

# Collision Resolution and Interference Elimination in Multiaccess Communication Networks

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**Abstract**—We define a multiaccess communication scheme that effectively eliminates interference and resolves collisions in many-to-one and many-to-many communication scenarios. Each transmitter is uniquely identified by a coding vector. Using these vectors, all signals issued from a specific transmitter will be aligned along a unique dimension at all receivers hearing this transmission. This dimension is characteristic of the transmitter. It also lies within a signal-and-noise subspace that is orthogonal to the noise-only subspace at the receiver. Signals along each dimension of the signal-and-noise subspace can be extracted separately using the properties of the Vandermonde matrix. The decoding algorithm is thus able to asymptotically achieve full network capacity at high signal-to-noise ratio (SNR) compared to 50% and 36.79% asymptotic throughputs for interference alignment and Ethernet respectively. Synchronization is assumed between the transmitters and the receiver(s). The number of transmitters is not necessarily known to each receiver.

**Index** — collision resolution, interference elimination, interference alignment, Vandermonde matrix, coding vectors

## I. INTRODUCTION

Communication resources are scarce relative to the data requirements of the system users. A multiaccess communication scheme is needed to control the sharing of these resources among the different users. If a scheme allows several transmitters to communicate with the same receiver using the same resources, the scheme incurs collisions and retransmissions become necessary. In addition, if a scheme allows several transmitters to communicate with distinct receivers but a signal targeted to one receiver suffers from interference of signals targeted to others, this signal gets deteriorated and its decoding incurs errors if feasible in the first place. In both many-to-one and many-to-many communication scenarios, one category of schemes regulates multiaccess communication by preventing collisions and non-negligible interference. The other category advises collision-resolution protocols and signal-decoding algorithms in presence of interference.

Time and frequency division multiple access (TDMA and FDMA) are two conflict-free multiaccess schemes. A major drawback of both schemes is that only a portion of the available resources (time and frequency) is utilized when the network is lightly loaded while the other portion remains idle. Code division multiple access (CDMA) is another scheme that orthogonalizes the channel access via use of codes. Yet it is interference-limited in practice [1]. On the other hand, slotted Aloha, carrier sense multiple access (CSMA), and CSMA with collision detection and collision avoidance (CSMA/CD and CSMA/CA) are contention-based protocols. Transmitters with data compete to access the channel after random waiting periods while bearing the risk of collisions. This access mode is useful to accommodate variable bit rate data streams but is less efficient in heavily loaded networks. CSMA/CD is used in Ethernet and achieves a throughput of

$1/e = 36.79\%$ . It can be shown that an upper bound on the throughput of any collision resolution algorithm is 58.7%. Among other assumptions, this upper bound holds true for the case where all packets involved in a collision should be retransmitted [2]. A throughput of 48.78% is achieved using a tree algorithm for collision resolution [2].

Interference alignment, first introduced in [3], is a recent interference management technique that is neither resource-reservation-based nor contention-based. Instead, each transmitter steers its signal so that it lies within a reduced subspace along other interfering signals at every receiver except its desired receiver. Each receiver then looks outside its interference subspace and is able to extract the desired signal. For the fully connected  $K$  user time-varying interference channel and assuming global channel knowledge, the authors in [4] show that this channel has  $K/2$  degrees of freedom which is a 50% throughput. In [5], the authors study the feasibility of interference alignment given only local channel state information (CSI) at each node. Practical challenges for interference alignment are described in [6]. Recent applications of interference alignment in network design and optimizations can be found in [7], [8], [9]. Interference alignment is not designed for the many-to-one communication scenario since if all the transmitted signals need to be decoded by a receiver, no two signals may be aligned with respect to that receiver.

In this paper we describe an algorithm that works in both the many-to-one and many-to-many communication scenarios. Having said that, a receiver manages collisions and interference by decoding the collided packets and the interfering signals. Here we overlook security or privacy concerns whenever the decoder is not the target receiver. The channel is assumed packet-switched so that we avoid the inefficiencies of resource reservation under low load. In addition, while interference alignment and best collision resolution algorithms achieve a 50% throughput, our algorithm asymptotically utilizes the full capacity of the network when the number of users grows infinite, provided that any communication (desired and undesired) between the transmitters and the receiver(s) is synchronized and the signal-to-noise ratio (SNR) is high.

