Unsupervised Time Domain Nonlinear Post-Equalization for ACO-OFDM Visible Light Communication Systems

Flávio R. Ávila, Lisandro Lovisolo
Electronics and Telecommunications Departament (DETEL)
Rio de Janeiro State University (UERJ)
Email: flavio.avila@uerj.br, lisandro@uerj.br

***Abstract*—LED nonlinearity is an important issue limiting the performance of Visible Light Communication (VLC) systems. This form of distortion is particularly problematic when the system employs Optical-OFDM because of the high Peak-to-Average Power Ratio (PAPR) of its time domain symbol. This paper proposes using ancillary statistical properties of the O-OFDM signal in order to mitigate LED nonlinearities in an unsupervised fashion. By exploring the Gaussianity of the time domain OFDM signal and the idea of distribution equalization, we propose a semi-parametric approach to blind nonlinear post-equalization for asymmetrically clipped O-OFDM (ACO-OFDM) VLC systems. In addition to not requiring training data, the equalizer is robust to different LED types and it is adaptive to time-varying nonlinearities. Simulations with a realistic LED model show that the developed tool is capable of substantially mitigating the effects of nonlinear distortion on system performance.**

I. Introduction

Visible Light Communication (VLC) [1] has attracted substantial recent attention due to its potential as a very low-cost solution for alleviating the RF spectrum shortage. In VLC, an LED in the transmitter performs electrical-to-optical power conversion, and a photodetector implements the reverse operation in the receiver. Unlike most traditional RF systems, VLC is based on the concept of intensity modulation and direct detection (IM/DD), in which the information is encoded in the intensity of the light wave rather than in its amplitude or phase. For this reason, the signal must be real and positive before being input to the LED, which in practice imposes constraints on the system.

Orthogonal Frequency Division Multiplex (OFDM) has emerged as a technology suitable for VLC due to its ability to very efficiently remove Intersymbol Interference (ISI) that results from a multipath dispersive channel. A well known drawback of OFDM is its high Peak-to-Average Power Ratio (PAPR), which in combination with LED nonlinearity causes time domain distortion that ultimately deteriorates system performance. A vast body of literature exists addressing PAPR reduction and nonlinearity mitigation strategies in RF OFDM systems [2], and a growing one is emerging in the VLC context [3]. Techniques such as selective mapping (SLM) and tone injection (TI) achieve PAPR reduction by manipulating the OFDM frequency domain (FD) symbol, while others such as predistortion [4], adaptive post-equalization [5] and nonlinear companding [6] attain the same goal via time domain processing.

As part of the latter category, this paper proposes a blind post-equalization technique for optical OFDM (O-OFDM). Its advantage with respect to static predistortion approaches such as [4] is that it is robust to different LED types and that it automatically adapts to time-vayring LED characteristics. Adaptive predistortions have also been devised, but they require complex feedback electronic circuits [2]. Unlike existing adaptive post-equalizers such as [5], our blind approach avoids the need of transmitting extra pilot data that do not carry useful information. Perhaps more importantly, the proposed equalizer opens up the possibility of designing adaptive companding curves in the transmitter without substantially increasing receiver complexity.

The premise of our approach is to explore the Gaussianity of the time domain OFDM symbol in order to find an inverting nonlinear curve that yields an output with the correct distribution. The Cumulative Distribution Function (CDF) of the distorted signal is first approximated by its empirical distribution. Then a distribution equalization procedure generates a nonlinear curve estimate. Later on, a simple least squares polynomial fitting yields a smother curve, which can then be used to recover the time domain signal. The present work can be seen as an adaptation of a previous work aimed at nonlinear distortion compensation for audio applications [7]. A related technique was proposed in [8] in the context of RF systems, in which the distribution of the amplitude of the time domain signal was Rayleigh rather than Gaussian.

While the ideas in this paper could be adapted to all forms of O-OFDM systems, the details would differ for each particular form. For the sake of clarity and conciseness, in this paper we decided to focus on the asymmetrically clipped optical OFDM (ACO-OFDM) scheme, and we leave the consideration of other O-OFDM schemes for future works.

