

# Incommensurable design descriptions

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**Abstract**—Data management is a persistent problem in design and manufacturing. This is because different processes require different descriptions of the same design concept. Descriptions can include geometry and/or topology as well as other non-spatial information, such as design intent, and over the course of a design and manufacturing process it is often necessary to convert between descriptions non-sequentially, to support development and realisation of a design concept. This paper highlights the difficulties that arise in managing different descriptions by exploring what are possibly unrealistically simple examples involving drawings of simple shapes. Although simple, the examples illustrate a fundamental truth; that the information embedded in the structures of different descriptions of a design are often incommensurable, and this can introduce challenges in the management of design data.

**Keywords**—design descriptions; shape structure, design data

## I. INTRODUCTION

Design descriptions come in many different forms and, throughout a design and manufacturing process, these are used individually, in parallel, sequentially and iteratively to support the design, analysis and realisation of emerging design concepts. However, working with multiple design descriptions can pose significant problems in data management [1]. For example, engineering design processes involve generating and coordinating descriptions in different domains, including product structures (bill of materials), manufacturing instructions, specifications for assembly, supply, service and maintenance, etc. Computationally, these different descriptions are realised as design data in systems developed for specific phases of the design process. Computer-aided design (CAD) is used for defining form and for representing and simulating a design concept, computer-aided engineering (CAE) is used for analysis of performance, computer-aided manufacturing (CAM) is used for realising a concept via manufacture, etc. These different design data are managed through product-lifecycle management (PLM) tools [2], but translating between them, so that the transitions between different phases of a design process is smooth, is an unresolved challenge.

As an illustrative example consider Fig 1, which is adapted from [1]. Here, a design concept is defined according to a solid model representation, and the concept has two descriptions, one in CAD, the other in CAM. In CAD, the structure is defined according to parametric form features, which specify the geometry of the concept, its material properties, and any constraints applied to the geometry. In CAM the same concept

is defined according to manufacturing features, which relate to specific manufacturing operations. The representations produced by both descriptions are visually the same, but their underlying structure is significantly different because different types of features are identified. Additionally, embedded in these structures is other information which may not be immediately apparent in the visual representation, such as design intent, which in CAD is recorded via model history and/or geometric constraints.

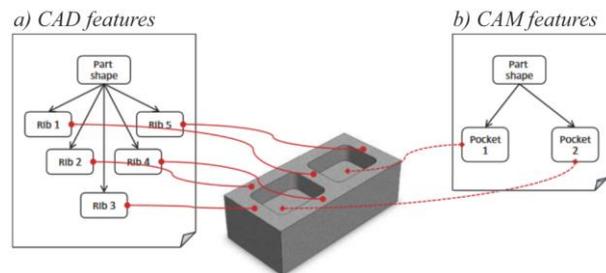


Fig. 1. Descriptions of a concept according to features, adapted from [1]

The scenario illustrated in Fig. 1 is replicated at all phases of the design and manufacturing process, because each phase is concerned with a different aspect of the design concept, e.g. its form, its behaviour, its manufacture, its assembly, etc. Consequently, design descriptions at each phase have different structures defined according to different types of features, and different information embedded in the structure [3]. Issues of geometric interoperability between structures make it difficult to integrate models used in different phases, e.g. between CAD and CAE, for analysis of the behaviour of a concept [4], or between CAD and CAM, for realisation of the concept through manufacture [5]. Information, such as design intent, that is readily available in one design description may be difficult to recover in other design descriptions [2]. This issue is not exclusive to engineering design; similar problems also arise in other areas. For example, in architectural design and construction, building information modelling (BIM) systems aim for consistency across different descriptions of building designs, including structural, lay-out, service and environmental [6].

In this paper, the problem of integrating design descriptions is explored via consideration of drawings of simple shapes, and their symmetric properties. The range of shapes is illustrated in Fig. 2. Each shape is composed of two squares, and these are

used as an analogy for design concepts. Different descriptions of these shapes give rise to different structures, defined according to sets of lines. The symmetry of the constituent shapes is used as an analogy for design intent, and it will be shown that integrating different descriptions of a design whilst retaining pertinent information, such as design intent, is not necessarily possible. By focussing on descriptions of simple shapes it is possible to show that incommensurable structures arise readily. Further, it is suggested that this is a potential source of integration issues in design data management, resulting in the need to adopt a multi-representation approach [3].