We take a different perspective than interference alignment. In interference alignment, each receiver looks at its own space uniquely: its desired signal lies in half the space while all the other signals lie in the second half. The split is thus unique to every receiver. Since the transmitters have to steer their signals so that they get aligned as desired by the receivers, they become over-constrained and the steering achieves only 50% efficiency. However, we let the receivers have a unified view of the signal space. This does not

mean cooperation among the transmitters nor the receivers. Instead, all the receivers view the space as two orthogonal subspaces: one subspace that holds all the signals and noise, and another subspace that holds only noise. The first subspace has dimensionality equal to the number of transmitters. The second subspace has enough dimensions to suppress noise, where a single dimension is enough at high SNR. We choose the coding vectors to uniquely identify each transmitter. Since the number of dimensions occupied by the transmitted signals always grows as their number increases, the problem is no more overconstrained and every transmitted signal can be decoded by every receiver. If an arriving signal is so weak, a receiver can always decide that this is an interfering signal and move the occupied dimension(s) to the noise subspace.

We select the same coding vectors as the columns of a Vandermonde matrix. For our algorithm, since these vectors lie in a high dimensional subspace, it is indifferent whether the extension over the dimensions occurs in time, frequency or space. For this paper the channel is a single carrier and every node has a single antenna, so the coding vectors extend over time. This paper is focused on describing the algorithm assuming a Gaussian channel and synchronization between the transmitters and the receiver(s). For the purpose of clarity and due to space considerations, we keep the description of the algorithm under fading and in the unsynchronized communication regime for later. Section III summarizes few properties of the Vandermonde matrix. The algorithm is defined incrementally in sections IV and V. The effect of the SNR is numerically illustrated in section VI. We first describe the system model.

## II. SYSTEM MODEL

Consider a single-carrier system in which  $K$  active transmitters within a set of  $\tilde{K}$  transmitters contact a single receiver, each node having a single antenna and  $\tilde{K} \geq K$ . Each transmitter accesses the channel whenever data is available without waiting for the channel to be idle. In the case of more than one receiver, each receiver performs the same functionality and is able to decode its unique set of arriving signals, whether desired or not. The minimum transmission unit is a packet of  $P$  symbols that could be real or complex. It takes one time slot to transmit a single packet. A symbol duration is  $\tau$  seconds, so 1 slot =  $P \times \tau$ . We impose the constraint  $P > K$  for all  $K$ , and so  $P > \tilde{K}$ .

We are interested in the case where  $K > 1$ . For  $K = 1$ , the transmitter identifies immediately upon sending the packet (through immediate feedback or channel sensing) that there are no concurrent transmissions and there is no need for further action. This is not absolutely necessary nor critical in order to achieve the full network asymptotic throughput but is assumed for simplicity. A feedback from the receiver is only possible if the packet has extra bits for error detection. We refer to these bits as CRC for cyclic redundancy check.

On the other hand, if there are concurrent transmissions, each transmitter will send its packet more than once before the receiver can decode these individual packets. During this time, some new transmitters might join and start sending new packets. The receiver in turn will need more time to

decode all the received packets. In such a scenario,  $K$  refers to the total number of active transmitters at the instant of successful decoding, which is greater than or equal to the number of transmitters upon arrival of the first few packets.

The time for successful decoding is measured in slots and is equal to  $N$ . In a synchronized communication regime this will be function of  $K$  and the SNR. It should be emphasized that it is not necessary for the receiver to know  $K$  beforehand in order to successfully decode the packets. If data availability at the transmitters is assumed random, then so  $K$  and  $N$  will be.