A. VLC OFDM systems

Many variations of Optical-OFDM (O-OFDM) have been proposed in order to make the time domain symbol to be real and positive, as required in VLC. Frequency domain Hermitian symmetry should be enforced so that the time domain symbol is real. In direct-current O-OFDM (DCO-OFDM), a DC bias

ISBN 978-0-9928626-7-1

is added to force the resulting signal to lie mostly in the positive range. In asymmetrically clipped O-OFDM (ACO), in constrast, the negative parts of the time domain signal are clipped at zero and the positive ones are transmitted without modification. In order to avoid loss of information in this scheme, only the odd carriers are loaded with useful data and the even ones are loaded with zeroes. It can be shown that the error caused by negative clipping will affect only the odd frequencies, which will be discarded in the receiver. In ACO-OFDM, because of Hermitian symmetry and insertion of zeros, only one fourth of the carries transmit useful information. The advantage of ACO-OFDM in comparison with DCO-OFDM is that it requires less power for a given BER level [9].

The structure of ACO-OFDM is detailed in Fig. 1. For clarity reasons and to facilitate concentrating on the strucures relevant for this paper, the cyclic prefix and the frequency domain (FD) equalization blocks, typically employed in OFDM in order to remove ISI, are not shown in the figure. This simplified structure is appropriate for a Line-of-Sight (LOS) scenario. It is important to point out that the proposed equalizer can work in more general settings as long as a separate FD equalization is employed beforehand, in a similar fashion to [10].

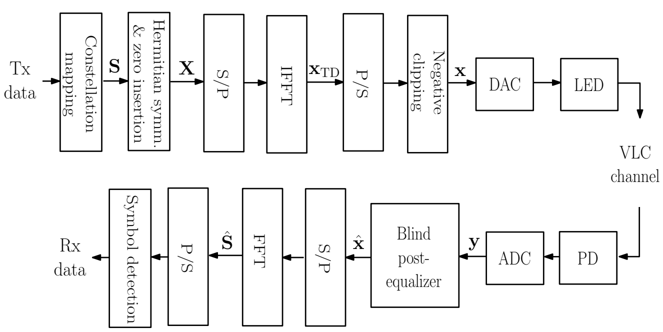

Figure 1. Structure of an ACO-OFDM VLC system.

B. The ACO-OFDM time domain signal

For an ACO-OFDM frame transmitting N (possibly complex) symbols $S_0, S_1, \dots, S_{N-1}$, the frequency domain signal of length $4N$, after inserting zeroes and enforcing Hermitian symmetry,

$$\mathbf{X} = \begin{bmatrix} 0 & S_0 & 0 & \dots & S_{N-1} & 0 & S_{N-1}^* & 0 & \dots & S_0^* \end{bmatrix}^T, \tag{1}$$

S^* is the conjugate of S. The time domain symbol $\mathbf{x}_{\mathrm{TD}}$ of length $4N$ is generated as $\mathbf{x}_{\mathrm{TD}} = \mathbf{W}^T\mathbf{X}$, where $\mathbf{W}$ is the DFT matrix. Because every sample in $\mathbf{x}_{\mathrm{TD}}$ can be seen as the sum of many random variables (for typical values of N), its distribution approximates a Gaussian, according to the central limit theorem (CLT) [11]. The clipped version of this signal is generated by simply mapping negative samples of $\mathbf{x}_{\mathrm{TD}}$ into zero. To avoid clumsy notation later on, we refer to this clipped version signal as simply $\mathbf{x}$. Naturally, $\mathbf{x}$ will be distributed as a truncated Gaussian.