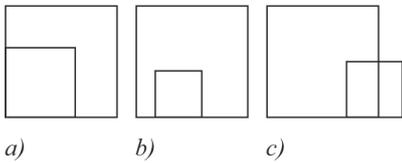


Fig. 2. A range of shapes composed of two squares

## II. DESCRIBING SHAPES AND DESIGN INTENT

Each of the shapes in Fig. 2 is composed of two squares, and despite their simplicity they can give rise to different descriptions, depending on how the shapes are viewed and/or constructed. For example, recognising the different squares in Fig. 2a produces different structures, as illustrated in Fig. 3, and each structure acts as a description of the shape, defined according to identified parts. Integrating these descriptions is analogous to the problem of integrating different descriptions of design concepts, defined according to features of different types.

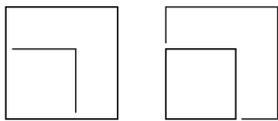


Fig. 3. Two descriptions of a shape

If the shape is taken simply as a set of lines, then integrating the two descriptions in Fig. 3 in a representation that incorporates both is relatively straightforward. The result is illustrated in Fig. 4, where edges of the large square have been decomposed to accommodate the edges of the small square. From this shape it is possible to account for both descriptions, although the two squares now have different representations: the small square is composed of four edges, and the large square is composed of six edges.

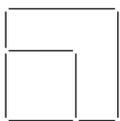


Fig. 4. Shape structure, accomodating different descriptions

However, design descriptions rarely include only geometry. They also include other information embedded in their structure, such as design intent, and this can further complicate the problem of integrating design descriptions. For the shape in Fig. 2, it can be argued that the intent is to represent two squares, but the structure illustrated in Fig. 4 has lost this intent, because important properties of the original squares, namely their symmetry, has been lost. The small square has retained its symmetry, but the large square has not, and the intent of the designer has been lost as a result of the process of integrating the two description. Constructing an integrated representation that incorporates the two descriptions of the shape whilst retaining symmetric properties is more challenging, and this explored in the next section for each of the types of shape illustrated in Fig. 2. In Fig. 2a, recognised squares share a point and vertex, in Fig. 2b recognised squares share a common edge, while in Fig. 2c recognised squares share parts of an edge but also overlap.

## III. INTEGRATING SHAPE DESCRIPTIONS

### A. Descriptions with a common vertex

Fig. 5 illustrates a visual process for integrating the two descriptions in Fig. 3. This process involves resolving the symmetries of the identified parts of the edges. The triangles are included on the bottom edge of the two squares to illustrate the part structures while highlighting their symmetry. For the sake of clarity this structure is shown only on the bottom edge, but it can be assumed that it applies to all edges of the squares. Each triangle correlates with a line segment embedded in an edge, and these are subdivided into finer structures, representing lines embedded in lines. Embedded lines associated with triangles are symmetrical, and their subdivision into embedded parts is symmetrical; in this sense, the triangles represent the structure of the edges as visual palindromes.

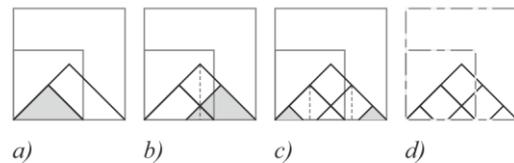


Fig. 5. Deriving the structure resulting from a combined description

Fig. 5a illustrates the initial structure of the edges where, to account for the symmetric properties of the squares, each edge is identified as a visual palindrome, represented by a triangle. The edge of the small square is embedded in the edge of the large square. As a consequence, the structure of the longer edges now incorporates an embedded line that is the length of the shorter edge, represented by the triangle highlighted in grey. This new structure breaks the symmetry of the longer edges, as identified in Fig. 4. The symmetry is resolved in Fig. 5b by reflecting the small triangle in the illustrated axis of symmetry (dashed line) of the large triangle. A new triangle is defined by the overlap, and this represents further subdivision of the visual palindrome. Fig. 5c resolves the symmetry of the longer edge by reflecting the emergent triangle in the illustrated axes of symmetry. The resulting part structure is illustrated in