While  $K$  might be unknown beforehand to the receiver, the receiver is aware that there are  $\tilde{K}$  transmitters of which any  $K$ -subset might be active. In particular, each transmitter  $\tilde{k}$  of the  $\tilde{K}$  transmitters that might contact the receiver is assigned a unique complex exponential  $r_{\tilde{k}} = e^{j\angle r_{\tilde{k}}}$  lying on the unit circle, where  $0 \leq \angle r_{\tilde{k}} < \pi$ . This assignment might be static or dynamic, but the receiver needs to know this assignment before any  $K$ -subset of the  $\tilde{K}$  transmitters contacts the receiver ( $K$  potentially unknown). Define the  $N$ -time extension of the coding vector of transmitter  $\tilde{k}$  as

$$\vec{w}_{\tilde{k},N} = [r_{\tilde{k}}^0, r_{\tilde{k}}^1, \dots, r_{\tilde{k}}^{N-1}]^T \quad (1)$$

All vectors  $\vec{v}$  are column vectors and have arrow symbols on top. The transpose of  $\vec{v}$  is  $\vec{v}^T$  and the conjugate transpose of  $\vec{v}$  is  $\vec{v}^H$ . Similar transpose notation is used for matrices. If  $\vec{v}$  has length  $L$ , then  $\vec{v}[l]$  is the  $l^{\text{th}}$  element of  $\vec{v}$ ,  $1 \leq l \leq L$ . A packet is represented as a vector of symbols, where  $\vec{s}_k$  is the packet to be transmitted by transmitter  $k$ ,  $1 \leq k \leq K$ . During time slot  $n$  and whenever at least one transmitter is active, the receiver collects a vector of symbols

$$\vec{y}_n = [\vec{y}_n[1], \vec{y}_n[2], \dots, \vec{y}_n[P]]^T \quad (2)$$

The noise over all channels is complex normal  $\mathcal{CN}(0, \sigma^2 I)$  of mean 0 and covariance  $\sigma^2 I$ . A collection of  $p \times q$  noise samples is denoted as  $\mathcal{N}_{p,q}$ .

## III. VANDERMONDE MATRIX

This section summarizes important properties of the Vandermonde matrix that will be used in the design of the transmission and decoding schemes. Consider matrix  $A$

$$A = \begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ \alpha_1 & \alpha_2 & \alpha_3 & \dots & \alpha_M \\ \alpha_1^2 & \alpha_2^2 & \alpha_3^2 & \dots & \alpha_M^2 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ \alpha_1^{N-1} & \alpha_2^{N-1} & \alpha_3^{N-1} & \dots & \alpha_M^{N-1} \end{pmatrix} \quad (3)$$

$A$  is an  $N \times M$  Vandermonde matrix where  $N > M$ . Each column of  $A$  is a geometric series. Assuming all  $\{\alpha_m\}_{m=1}^M$  are distinct complex numbers, any subset of the columns of  $A$  is full rank. In addition, since  $N > M$ ,  $A$  has a non-trivial left null space  $A_{\perp}$ . For a Vandermonde matrix  $A$ ,  $A_{\perp}$  fully identifies the elements  $\{\alpha_m\}_{m=1}^M$ . This is because  $A_{\perp}^H A = 0$  and so equation

$$Z^H A_{\perp} A_{\perp}^H Z = 0 \quad (4)$$

admits roots  $\{\alpha_m\}_{m=1}^M$ , where  $Z = [1, z, z^2, \dots, z^{N-1}]^T$ .

IV. ALIGNMENT AT TIME  $t = 0$ 

In this section we assume synchronized collisions: all the packets from the  $K$  active transmitters arrive at the receiver at the same instant  $t = 0$ . This will be relaxed in section V. We first describe the transmission scheme. Then we show how the receiver detects  $K$  if unknown, identifies the  $K$  transmitters and decodes the received packets.

## A. Transmission scheme

As mentioned previously, for the case  $K = 1$  both the transmitter and the receiver detect a contention-free channel (CRC checking, carrier sensing, etc.). No collision occurs and the packet is decoded correctly at high SNR. The transmitter does not have to do any retransmissions of the same packet.