C. LED model

In VLC, the LED is the element performing electrical-to-optical power conversion. This involves two steps: voltage to current (V-I) and current to optical (I-O) conversions. In order for the V-I to be properly performed, the LED voltage v should be above its Turn-On Voltage (TOV). For safety regulations reasons and to prevent damage to the device, the LED current i should be limited to a certain maximum value $i_{\max}$. A model that encapsulates the LED I-V nonlinear curve and the need to limit its current output was proposed in [12], defined by:

$$i(v) = \begin{cases} h(v), & \text{if} \quad v \geq 0 \\ 0, & \text{if} \quad v < 0 \end{cases}, \tag{2}$$

with

$$h(v) = \frac{f(v)}{\left(1 + \left(\frac{f(v)}{i_{\max}}\right)^{2k}\right)^{1/2k}}, \tag{3}$$

where $f(v)$ defines the LED I-V characteristics, which in practice could be obtained from the device datasheet or through measurements. This equation is referred to as the S-model and is based on the Rapps model used to describe power amplifiers. In the VLC context, parameter k controls the curve smoothness (higher values of k lead to less smooth curves). In our simulations, we defined the parameters of $h(v)$ similarly to [12].

D. Channel model

In addition to the LED nonlinear distortion, the received signal will be contaminated by electronic thermal noise. Therefore, a useful channel model for evaluating our proposal (that disregards multipath dispersion) is:

$$y_n = f(x_n) + v_n, n = 0, \dots, 4N - 1, \tag{4}$$

where x_n and y_n are the transmitted and received time domain signals, and v_n is the noise, assumed to be AWGN with variance σ_v^2. Function $f(\cdot)$ in practice is obtained from Eq. (3), but in the following analysis it will be considered a generic memoryless invertible smooth function. One observes that due to free space fading, the received signal has much lower power than the transmitted one, and thus the nonlinear effects of the photodetector can be disregarded.

II. Proposed Blind Post-Equalizer

For derivation of the proposed tool, we assume a noise-free scenario, leaving the consideration of noise when assessing the performance of the method in Sec. III. In a statistical framework, signals x_n and y_n can be interpreted as samples from discrete stationary i.i.d. random processes X_n and Y_n, respectively. For clarity reasons, and to explore the i.i.d. assumption, we will drop time index n and write:

$$Y = f(X). \tag{5}$$

The proposed equalizer is based on finding a compensating nonlinear curve $f_{eq}(\cdot)$ which produces in its output a random process $\hat{X}$ that is statistically similar to X. More precisely,

ISBN 978-0-9928626-7-1 © EURASIP 2017

by exploring the near-Gaussianity of X, the equalizing curve is obtained in such a way that the CDF of $\hat{X} = f_{eq}(Y)$ is close to a normal. The practical design of the equalizer can be divided into two main steps:

1) Find via distribution equalization a non-parametric estimate $\hat{f}^{-1}(\cdot)$ of the LED inverse nonlinear curve;
2) Obtain a smooth parametric polynomial estimate of $f_{eq}(\cdot)$ that best fits function $\hat{f}^{-1}(\cdot)$ in the least squares sense.

We detail each step in the next two subsections.

A. Non-parametric curve estimation via distribution equalization

The monotonically increasing behavior of the LED curve (Fig. 2, i.e. $P\{X < x\} = P\{Y < y\}$) implies:

$$P\{X < f^{-1}(y)\} = P\{Y < y\}. \tag{6}$$

We recognize the left hand side as the CDF of X applied at point $f^{-1}(y)$ and the right hand side as the CDF of Y applied at point y. Hence:

$$F_X(f^{-1}(y)) = F_Y(y). \tag{7}$$

By applying $F_X^{-1}(\cdot)$ to both sides in the above equation, we finally obtain $f^{-1}(y) = F_X^{-1}(F_Y(y))$. If the distributions of X and Y were known, the inverse LED curve would be obtained exactly. Because neither $F_X(\cdot)$ nor $F_Y(\cdot)$ is perfectly known, we approximate them, yielding:

$$\hat{f}^{-1}(y) = \hat{F}_X^{-1}(\hat{F}_Y(y)). \tag{8}$$

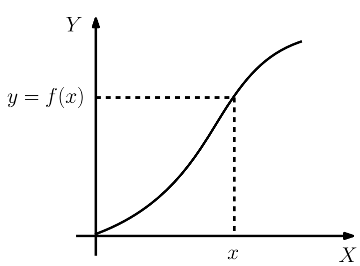

Figure 2. Generic LED nonlinear curve.