Fig. 5d. It is an integrated description that accounts for the symmetric properties of both squares, and allows the edges of the small square to be embedded in the edges of the large square. The two squares still have different representations but in these, symmetric properties are preserved. The result is a periodic palindromic structure where the shorter edges can be described by the string  $UVU$  and the longer edges can be described by the string  $UVUVU$ , where  $U$  and  $V$  represent line segments of different lengths, determined by the ratio of the lengths of the edges of the squares. Intuitively, it might be expected that embedding a short edge as part of a longer edge would simply result in a decomposition of the longer edge to accommodate the shorter. But this is not what is shown in Fig. 5, and the reason for this is apparent in Fig. 5b, where the emergent triangle requires a finer decomposition of the edges than might be intuitively expected.

This result can be generalised by considering the edges of the squares as combinatorial arrangements of words representing line elements, as explored in [7]. Let the word  $A$  represent an edge of the small square, and  $AB$  represent an edge of the large square, as illustrated in Fig. 6a. The symmetric properties of the two squares require that their edges are equal to their reflections, i.e.

$$\begin{aligned} A &= \overline{A} \\ AB &= \overline{AB} \end{aligned}$$

where the bar notation indicates a reflected word. Together these give

$$AB = \overline{BA}$$

This is a combinatorial configuration that is addressed in Lyndon and Schützenberger [8], and gives rise to the structure illustrated in Fig. 6b, with  $A$  and  $B$  defined as follows:

$$\begin{aligned} A &= (UV)^k U \\ B &= VU \end{aligned}$$

Here,  $U$  and  $V$  are both words and  $k$  is an integer defined by the ratio of the lengths of  $A$  and  $AB$ . For the specific example illustrated in Fig. 2a,  $k$  is equal to 1, but as the length of  $B$  decreases so that the length of  $A$  gets closer to the length of  $AB$ , the value for  $k$  increases resulting in finer and finer decompositions of the edges, with the length of  $U$  and  $V$  getting shorter, and the number of line segments increasing.

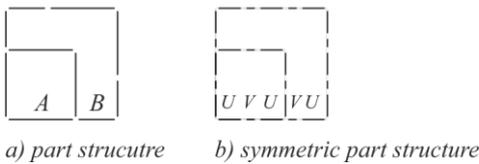


Fig. 6. Combinatorial structure of shape from Fig. 2a

The structure is always defined according to line segments of two alternating lengths, which can be described as a periodic palindrome over two atoms,  $U$  and  $V$ . In general [7], the

structure of the shorter edge is always described by the string  $(UV)^k U$ , and the structure of the longer edge can be described by the string  $(UV)^{k+1} U$ . The key characteristic of the shape in Fig. 2a is that the two squares share a common edge and a common vertex. The integrated description has to take into consideration the embedded lines (shared geometry) and the symmetry of the lines (design intent) and it gives rise to a periodic palindromic structure  $(UV)^{k+1} U$  for the resulting shape (Fig. 6b).

### B. Descriptions with common lines

Recognising the two squares in the shape in Fig. 2b also gives two different descriptions of the shape, and these can also be integrated by considering the edges of the squares as combinatorial arrangements of lines, as illustrated in Fig. 7. In Fig. 7a, the word  $B$  represents an edge of the small square, and the word  $ABC$  represents an edge of the large square. As before, the symmetric properties of the two squares require that their edges are equal to their reflections, i.e.

$$\begin{aligned} B &= \overline{B} \\ ABC &= \overline{ABC} \end{aligned}$$

Together these give

$$ABC = \overline{CBA}$$

and, without loss of generality if we assume  $|A| < |C|$ , resolving this combinatorial configuration gives rise to the structure illustrated in Fig. 7b, with  $B$  and  $C$  defined as follows:

$$\begin{aligned} B &= (UV)^k U \\ C &= VU\overline{A} \end{aligned}$$

Again,  $U$  and  $V$  are both words and  $k$  is an integer defined by the ratio of the lengths of  $B$  and  $ABC$ . For the specific example illustrated in Fig. 2b,  $k$  is equal to 1, but as the length of  $C$  decreases so that the length of  $B$  gets closer to the length of  $ABC$ , the value for  $k$  increases resulting in finer and finer decompositions of the edges, with the parts  $U$  and  $V$  getting shorter, and the number of line segments increasing.