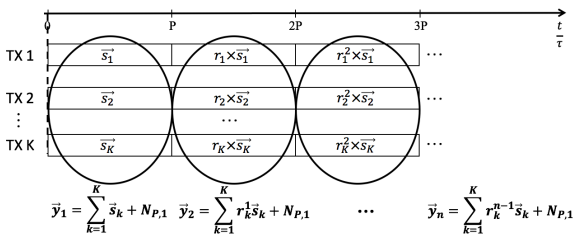


Fig. 1. Transmission scheme of  $K$  packets that collide synchronously at the receiver at  $t = 0$ .

On the other hand, figure 1 shows the transmission scheme adopted by each transmitter if  $K > 1$ . Each transmitter  $k$  sends its packet  $\vec{s}_k$ . The  $K$  packets arrive at the receiver at exactly time  $t = 0$ , i.e. the start of slot  $n = 1$ . During this slot, the receiver collects packet  $\vec{y}_1 = \sum_{k=1}^K \vec{s}_k + \mathcal{N}_{P,1}$  that has a corrupted CRC due to collision, even though the SNR is high. Collision is detected by the receiver and the transmitters.

The receiver will not be able to decode the  $K$  packets within a single slot since  $K > 1$  and all the packets occupy the same frequency band. In this case, each transmitter  $k$  sends a contiguous packet  $r_k \times \vec{s}_k$  that will exactly fit within slot  $n = 2$ . Recall that  $r_k$  is characteristic of transmitter  $k$ , but the identity of the  $K$  active transmitters might be ambiguous to the receiver. During slot  $n = 2$ , the receiver collects  $\vec{y}_2 = \sum_{k=1}^K r_k \vec{s}_k + \mathcal{N}_{P,1}$ . Suppose the receiver fails again in decoding the original  $K$  packets  $\{\vec{s}_k\}_{k=1}^K$ . The transmitters continue to send their contiguous transmissions of the weighted packets. In its  $n^{\text{th}}$  transmission, transmitter  $k$  sends packet  $r_k^{n-1} \vec{s}_k$  and the receiver collects during slot  $n$  packet  $\vec{y}_n = \sum_{k=1}^K r_k^{n-1} \vec{s}_k + \mathcal{N}_{P,1}$ .

 B. Detection of  $K$ 

At the end of each time slot  $n$ , the receiver stacks the  $n$  already collected vectors  $\{\vec{y}_1, \vec{y}_2, \dots, \vec{y}_n\}$  horizontally into matrix  $Y_n$ . Note that matrix  $Y_n$  can be expressed as

$$Y_n = \begin{pmatrix} \vec{y}_1^T \\ \vec{y}_2^T \\ \vdots \\ \vec{y}_n^T \end{pmatrix} = \begin{pmatrix} 1 & 1 & \dots & 1 \\ r_1 & r_2 & \dots & r_K \\ \vdots & \vdots & \dots & \vdots \\ r_1^{n-1} & r_2^{n-1} & \dots & r_K^{n-1} \end{pmatrix} \times \begin{pmatrix} \vec{s}_1^T \\ \vdots \\ \vec{s}_K^T \end{pmatrix} + \mathcal{N}_{n,P} \quad (5)$$

or shortly

$$Y_n = W_n \times S + \mathcal{N}_{n,P} \quad (6)$$

Since  $P > \tilde{K} \geq K$  and the packets  $\{\vec{s}_k\}_{k=1}^K$  of the  $K$  transmitters are independent,  $\text{rank}(S) = K$ . But  $S$  is a  $K \times P$ -matrix, so  $\text{rank}(W_n \times S) = \text{rank}(W_n)$ . However,  $W_n$  holds the coding vectors  $\{\vec{w}_{k,n}\}_{k=1}^K$  of the  $K$  transmitters as its columns. Since each transmitter  $k$  is assigned a different complex number  $r_k$ ,  $W_n$  is a Vandermonde matrix whose rank is  $K$  whenever  $n > K$ . From (6), if  $n > K$  then  $K$  is also the rank of  $Y_n$  in the noiseless case. In presence of noise, since the SNR is high, the receiver is still able to detect the actual rank of  $Y_n$  as that of the noiseless case by thresholding the small singular values of  $Y_n$ . Therefore, the receiver builds matrix  $Y_n$  and checks its true (noiseless) rank at the end of every slot  $n$ . Once  $\text{rank}(Y_n)$  stops growing with  $n$ , the receiver detects  $K$  as

