf{S}_c$ and $\mathbf{S}_n$, by solving:

$$\underset{\mathbf{D},\mathbf{W}}{\text{argmin}}||\bar{\mathbf{S}} - \bar{\mathbf{D}}\mathbf{W}||_F + \lambda||\mathbf{W}||_1 \quad (3)$$
$$\text{subject to } ||\bar{\mathbf{D}}(:,j)||_2^2 \leq 1, \quad j = \{1,...,K\},$$

where $\bar{\mathbf{S}} = \begin{bmatrix}\mathbf{S}_c \\ \mathbf{S}_n\end{bmatrix}$, $\bar{\mathbf{D}} = \begin{bmatrix}\mathbf{D}_c \\ \mathbf{D}_n\end{bmatrix}$, and λ is the sparsity regularization term. As a result, the problem posed in (2) is converted into a standard sparse dictionary learning problem, which can be efficiently solved via existing algorithms, such as the K-SVD [11]. However, a major limitation of single dictionary learning strategies, is their inability to guarantee that the same sparse coding can be independently utilized by the different signal resolutions.

In order to overcome this limitation, we propose a computationally efficient *Coupled Dictionary Learning* technique, based on the Alternating Direction Method of Multipliers (ADMM) [8], [9], that converts the constrained dictionary learning problem in (2), into an unconstrained version which can be efficiently solved via alternating minimizations. The main task of coupled dictionary learning is to recover both the dictionaries $\mathbf{D}_n$ and $\mathbf{D}_c$ with their corresponding sparse codes $\mathbf{W}_n$ and $\mathbf{W}_c$, under the constraint $\mathbf{W}_n = \mathbf{W}_c$, by solving the following sparse matrix decomposition problems:

$$\underset{\mathbf{D}_c,\mathbf{W}_c}{\text{argmin}}||\mathbf{D}_c\mathbf{W}_c - \mathbf{S}_c||_F + \lambda_c||\mathbf{W}_c||_1, ||\mathbf{D}_c(:,j)||_2^2 \leq 1 \quad (4)$$
$$\underset{\mathbf{D}_n,\mathbf{W}_n}{\text{argmin}}||\mathbf{D}_n\mathbf{W}_n - \mathbf{S}_n||_F + \lambda_n||\mathbf{W}_n||_1, ||\mathbf{D}_n(:,j)||_2^2 \leq 1,$$

To apply the ADMM technique in our dictionary learning scheme, we reformulate the minimization problem in (4) as:

$$\min_{\mathbf{D_c},\mathbf{W_c},\mathbf{D}_n,\mathbf{W}_n} ||\mathbf{S}_c - \mathbf{D}_c\mathbf{W}_c||_F^2 + ||\mathbf{S}_n - \mathbf{D}_n\mathbf{W}_n||_F^2 + \quad (5)$$
$$\lambda_n||\mathbf{Q}||_1 + \lambda_c||\mathbf{P}||_1, \text{ subject to: } \mathbf{P} = \mathbf{W}_c, \mathbf{Q} = \mathbf{W}_n,$$
$$\mathbf{W}_c = \mathbf{W}_n, ||\mathbf{D}_c(:,i)||_2^2 \leq 1, ||\mathbf{D}_n(:,i)||_2^2 \leq 1$$

The ADMM technique takes into account the separate structure of each variable in (5), relying on the minimization of its augmented Lagrangian function:

$$\mathcal{L}(\mathbf{D}_c, \mathbf{D}_n, \mathbf{W}_c, \mathbf{W}_n, \mathbf{P}, \mathbf{Q}, Y_1, Y_2, Y_3) =$$
$$\frac{1}{2}||\mathbf{D}_c\mathbf{W}_c - \mathbf{S}_c||_F^2 + \frac{1}{2}||\mathbf{D}_n\mathbf{W}_n - \mathbf{S}_n||_F^2 + \lambda_c||\mathbf{P}||_1 +$$
$$\lambda_n||\mathbf{Q}||_1 + < Y_1, \mathbf{P} - \mathbf{W}_c > + < Y_2, \mathbf{Q} - \mathbf{W}_n > +$$
$$< Y_3, \mathbf{W}_c - \mathbf{W}_n > + \frac{c_1}{2}||\mathbf{P} - \mathbf{W}_c||_F^2 +$$
$$\frac{c_2}{2}||\mathbf{Q} - \mathbf{W}_n||_F^2 + \frac{c_3}{2}||\mathbf{W}_c - \mathbf{W}_n||_F^2, \quad (6)$$

where $\mathbf{Y}_1$, $\mathbf{Y}_2$, and $\mathbf{Y}_3$ stand for the Lagrange multiplier matrices, while $c_1 > 0$, $c_2 > 0$, and $c_3 > 0$ denote the step size parameters. We empirically set the step size parameters to $c_1 = c_2 = 0.4$ and $c_3 = 0.8$. Following the algorithmic strategy of the ADMM scheme, we seek for the stationary point, solving iteratively for each one of the variables, while keeping the others fixed. The overall algorithm for learning the coupled dictionaries is summarized in **Algorithm 1**.