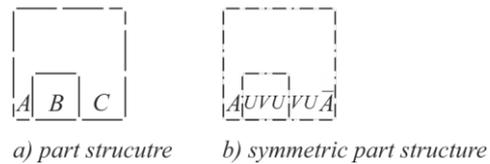


Fig. 7. Combinatorial structure of shape from Fig. 2b

The structure is always defined according to line segments of three different lengths. The structure is described as a periodic palindrome over two atoms,  $U$  and  $V$ , contained within a border  $A$ . In general [7], the structure of the shorter edge is always described by the string  $(UV)^k U$ , and the structure of the longer edge can be described by the string  $A(UV)^{k+1} U\overline{A}$ . The key characteristic of the shape in Fig. 2b is that the two squares share a common edge with the edge of the small square

embedded in the edge of large square. The integrated description has to take into consideration the embedded lines (shared geometry) and the symmetry of the lines in the square configurations (design intent). This gives rise to the resulting integrated description of the shape shown in Fig. 7b, which is composed of the periodic palindromic structure contained within a border.

### C. Overlapping descriptions

The problem of integrating descriptions of the shape in Fig. 2c is more difficult. Recognising the two squares in the shape gives the two different descriptions of the shape, as illustrated in Fig. 8a. In the previous two cases, the edge of one square was embedded in the edge of another, so that the structure of the two edges were always commensurable. In Fig. 2c, the edges of the squares share a common part, so only that part of the structures is required to be commensurable, as illustrated in Fig. 8b.

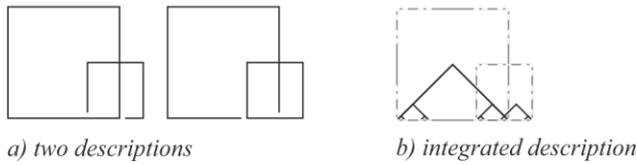


Fig. 8. Describing overlapping squares

Combinatorially, this is equivalent to satisfying two separate relations between arrangements of words. If the edge of the large square is represented by the word  $AB$  and the edge of the small square is represented by the word  $BC$ , as illustrated in Fig. 9, then the symmetric properties of the two squares requires that

$$AB = \overline{BA}$$

and

$$BC = \overline{CB}$$

So, each of these two conditions gives rise to a different combinatorial structure of the form found for the shape in Fig. 2a. So,

$$\begin{aligned} A &= UV \\ B &= (UV)^k U \end{aligned}$$

and

$$\begin{aligned} B &= (XY)^j X \\ C &= YX \end{aligned}$$

Here,  $U$ ,  $V$ ,  $X$  and  $Y$  are all words and  $k$  and  $j$  are integers where  $k$  is defined by the ratio of the lengths of  $B$  and  $AB$ , and  $j$  is defined by the ratio of the lengths of  $B$  and  $BC$ . For the specific example illustrated in Fig. 2c,  $k$  is equal to 0 and  $j$  is

equal to 1, and an integrated description has been identified that includes both descriptions illustrated in Fig.8a. However, this is not always possible, and [9] explores an example where an integrated description cannot be found. Problems arise because the descriptions that result from recognising the different squares give rise to structures that are not necessarily commensurable.