$$K = \text{rank}(Y_n) \quad (7)$$

The receiver stops expanding  $Y_n$  at  $n = N > K$ :

$$Y_N = W_N \times S + \mathcal{N}_{N,P} \quad (8)$$

 C. Identification of the  $K$  transmitters

At high SNR,  $Y_N$  in (8) has a non-trivial left null space of dimension  $N - K$ . Let  $U_\perp$  hold as columns the basis vectors of the left null space of  $Y_N$ . The receiver computes  $U_\perp$  by performing the singular value decomposition (SVD) of  $Y_N$ :

$$Y_N = ([U_\parallel \quad U_\perp]) \Sigma V^H \quad (9)$$

From (8),  $Y_N$  and  $W_N$  have the same left null space in the noiseless case:

$$U_\perp^H W_N \xrightarrow{\sigma \rightarrow 0} 0 \quad (10)$$

However,  $W_N$  is a Vandermonde matrix. As discussed in section III,  $U_\perp$  thus fully identifies the elements  $\{r_k\}_{k=1}^K$ . Therefore, after computing  $U_\perp$  from the SVD of  $Y_N$ , the receiver solves equation

$$J(z) = \vec{w}_N^H \times U_\perp U_\perp^H \times \vec{w}_N = 0 \quad (11)$$

for  $z$ , where coding vector  $\vec{w}_N$  is given by

$$\vec{w}_N = [1, z^1, \dots, z^{N-1}]^T \quad (12)$$

Equation (11) yields  $K$  unit complex exponentials  $\{r_k\}_{k=1}^K$  with angles  $\leq \angle z < \pi$  that indicate to the receiver the identity of the  $K$  transmitters. On the other hand, in presence of noise,  $U_\perp$  will not exactly describe the left null space of  $W_N$ . In this case, the receiver still computes  $U_\perp$  from  $Y_N$  and solves (11). Then the receiver chooses the  $K$  solutions closest to the unit circle (upper half) and the individual elements of set  $\{r_k\}_{k=1}^K$  to identify the  $K$  active transmitters.

 D. Decoding of the  $K$  packets

Having identified the  $K$  unit exponentials  $\{r_k\}_{k=1}^K$ , the receiver constructs the Vandermonde matrix  $W_N$  whose columns are  $\{\vec{w}_{k,N}\}_{k=1}^K$  as defined in (1). The order of the columns of  $W_N$  is unimportant. From (8), the matrix of decoded packets is obtained as

$$\hat{S} = (W_N^H W_N)^{-1} W_N^H Y_N \quad (13)$$

Matrix  $W_N^H W_N$  is full rank and thus admits an inverse. In the noiseless case  $\hat{S}$  is exactly  $S$ . The  $k^{\text{th}}$  row of  $\hat{S}$  is the decoded packet of the transmitter whose coding vector is the  $k^{\text{th}}$  column in constructed matrix  $W_N$ .

### E. Throughput and delay

As mentioned previously,  $N - K$  is the number of columns of  $U_{\perp}$  which defines the left null space of  $Y_N$ . Referring to the SVD of  $Y_n$  in (9), these columns correspond to the  $N - K$  singular values of the noise-only subspace. Since  $K$  is fixed in a given communication scenario, the receiver gains better approximation of the noise-only subspace by stacking more packets  $\vec{y}_n$  to matrix  $Y_n$  in order to increase  $N$  and consequently  $\text{rank}(U_{\perp}) = N - K$ . Lower SNR requires extra packets  $\vec{y}_n$  to be collected for the same performance. At high SNR,  $N - K \sim \mathcal{O}(1)$ . This implies that the decoding delay  $N$  (measured in slots) is of the order of the number of active transmitters  $K$ . During this time,  $K$  distinct packets are correctly decoded. The asymptotic throughput becomes