As we stated before, the CLT implies that the time domain ACO-OFDM signal (before clipping) is approximately normally distributed for typical OFDM systems. After negative clipping, only the rightmost half of the distribution preserves its Gaussian shape, while the left part collapses in the origin. Therefore:

$$\hat{F}_X(x) = \begin{cases} F_N(x|0, \sigma_X^2), & x \geq 0 \\ 0, & x < 0 \end{cases}, \tag{9}$$

where $F_N(x|0, \sigma_X^2)$ is the CDF of a normally distributed random variable (R.V.) of zero mean and variance σ_X^2.

Furthermore, the CDF $F_Y(\cdot)$ can be approximated by the empirical distribution of R.V. Y, which is calculated as the fraction of samples that lie below a certain point, or, more precisely [11]:

$$\hat{F}_Y(y) = \frac{1}{4N} \sum_{n=0}^{4N-1} I_{(-\infty,y]}(y_n), \tag{10}$$

where $I_A(x)$ is the indicator function, which returns 1 if $x \in A$ and 0 otherwise. Because of the Glivenko-Cantelli theorem [11], the empirical distribution converges uniformly almost surely to the correct CDF. By replacing Eqs. (10) and (9) in Eq. (8), we obtain a non-parametric LED inverse function estimate.

B. Least squares parametric smooth curve estimation

At this point, one could use the nonparametric LED inverse curve estimate $\hat{f}^{-1}(\cdot)$ as the equalizing function $f_{eq}(\cdot)$; however, due to inaccuracies of the empirical distribution for finite N, this estimate would be less smooth than expected for a typical LED curve. A simple and computationally efficient way to explore this prior information is by an LS fitting of a parametric polynomial model to the non-parametric curve estimate of Eq. (8). The following parametrization of the equalizing curve is adopted:

$$\hat{x}_n = f_{eq}(y_n) = w_1 y_n + w_2 y_n^2 + \ldots + w_P y_n^P, \tag{11}$$

where w_i, $i = 1, \ldots, P$ are the unknown polynomial weights; the number of free parameters in the model, P, should be chosen beforehand according to the expected level of LED nonlinearity. This linear-in-parameters model can be adjusted by minimizing with respect to weights w_i the squared Euclidean distance between the curves. The resulting quadratic cost function $C(\mathbf{w})$ can be written in vector form as:

$$C(\mathbf{w}) = \left\| \mathbf{Y}\mathbf{w} - \hat{\mathbf{f}}^{-1}(\mathbf{y}) \right\|_2^2, \tag{12}$$

where $\hat{\mathbf{f}}^{-1}(\mathbf{y}) = \begin{bmatrix} \hat{f}^{-1}(y_0) & \hat{f}^{-1}(y_1) & \ldots & \hat{f}^{-1}(y_{4N-1}) \end{bmatrix}^T$, $\mathbf{w} = \begin{bmatrix} w_1 & w_2 & \ldots & w_P \end{bmatrix}^T$, and matrix $\mathbf{Y}$ is defined as:

$$\mathbf{Y} = \begin{bmatrix} y_0 & y_0^2 & \cdots & y_0^P \\ y_1 & y_1^2 & \cdots & y_1^P \\ \vdots & \vdots & \ddots & \vdots \\ y_{4N-1} & y_{4N-1}^2 & \cdots & y_{4N-1}^P \end{bmatrix}. \tag{13}$$

Minimization of the quadratic function $C(\mathbf{w})$ leads to:

$$\mathbf{w} = \left(\mathbf{Y}^T\mathbf{Y}\right)^{-1} \mathbf{Y}^T \left(\hat{\mathbf{f}}^{-1}(\mathbf{y})\right). \tag{14}$$

The equalizer design is completed by substituting in Eq. (11) the weights w_i obtained in Eq. (14).

