

# Electromagnetic Metamaterial Design using Finite Element Based Optimization Technique: A Case Study for a SATCOM Antenna

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**Abstract**—The development of artificial materials with new features has been a key to the spectacular technological advances of the last decades. In particular, in addition to the analysis and synthesis of materials with special electric and magnetic properties, their fabrication and characterization are topics of high strategic relevance, in view of their potential impact on a variety of leading edge technologies, including not only telecommunications but also aerospace, RF-MEMS, biomedical engineering and the science of materials. Advances in electromagnetic metamaterial design studies based on the integration of robust and fast Finite Element Based simulation tools and topology design algorithms should allow the generation of totally novel and yet unthinkable designs that will lead to a new paradigm in design. Such a case study is presented in this paper for an antenna design for Satellite Communications (SATCOM) working in the Ultra High Frequency (UHF) band. More specifically, in this paper we present the design and fabrication of a SATCOM antenna with material compositions varying in full 3 dimensions to increase the electrical size of a 3-by-3 spiral array antenna and yield an optimum meta-material profile to achieve broadband radiation performance.

**Keywords**—*electromagnetic metamaterials; topology optimization; Finite Element Analysis, SATCOM antenna, UHF*

## I. INTRODUCTION

Increasing demand for wideband devices and stringent metrics for multi-functionality within the telecommunications and microwave industry led to significant developments in novel components and sub-systems required for re-configurable, multifunctional, and dynamic devices. Parallel to these developments, complex electromagnetic devices with unique electromagnetic properties, such as magnetic photonic crystals and left handed media [1], has attracted considerable research interest in recent years. As a result, extensive studies on the theory and application of artificially engineered materials or metamaterials (MTM) have been conducted. Interest in MTMs has increased substantially since their discovery in the late 1980's. The potential of these structures in applications like communication and sensing systems is primarily due to their capability of controlling amplitudes, group and phase velocities, frequencies, wave-numbers, etc. of propagating and non-propagating electromagnetic modes to the extent that was not previously achievable.

Among these, strong controllability of dispersion in waveguides is an extremely attractive MTM characteristic to realize compact and therefore cost efficient phase shifters and

delay lines for various applications including communication systems. In addition, MTMs can reduce the size and enhance the communication performance of integrated planar filter components, especially where loss and bandwidth issues of these components is critical. Dielectric substrate designs of microstrip based patch antennas and filters [2] and electromagnetic bandgap (EBG) structures [3] are important examples. Also, MTMs are used in frequency selective surfaces (FSS), e.g. for radomes, stealth planes, cellular phones. EBG, FSS, and other MTMs gives the opportunity to mold the radiation characteristics of antennas [4]. MTMs have also demonstrated benefits for sub-diffraction imaging applications [5] and presented the opportunity to optimize the radiation characteristics of light emitting diodes to couple light generated in high refractive index materials into air effectively. Similarly, MTMs, more specifically photonic crystal slab lasers with defect modes, were employed to lower the threshold of lasers [6]. This is not only important in terms of efficiency and modulation bandwidth enhancement, but also to realize deterministic single photon sources for quantum computing/communication technologies. Another potential application is the capability of Left Handed Materials (LHM) to be used for super-resolution imaging lenses [5]. Incorporating tunable materials into MTMs or having materials with active and controllable inclusions leads to tunable properties, regardless of the origin of the modulation. Under proper MTM configurations, small variations of the constituents may lead to extreme changes of MTM characteristics. Consequently, tunable components can be realized effectively in MTMs [4, 7]. Practical applications include highly flexible and nimble cellular phones and low-cost steerable antennas.