$$\lim_{K \rightarrow \infty} \frac{K}{N} = 100\% \quad (14)$$

### V. ALIGNMENT AT THE START OF A TIME SLOT

In the previous section we assumed tight synchronization requirements: packets of all transmitters should arrive at the same time instant. All inactive transmitters at  $t = 0$  have to stay idle until the packets of the active transmitters get decoded. In this section we relax the synchronization requirements: transmitters are synchronized to the receiver, but an idle transmitter may join the active set of transmitters on condition that its packet is received at the start of a time slot, i.e.  $t = (n - 1)P\tau$ ,  $n \in \mathcal{Z}^+$ .

#### A. Transmission scheme

All transmitters follow the same transmission scheme as in the previous section. Figure 2 illustrates a specific communication scenario which we use to derive a general expression of the received matrix of packets  $Y_n$  afterwards.

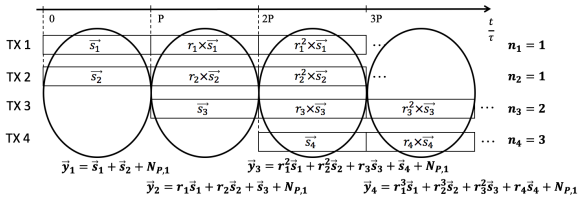


Fig. 2. Transmission scheme of  $K = 4$  packets synchronized to the start of a time slot.

Transmitters 1 and 2 initially send their unweighted packets. At  $t = 0$ , the receiver collects  $\vec{y}_1 = \vec{s}_1 + \vec{s}_2 + \mathcal{N}_{P,1}$ . Due to collision, the receiver is unable to decode packets  $\vec{s}_1$  and  $\vec{s}_2$ . Transmitters 1 and 2 then send packets  $r_1 \vec{s}_1$  and  $r_2 \vec{s}_2$  respectively. In addition, transmitter 3 joins the set of active transmitters. Since this is the first time transmitter 3 sends its packet, transmitter 3 sends  $\vec{s}_3$ . The receiver collects at  $t = P\tau$  packet  $\vec{y}_2 = r_1 \vec{s}_1 + r_2 \vec{s}_2 + \vec{s}_3 + \mathcal{N}_{P,1}$ . In the next time slot, transmitter 4 joins the active set. At  $t = 2P\tau$  the receiver collects  $\vec{y}_3 = r_1^2 \vec{s}_1 + r_2^2 \vec{s}_2 + r_3 \vec{s}_3 + \vec{s}_4$ . Notice that all transmitters follow the same transmission algorithm independent of the time they join the active set. No more transmitters get involved in the scenario of figure 2. By the time the receiver manages to decode the packets, only 4 transmitters are active, i.e.  $K = 4$ .

### B. Detection of $K$

At the end of each time slot  $n$ , the receiver stacks the  $n$  already collected vectors  $\{\vec{y}_1, \vec{y}_2, \dots, \vec{y}_n\}$  horizontally into matrix  $Y_n$ . A general expression of  $Y_n$  is given by

$$\begin{aligned} Y_n &= \left( \vec{w}_{1,n}^{(n_1-1)} \quad \vec{w}_{2,n}^{(n_2-1)} \quad \dots \quad \vec{w}_{K,n}^{(n_K-1)} \right) \times S + \mathcal{N}_{n,P} \\ &= W_n^{(n-1)} \times S + \mathcal{N}_{n,P} \end{aligned} \quad (15)$$