III. Performance Assessment

To assess the merits and limitations of the proposed tool, we use as performance measure the Symbol Error Rate (SER) and time domain Signal-to-Noise-plus-Distortion Ratio (SNDR):

$$\text{SNDR} = 10 \log_{10} \left(\frac{\|\mathbf{x}\|_2^2}{\|\mathbf{x} - \hat{\mathbf{x}}\|_2^2} \right), \tag{15}$$

ISBN 978-0-9928626-7-1

where $\mathbf{x}$ and $\hat{\mathbf{x}}$ are vectors containing all $4N$ samples of x_n and $\hat{x}_n$ (obtained from Eq. (11)). This figure-of-merit encompasses all system imperfections, including inaccuracies in the curve estimate, channel noise and the possible effect of noise enhancement caused by the equalizer.

For the conventional receiver (without equalizer) SNDR is computed by replacing $\hat{\mathbf{x}}$ with $\mathbf{y}$, the vector containing samples of the received signal y_n. Since nonlinear distortion in time domain causes carrier interferences in frequency domain that is indistinguishable from Gaussian noise, the SNDR is the main variable defining the symbol error rate (SER) for a given constellation. In next two subsections, we consider the effect of the OFDM frame length N and parameter k on the SNDR; later on, systemic performance in terms of SER is analyzed for different optical power and noise variances. In all simulations, we used $i_{\max} = 0.5$, $P = 5$ and we considered 64-QAM as the OFDM constellation.

A. Effect of frame length N

In a noise-free scenario, the performance of the method depends on two crucial factors: (1) how accurately the empirical distribution of Y estimates its actual CDF, and (2) how close the time domain OFDM signal x_n is to normal. While the CLT and the Glivenko-Cantelli theorem guarantee asymptotic consistency of these approximations, the equalizer performance for finite N is hard to predict theoretically. In this section we investigate via Monte Carlo simulations how the SNDR varies with respect to N for $k = 2$, $\sigma_X^2 = 0.1$ and $\sigma_v^2 = 0$ (i.e., no noise).

We first evaluate the effect of frame length N on the SNDR for N ranging from 2^6 to 2^{14}. Fig. 3 allows us to conclude that larger values of N imply higher median SNDR and lower spread around the median. In this range of variation of N, the equalizer typically achieves a substantial improvement in SNDR when compared with a system without equalization. It is important to point out that, in practice, many OFDM frames could be combined and used in the design of the equalizer, thus improving the accuracy of the curve estimate. This strategy depends on the LED curve being fixed for more than one frame, which tends to be the case in practice.

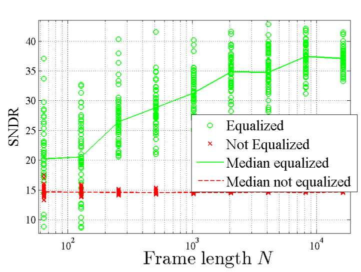

Figure 3. Analysis of effect of frame length holding the remaining parameters constant at $k = 2$, $\sigma_X^2 = 0.1$, in a noiseless scenario. Median performance of the equalizer improves substantially with increasing frame length N.

B. Effect of parameter k

Parameter k in Eq (3) determines how soft the clipping operation is: lower values of k mean softer clipping and thus a function that is easier to equalize. Fig. 4 shows the SNDR obtained with the proposed equalizer in comparison with a standard receiver, for different values of k. These graphs consider $N = 1024$, $\sigma_X^2 = 0.2$ and $\sigma_v^2 = 0$ (again, no noise). They show that higher values of k reduce the SNDR for both the proposed receiver and the conventional one – i.e. independently of the equalizer. The reason is that the curve becomes less linear as k increases and the accuracy of the curve estimate degrades. It should be noted, however, that lower values of k, while reducing the level of nonlinearity, imply lower transmitted power, which could degrade system performance depending on the noise level.