Algorithm 1 Coupled Dictionary Learning

Input: training examples $\mathbf{S}_c$ and $\mathbf{S}_n$, iterations: K, step size parameters: c_1, c_2, c_3.
Initialize: $\mathbf{D}_c \in \mathbb{R}^{M \times N}$ and $\mathbf{D}_n \in \mathbb{R}^{M \times N}$: random selection of the columns of $\mathbf{S}_c$ and $\mathbf{S}_n$; Initialization of Lagrange multiplier matrices: $\mathbf{Y}_1 = \mathbf{Y}_2 = \mathbf{Y}_3 = \mathbf{0}$.
for $k = 1, \cdots, K$ do

1) Update $\mathbf{W}_c$ and $\mathbf{W}_n$:
$$\mathbf{W}_c = (\mathbf{D}_c^T\mathbf{D}_c + c_1 I + c_3 I)^{-1} \cdot (\mathbf{D}_c^T\mathbf{S}_c + Y_1 - Y_3 + c_1\mathbf{P} + c_3\mathbf{W}_n)$$
$$\mathbf{W}_n = (\mathbf{D}_n^T\mathbf{D}_n + c_2 I + c_3 I)^{-1} \cdot (\mathbf{D}_n^T\mathbf{S}_n + Y_2 + Y_3 + c_2\mathbf{Q} + c_3\mathbf{W}_c)$$
2) Update $\mathbf{P}$ and $\mathbf{Q}$:
$$\mathbf{P} = S_{\lambda_c}\left(\left|\mathbf{W}_c - Y_1/c_1\right|\right)$$
$$\mathbf{Q} = S_{\lambda_n}\left(\left|\mathbf{W}_n - Y_2/c_2\right|\right),$$
3) **for** $j = 1, \cdots, N$ do
   - Update ϕ_c and ϕ_n:
   $$\phi_c = \mathbf{W}_c(j,:)\mathbf{W}_c(j,:)^T$$
   $$\phi_n = \mathbf{W}_n(j,:)\mathbf{W}_n(j,:)^T$$
   - Update the dictionaries $\mathbf{D}_c$ and $\mathbf{D}_n$:
   $$\mathbf{D}_c^{(k+1)}(:,j) = \mathbf{D}_c^{(k)}(:,j) + (\mathbf{S}_c\mathbf{W}_c(j,:))/(\phi_c + \delta)$$
   $$\mathbf{D}_n^{(k+1)}(:,j) = \mathbf{D}_n^{(k)}(:,j) + (\mathbf{S}_n\mathbf{W}_n(j,:))/(\phi_n + \delta)$$

   end
   - Normalize $\mathbf{D}_c$ and $\mathbf{D}_n$ between $[0, 1]$
   - Update Lagrange multiplier matrices Y_1, Y_2 and Y_3:
   $$Y_1^{(k+1)} = Y_1^{(k)} + c_1(\mathbf{P} - \mathbf{W}_c)$$
   $$Y_2^{(k+1)} = Y_2^{(k)} + c_2(\mathbf{Q} - \mathbf{W}_n)$$
   $$Y_3^{(k+1)} = Y_3^{(k)} + c_3(\mathbf{W}_c - \mathbf{W}_n)$$

end

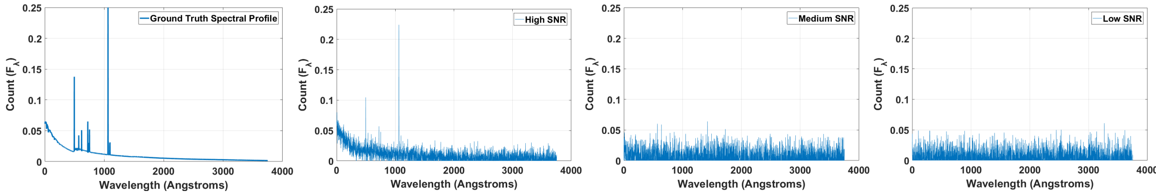
IV. Experimental Setup

In this Section, we evaluate the performance of the proposed denoising scheme when applied to spectroscopic data in terms of the quality of the reconstructed signals. Experimental results provided in this work consider noisy Euclid-like simulated spectral templates. Specifically, we examine the proposed algorithm's reconstructions under various noise scenarios. In the next paragraph, we explicitly describe the dataset and the generated noisy spectra templates.