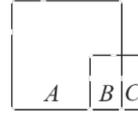


Fig. 9. Combinatorial structure of shape from Fig. 2c

Both small and large squares are reduced to periodic palindromic structures, and for the large square this is of the form  $(UV)^{k+1}U$ , while for the small square it is of the form  $(XY)^{j+1}X$ . The period of the first is given by the length  $|UV|$  while the period of the second is given by the length  $|XY|$ . Words have atomic parts which are the constituent letters, but shapes do not. Shapes have parts, which are themselves shapes, and these are continuous. Although presented as words, the lengths  $|UV|$  and  $|XY|$  do not necessarily take integer, or indeed rational, values. Consequently, describing a shape according to recognised parts, whether these are lines or features, makes it necessary to engage with the peculiarities of the real numbers, which unlike words, do not necessarily align in combination. This is particularly apparent in the examples explored here, where multiple descriptions are combined with the aim to find an integrated description. If the resulting periodic palindromic structures have incommensurable periods, then descriptions cannot be combined in an integrated description. This is an unavoidable and fundamental truth which results in the problems inherent in managing design data.

## IV. DISCUSSION

This paper has explored the geometric interoperability of different descriptions of a design concept. It has shown, via consideration of drawings of simple shapes, that such descriptions can be incommensurable. The issue of interoperability has been formulated, for these drawings, in terms of the existence of a valid ‘covering’ description. This ‘covering’ includes the separate descriptions as well as conforming to overall properties of the pertinent parts selected for use in the descriptions. This paper has used the symmetry properties of two squares; a large and a small square, aligned along one edge. Each square is used to identify a possible description. The paper explored cases where a covering description can be constructed which maintains the symmetries of the component squares as well as cases in which such covering descriptions cannot be constructed. In this latter case, the two descriptions are incommensurable. Essentially, an attempt to construct a covering description which works for one of the squares, leads to further refinements in the description of the other square, and so on back and forth between the descriptions possibly without resolution.

This shape-based perspective has revealed, albeit in simple examples, a critical problem. This is that individual shape

descriptions cannot necessarily be integrated within a covering description. For shapes this means that if a description corresponds to a way 'to see' a shape then different ways to see a shape may not be consistent. This has implications for the descriptions used across the design and manufacturing process.

It is important to recognise that this incommensurability of descriptions is more than incompatibility between different linguistic or semantic data associated with descriptions. Rather, this paper indicates that the underlying structure of two descriptions may not be integrated in a single covering structure. The association of different data types has led to systems for design data management, e.g. in PLM or BIM, typically adopting a multiple-representation approach. This approach avoids the need to integrate different design descriptions [3]. However, this also introduces other complications, because when changes to the design concept are introduced they need to be managed in all descriptions of a concept [2], and when changes to process are introduced new descriptions need to be included [10].

An alternative to the multiple representations' approach is proposed by Stiny [11], and implemented by Behara et al. [12]. This is based on shape computations on embedded parts. Descriptions do not correspond with fixed parts of a shape. Instead descriptions are constructed for a shape via recognition of pertinent parts and properties, e.g. features and design intent. The present paper has shown that this approach does not escape the issue of underlying structural differences between descriptions.

The descriptions used here arise from recognising parts and properties. Individual squares together with their shape complements are identified as parts and shape symmetry is employed as the relevant design property. In the examples a designer might see a 'two squares' shape in two alternative ways depending on which square is the focus of attention. The resulting data to describe these alternative views may not admit a covering structure which encompasses both views. The reasons for this incommensurability between descriptions and its effects on data representation in CAD pose immediate and pressing questions. One of these is about how parts and properties are identified for descriptions? Further, how are these descriptions constructed? One attractive route is to use rules as the mechanism of identifying parts in individual descriptions. Repeated applications of the rules associated with the individual descriptions may offer the means to explore the possibilities for covering descriptions. In the examples presented here the rule is an identity, from square to square, while repeated application of the rule applies to both identified squares as well as to their symmetry transformations.

This paper has prompted questions about design descriptions; where they come from and how they are used. It has highlighted the special properties of shape computations that puts them beyond standard analytical techniques in design data management systems that require explicit, finite compositions beforehand. The results here, specifically from shape descriptions, suggest that composing design descriptions is more complex. They indicate that compositions of descriptions are emergent rather than predefined. The paper suggests both the difficulty of shape computation and the

importance of re-examining the algorithmic approaches to design. This will inform practice in creating design descriptions, e.g. in CAD, CAM or CAE, and managing them, e.g. in PLM or BIM.

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