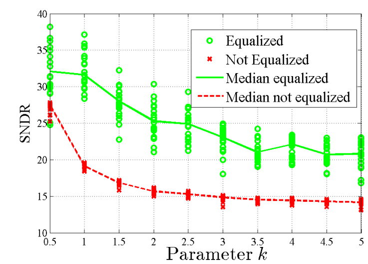

Figure 4. Analysis of the effect of parameter k of LED S-model holding the remaining ones constant at $N = 1024$, $\sigma_X^2 = 0.2$, in a noiseless scenario. Higher values of k (less smoth curve) are associated with reduced median SNDR.

C. Effect of noise variance σ_v^2

We consider now the effect of the noise power. In order to facilitate the interpretation of the results of this test, we define an idealized SNR measure based on the ratio of the signal variance σ_X^2 before clipping and the noise variance in the receiver, that is $\text{SNR} = 10\log_{10}(\sigma_X^2/\sigma_V^2)$. Fig. 5 shows the SER as a function of SNR for the equalized receiver and the conventional one, considering $N = 1024$, $k = 0.8$ and $\sigma_X^2 = 0.15$. For high SNR, the proposed equalizer achieves a substantial SER reduction. On the other hand, for low SNR, the equalizer can sometimes deteriorate slightly the performance.

D. Effect of transmission power

The optical power in VLC is directly related to the average time domain signal $E[X]$ while the electrical power, on which the performance ultimately depends, is proportional to $E[X^2]$ [9]. We investigate in this section how variations in $E[X]$ affect system SER performance. Fig. 6 was generated using parameters $k = 0.8$, $N = 1024$ and $\sigma_v^2 = 1.1 \times 10^{-5}$. This graph indicates that the equalizer achieves good performance when $E[X]$ (transmission power) is high, and becomes less impressive when optical power decreases. However, when $E[x]$ is too high, the time domain samples are more severely affected by the LED nonlinearity, and the curve inverse

ISBN 978-0-9928626-7-1

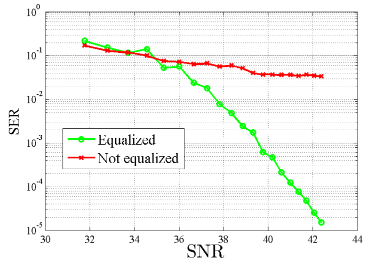

Figure 5. SER performance analysis with respect to SNR when σ_V^2 is varied, and frame length, k parameter and signal variance are kept constant at $N = 1024$, $k = 0.8$ and $\sigma_X^2 = 0.15$, respectively.

estimate becomes less accurate. That is why the performance of the equalizer starts to deteriorate when transmission power surpass a certain point (about $E[X] = 0.4$). On the other hand, when $E[x]$ is low, the nonlinearity is milder and the equalizer becomes less relevant at reducing SER. For very low $E[X]$ values, the proposed receiver slightly worsen performance in comparison with a system without equalization.

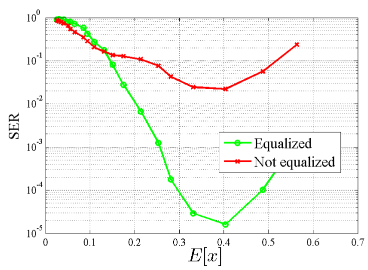

Figure 6. SER performance for different transmission power levels $E[X]$ with remaining variables fixed at $N = 1024$, $k = 0.8$ and $\sigma_v^2 = 1.1 \times 10^{-5}$.

IV. Discussion

As the experiments clearly indicate, the proposed tool is effective at improving system performance for conditions that do not depart excessively from the scenario assumed for its design. Its performance deteriorates when the nonlinear curve becomes close to a hard-limiter (high values of k), as illustrated in Fig. 4. Additionally, the presence of noise can sometimes deteriorate performance in comparison with a conventional receiver. While the former problem could be handled by changing the value of k or controlling the transmitted power, the second one is more challenging. Noise deteriorates the performance of the equalizer in two different ways: (1) it causes bias in the inverse LED curve estimate of Eq. (8), and (2) it makes the system susceptible to the noise enhancement phenomenon due to the typical expanding shape of the equalizing curve. For ACO-OFDM, these effects are more relevant because a large fraction of the signal lies near the origin where the LED curve is highly nonlinear.