A. Generating Euclid-like simulated templates

Euclid will observe an estimated 50 million spectra through slitless spectroscopy, in the wavelength range $1.1 - 2.0\mu m$ with a mean resolution $R = 250$, where $R \equiv \lambda/\Delta\lambda$. The required sensitivity is defined in terms of the significance of the detection of the Hα Balmer transition line: an unresolved (i.e. sub-resolution) Hα line of spectral density flux $3 \times 10^{-16}\, erg\, cm^{-2}\, s^{-1}$ is to be detected at 3.5σ above the measurement's noise. These requirements imply a detection rate that depends on magnitude and redshift, and Euclid will mostly detect galaxies in the redshift range $0.7 < z < 2.0$ [1].

According to these requirements, we simulate a distribution of galaxies with realistic photometric observational distributions, such as redshift, color, magnitude and spectral type, and



(a) *Clean Spectral Profile* **(b)** *High SNR* **(c)** *Medium SNR* **(d)** *Low SNR*

Fig. 2: *High, medium, and low SNR uncorrelated Gaussian noise added to a simulated resampled and integrated spectrum*

also taking into consideration the spectral energy distributions and the emission-line strengths. First, we define a master catalog for the analyses, with the COSMOSSNAP simulation pipeline [12], which calibrates property distributions with real data from the COSMOS survey [13], thereby ensuring that realistic relationships between galaxy type, color, size, redshift and SED are preserved. As a result, we have constructed a master galaxy catalogue with magnitudes, colors, shapes and photometric redshifts for 538.000 galaxies on a $1.38\ deg^2$ region of the sky down to an i-band magnitude of ~ 24.5.

B. Simulating the noise conditions

In order to create the noisy dataset, we select a 50% random subset of the galaxies that are below redshift $z = 1$ with Hα flux above $10^{-16}\, erg\, cm^{-2}\, s^{-1}$, and bring them to rest-frame values ($z = 0$). We then create three matched noisy cases, by adding white Gaussian noise, of high SNR, medium SNR and low SNR, all sharing a common clean dataset. This simplified noise is not fully representative of all the observational aspects of Euclid observations. Nonetheless, we calibrate the noise variance in terms of the SNR of the Hα flux, ensuring that these values correspond to expected Euclid SNR values. We split each of these three coupled datasets into 70% for training, and 30% for testing sets. The training sets use both the clean and noisy versions to train the proposed dictionary learning technique, while the testing is only composed by the noisy signals, to which the trained pipeline will be applied in order to evaluate the performance of the proposed algorithm. Fig. 2 demonstrates an example of a simulated spectroscopic template, degraded by the three different noise cases.

C. Simulation Results

In this Section, we evaluate the performance of the proposed denoising coupled dictionary learning scheme under various noise scenarios, each one corresponding to a different SNR level, namely, the low, medium, and high SNR. Proper normalization of the input data is essential in order to achieve a valid reconstruction. Hence, to distribute the spectral profiles evenly and scale them into an acceptable range, we normalized the input spectral profiles by mapping them into the predefined range of $[-1, 1]$.

During the dictionary training phase, three pairs of noisy and clean dictionaries were prepared, one for each SNR value. For all three cases, we utilized 10.000 training spectral templates, and we have experimented with different number of representative dictionary atoms. Specifically, we examined the impact of using 4000, 6000, and 8000 dictionary atoms. The best performance is achieved in the case of 8000 dictionary atoms. In order to quantify the proposed algorithm's performance, we investigate the empirical convergence behaviour of the Augmented Lagrangian function $\mathcal{L}$. Figure 3 depicts the normalized reconstruction error for the Augmented Lagrangian function as a function of the iterations number.

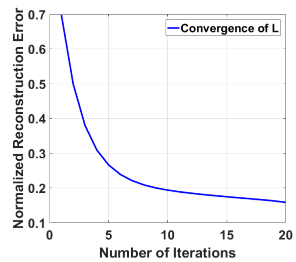


Fig. 3: *Convergence Behaviour of the proposed Dictionary Learning Algorithm: Convergence of the Augmented Lagrangian function.*

In run-time, we applied our coupled dictionary learning denoising scheme in 3000 spectral profiles. In order to evaluate the reconstruction quality, we measure the normalized root mean square error (NRMSE), between the reconstructed and the original spectral profiles. The normalized root mean square error between an original signal $\mathbf{y}$ and its reconstruction $\hat{\mathbf{y}}$ is defined as: $NRMSE = \sqrt{\frac{\|\mathbf{y}-\hat{\mathbf{y}}\|_2^2}{\|\mathbf{y}\|_2^2}}$.