In (15),  $n_k$  refers to the slot index in which transmitter  $k$  joins the set of active transmitters in a given communication scenario, i.e. packet of transmitter  $k$  is received for the first time at  $t = (n_k - 1)P\tau$ . In the example of figure 2,  $n_1 = n_2 = 1$ ,  $n_3 = 2$  and  $n_4 = 3$ . Moreover,  $\vec{w}_{k,n}^{(n_k-1)}$  is a shifted version of the coding vector  $\vec{w}_{k,n}$  of transmitter  $k$  defined as follows

$$\vec{w}_{k,N}^{(n_k-1)} = \left[ \underbrace{0, \dots, 0}_{(n_k-1) \text{ zeros}}, r_k^0, r_k^1, \dots, r_k^{n-n_k} \right]^T \quad (16)$$

Notice that (15) is a general expression for  $Y_n$  that applies to all values of  $n$  even if not all  $K$  transmitters have yet joined the active set. It also applies to an arbitrary value of  $K$ . We abuse notation and denote the matrix of shifted coding vectors in (15) as  $W_n^{(n-1)}$ .

The receiver builds matrix  $Y_n$  and checks its true (noiseless) rank at the end of every slot  $n$ . This rank will be growing as more transmitters join the active set, or if  $n$  is still less than the transmitters in the active set by the end of slot  $n$ . Once  $\text{rank}(Y_n)$  stops growing with  $n$ , the receiver detects  $K$  as  $\text{rank}(Y_n)$ . At stopping time  $n = N > K$ , the received matrix is

$$Y_N = W_N^{(N-1)} \times S + \mathcal{N}_{N,P} \quad (17)$$

We do not discuss here the properties of pseudo-Vandermonde matrix  $W_n^{(n-1)}$  that validate the receiver's approach to detect  $K$ .

#### C. Identification of the $K$ transmitters

$Y_N$  and  $W_N^{(N-1)}$  have the same left null space of dimension  $N - K$  at high SNR. The receiver computes  $U_{\perp}$  from the SVD of  $Y_N$ . Instead of solving (11), the receiver solves a system of  $N - 1$  equations

$$J(z)^{n^*-1} = (\vec{w}_N^{(n^*-1)})^H \times U_{\perp} U_{\perp}^H \times \vec{w}_N^{(n^*-1)} = 0 \quad (18)$$

for  $1 \leq n^* \leq N - 1$ , where  $\vec{w}_N^{(n^*-1)} = \left[ \underbrace{0, \dots, 0}_{(n^*-1) \text{ zeros}}, 1, z^1, \dots, z^{N-n^*} \right]^T$ . Among all the sets of solutions of (18), the receiver selects the  $K$  solutions closest to the unit circle (upper half) and the individual elements of set  $\{r_k\}_{k=1}^K$  to identify the  $K$  active transmitters. Based on what equation in set (18) generates the  $k^{\text{th}}$  solution  $r_k$ , the receiver also recovers the corresponding shift  $n_k - 1$ .

#### D. Decoding of the $K$ packets

Given  $\{r_k\}_{k=1}^K$  and  $\{(n_k - 1)\}_{k=1}^K$ , the receiver constructs matrix  $W_N^{(N-1)}$  and decodes the packets as

$$\hat{S} = ((W_N^{(N-1)})^H W_N^{(N-1)})^{-1} (W_N^{(N-1)})^H Y_N \quad (19)$$

Note that (13) is a special case of (19) by setting  $n_k = 1$  for all  $k$ .

### E. Throughput and delay

Suppose transmitters 3 and 4 in the scenario of figure 2 do not join the active set of transmitters until  $t = 3P\tau K$ . In this case, during the first three slots the receiver only collects packets of transmitters 1 and 2. At high SNR, the receiver is able to decode  $\vec{s}_1$  and  $\vec{s}_2$  by the end of the third time slot using (19), where  $n_1 = n_2 = 1$ ,  $K = 2$  and  $N = 3$ . The arriving packets of transmitters 3 and 4 are then considered to belong to a separate communication scenario. This tells that for a fixed number of transmitters  $K$ , the delay  $N$  does not grow arbitrarily large.  $K$  packets are decoded within  $N$  time slots where  $N - K \sim \mathcal{O}(1)$  at high SNR. The asymptotic throughput is 100%.