V. Conclusions and Future Works

The main conclusion of this paper is that the statistics of the time domain OFDM symbol are useful to perform blind equalization of the LED nonlinear curve. From this insight, a tool capable of significantly improving system performance provided that the original signal does not undergo hard-clipping and the noise level in the receiver is sufficiently low is proposed.

A natural extension of this work is to consider other forms of OFDM systems, such as DCO-OFDM, for which the proposed equalizer appears to be even more suitable. Another important research direction is to investigate forms of avoiding bias in the curve estimate caused by noise, and its enhancement caused by the typical shape of the equalizing curve. The fact that noise statistics are easily available in the receiver could be used to obtain an unbiased estimate of the CDF of the received signal. Furthermore, noise enhancement could be mitigated by using a maximum likelihood estimate of the transmitted time domain signal. Finally, Bayesian model-based approaches are another promising strategy for blind nonlinear equalization in VLC.

References

[1] Aleksandar Jovicic, Junyi Li, and Tom Richardson, "Visible light communication: opportunities, challenges and the path to market," *IEEE Communications Magazine*, vol. 51, no. 12, pp. 26–32, 2013.

[2] Tao Jiang and Yiyan Wu, "An overview: Peak-to-average power ratio reduction techniques for OFDM signals," *IEEE Transactions on broadcasting*, vol. 54, no. 2, pp. 257–268, 2008.

[3] Kai Ying, Zhenhua Yu, Robert J Baxley, Hua Qian, Gee-Kung Chang, and G Tong Zhou, "Nonlinear distortion mitigation in visible light communications," *IEEE Wireless Communications*, vol. 22, no. 2, pp. 36–45, 2015.

[4] Hany Elgala, Raed Mesleh, and Harald Haas, "Non-linearity effects and predistortion in optical OFDM wireless transmission using LEDs," *International Journal of Ultra Wideband Communications and Systems*, vol. 1, no. 2, pp. 143–150, 2009.

[5] Jianfeng Li, Zhitong Huang, Xiaoshuang Liu, and Yuefeng Ji, "Hybrid time-frequency domain equalization for LED nonlinearity mitigation in OFDM-based VLC systems," *Optics express*, vol. 23, no. 1, pp. 611–619, 2015.

[6] Hongming Zhang, Lie-Liang Yang, and Lajos Hanzo, "Piecewise companding transform assisted optical-OFDM systems for indoor visible light communications," *IEEE Access*, 2016.

[7] Flávio R Avila and Luiz WP Biscainho, "ML estimation of memoryless nonlinear distortions in audio signals," in *Acoustics, Speech and Signal Processing (ICASSP), 2014 IEEE International Conference on*. IEEE, 2014, pp. 4493–4497.

[8] Congying Xia and Jacek Ilow, "Blind compensation of memoryless nonlinear effects in OFDM transmissions using cdf," *nonlinearity*, vol. 7, pp. 8, 2003.

[9] Jean Armstrong and Brendon JC Schmidt, "Comparison of asymmetrically clipped optical OFDM and dc-biased optical OFDM in AWGN," *IEEE Communications Letters*, vol. 12, no. 5, 2008.

[10] Cuiwei He and Jean Armstrong, "Clipping noise mitigation in optical OFDM systems," *IEEE Communications Letters*, 2016.

[11] Larry Wasserman, *All of statistics: a concise course in statistical inference*, Springer Science & Business Media, 2013.

[12] Hany Elgala, Raed Mesleh, and Harald Haas, "An led model for intensity-modulated optical communication systems," *IEEE Photonics Technology Letters*, vol. 22, no. 11, pp. 835–837, 2010.

ISBN 978-0-9928626-7-1 © EURASIP 2017