In terms of a quantitative comparison, we averaged the NRMSE reconstruction errors along all 3000 testing spectral templates, subject to their corresponding original spectral profiles, and for all three different noise scenarios. Table I highlights the proposed system's performance, when different number of dictionary learning atoms are used. The best obtained averaged reconstruction errors are $\mathbf{0.0026}$, $\mathbf{0.0069}$, and $\mathbf{0.0091}$ for the high, medium, and low SNR noise levels, respectively. In all cases, one may observe that the size of the reconstruction error indicates a valid recovery of the noisy signals. Fig. 4 demonstrates an example of the proposed system's reconstruction, when applied on the three different noise scenarios. We observe, that in all three cases, the reconstructed denoised signals preserve all the significant peaks with respect to their corresponding "clean" spectral profiles. Additionally, the resulting denoised reconstructions preserve accurate similarity with the ground truth templates, even in the challenging low-SNR noisy case.

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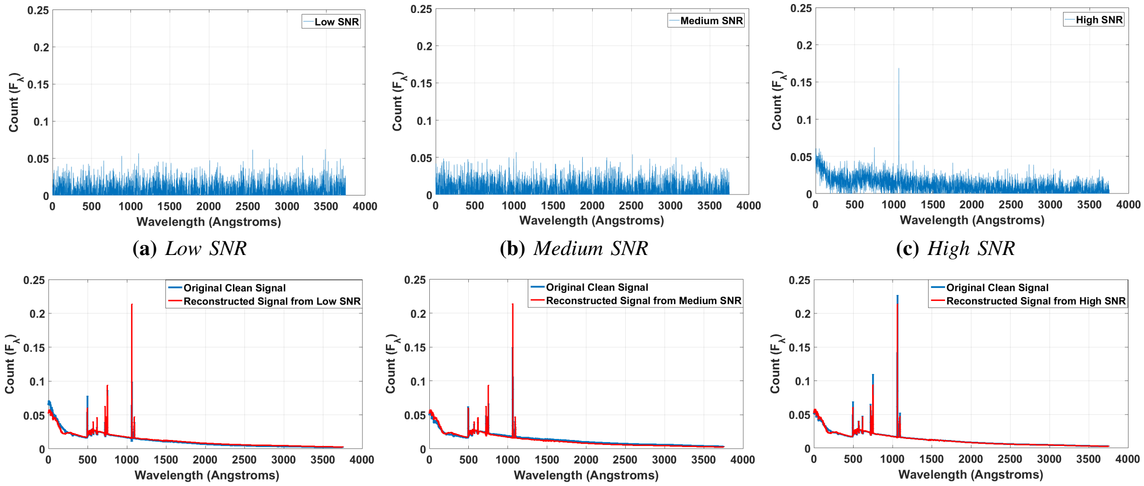

(a) *Low SNR* **(b)** *Medium SNR* **(c)** *High SNR*

(d) *Reconstruction from Low SNR* **(e)** *Reconstruction from Medium SNR* **(f)** *Reconstruction from High SNR*

Fig. 4: *Reconstructed Spectral Profile under the three noise scenarios: (Left) Reconstructed Spectral Profile under low SNR conditions, (Middle) Recovered Spectral Template from medium SNR noise scenario, (Right) Reconstructed Spectra from high SNR conditions. The number of selected dictionary atoms was set to* 8000*. In all three cases, the recovered templates preserve accurate similarity with the ground truth and clean spectral templates.*

TABLE I: *Quantitative evaluation of the proposed denoising scheme versus the different number of dictionary atoms in terms of NRMSE*

Dictionary Atoms	4000	6000	8000
Low SNR	0.0106	0.0099	**0.0091**
Medium SNR	0.0079	0.0077	**0.0069**
High SNR	0.0033	0.0028	**0.0026**

D. Conclusions

In this paper, we developed a novel technique that tackles the spectroscopic data denoising problem applied on Euclid's simulated spectra templates. The reported experimental results suggest that *Sparse Representations* and *Coupled Dictionary Learning* are powerful tools, able to reconstruct denoised spectral profiles from their corresponding low-resolution, and noisy versions. Additionally, we observed that the proposed denoising algorithm works successfully even with extreme noise scenarios. The developed coupled dictionary learning scheme can be efficiently used in other types of astronomical and spectral signal applications, such as deblurring and super-resolution [14].

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