## VI. EFFECT OF THE SNR

In this section we show through simulations the effect of the SNR on the ability of the receiver to correctly detect the identity of the transmitters on one hand, and to correctly decode the packets of the identified transmitters on the other hand. We assume the network has  $\tilde{K} = 32$  transmitters of which only  $K = 8$  are active. The 32 transmitters are assigned equally-spaced angles between 0 and  $\pi$ , and the  $K$  transmitters are randomly selected. The first packet from each of the  $K$  transmitters arrives at  $t = 0$  as in Section IV. Similar results hold for the general case of Section V. Each packet is of length  $P = 24$ , and so  $S$  is a matrix of  $8 \times 24$  random 8-bit integers between 0 and 255. Having each symbol hold 8 bits increases the sensitivity to the SNR. All elements of  $S$  are real. We vary  $\sigma^2$  on a log-scale between  $1e-6$  and  $1e3$ , and we define the SNR as  $\text{SNR} = 10 \log_{10}(1/\sigma^2)$ . We assume the noise power is equally distributed on the real and imaginary values of the received samples of  $Y_N$ . For each value of  $\sigma^2$  we run the simulation 1000 times and compute the mean statistics. In

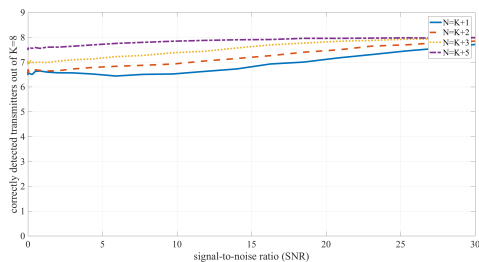


Fig. 3. Variation of the number of correctly detected transmitters out of  $K$  versus  $\text{SNR} = 10 \log_{10}(1/\sigma^2)$  for  $N - K = 1, 2, 3$  and 5 and aligned transmissions.

figure 3 we check the number of correctly detected transmitters out of  $K$  within the superset of 32 transmitters. Here we assume that the receiver knows at every SNR what threshold to use in order to decide whether a singular value of the left nullspace of  $Y_N$  corresponds to noise and should be nullified. Thus, the receiver knows  $K$  and the only error that might occur is that it misidentifies which  $K$ -selection of the 32 transmitters is the active set. We examine the performance of the receiver when it collects  $N - K = 1, 2, 3$ , and 5 extra packets. We can see that for all stopping times  $N$  the number of correctly identified transmitters is increasing with the SNR. Moreover, the labeling of the transmitters becomes more accurate for

higher  $N - K$  at all SNR. This is because the receiver acquires more dimensions of the noise subspace  $U_{\perp}$ . All  $K$  transmitters are detected at high SNR and  $N - K = 5$ . We now check

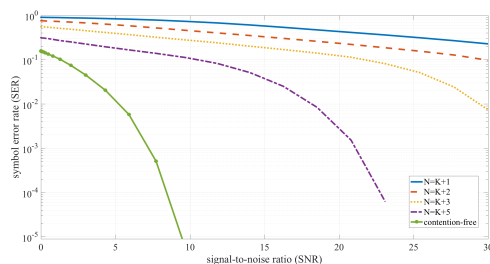


Fig. 4. Variation of symbol error rate versus  $\text{SNR} = 10 \log_{10}(1/\sigma^2)$  for  $N - K = 1, 2, 3$  and 5 and aligned transmissions.

the symbol error rate SER of the decoded packets for the case where all  $K$  transmitters are known to the receiver. As expected, the SER drops for higher SNR. It also drops when  $N - K$  increases and approaches the contention-free curve. The SER is less than  $1e - 4$  for  $N - K = 5$  and high SNR.

## VII. CONCLUSION

In this paper we show how a receiver correctly extracts a desired packet when it arrives simultaneously with other desired colliding packets and undesired interference in many-to-one and many-to-many communication scenarios. The algorithm achieves full asymptotic throughput at high SNR and slot-synchronization between each receiver and its candidate set of transmitters. In the simulations we show that high SNR is important. At a given SNR, performance of the receiver is always improved by collecting few extra packets.

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