However, as the variety of examples in the literature show, the perfect combination of materials is unique and extremely difficult to determine without a systematic design framework [2, 3, 8]. Until recently most literature studies dealing with design optimization for RF applications focused to a large extent on size and shape optimization. Material and topology optimization has not been pursued primarily due to the challenges associated with the fabrication of inhomogeneous materials and the limited access to analysis tools. Optimal topology design was demonstrated initially for RF applications in [2] and [9] by designing dielectric material distributions for bandwidth enhancements of a patch antenna and later in [10]. Jensen and Sigmund [11] successfully designed photonic crystal waveguides using topology optimization. Research for

material microstructure designs, on the other hand, using topology optimization started with elastic material studies [12], and has been extended to piezoelectric composites [13], thermal and multifunctional materials with simultaneous design for heat conduction and electrical conductivity [14]. Material design studies targeting effective electromagnetic properties based on coupled Maxwell's equations have been pursued in [15]. More specifically, the topology optimization problem is applied for the first time towards designing the unit cell topology of periodic electromagnetic materials from scratch with desired dielectric and magnetic tensors using off-the-shelf, i.e. readily available constituents obtained from isotropic ceramic powders. In this paper, we extend the application of topology optimization to the design of a metamaterial substrate supporting an inherently wideband spiral antenna unlike in earlier studies [2, 9] conducted primarily for the design of patch antennas. The objective is to reduce the size of the antenna to a maximum of 6 in x 6 in and targeting SATCOM operation at UHF frequencies (224-317MHz) with a particular emphasis on the higher 290-317MHz band.

## II. DESIGN METHODOLOGY

In any automated optimum design approach the goal is to identify in some automatic process/iteratively the device structure subject to some prescribed performance. Here, SIMP is integrated with fast hybrid Finite-Element Boundary Integral (FE-BI) simulations to develop a 6" aperture three-dimensional antenna design subject to the 290-317MHz SATCOM bandwidth constraint.

### A. Electromagnetic Analysis

As is the case with all numerical design optimization loops, the design process must incorporate a flexible and fast analysis module. It is also of crucial importance that this module be fast without compromising the generality of the geometry and material properties of the device.

For our case, a finite element (FE) simulator based on the well-established hybrid finite element-boundary integral (FE-BI) method is employed [16]. A periodic version of the FE-BI method is employed as described in [16] to provide for additional speed by using only a few modes and the FFT to carry out the matrix vector products. However, the optimization can be applied with any other analysis tool and for different applications other than printed antennas and/or arrays. As described in [16], the overall analysis method has  $O(N)$  memory demand and CPU complexity. By virtue of the finite-element method, the simulator is suitable for complex structures such as those involving inhomogeneous dielectrics, resistive patches, conducting patches and blocks, feed probes, impedance loads, etc. Accurate results have already been obtained for scattering and radiation by cavities, slots, and multilayer patch antennas and frequency selective surfaces, demonstrating the method's capability. The conventional implementation of the hybrid FE/BI method for doubly periodic arrays leads to a linear algebraic system in (1):

$$[A]\{E\} + [Z]\{E\} = \{f\} \quad (1)$$

The A matrix is sparse and is associated with the FE portion of the hybrid method. Contributions of dielectric blocks or volumes and resistive cards or metallic edges in the unit cell are embedded in the [A] matrix. The [Z] matrix is associated with the edges on the top and bottom surfaces of the discretized unit cell and is fully populated. The right hand side vector {f} contains, as usual, excitations in the FE volume or BI apertures.

### B. Topology Optimization via the Density Method

Topology optimization methods are general design methods used to obtain simultaneously the best geometric and topological configuration in terms of geometry, physical dimensions, connectivity of boundaries and material implants. They have reached a level of maturity and are being applied successfully to many industrial problems for almost 30 years [17]. In the last decade, some applications have also appeared in the electromagnetics, the majority of which are restricted to specific magneto-static applications [18] or passive devices. For our design problem, we employ the topology optimization method based on the SIMP method to design the dielectric substrate's volumetric material distribution of an inherently broadband spiral antenna for SATCOM applications. The proposed method is aimed at designing the inhomogeneous structure of the dielectric substrate in which the spiral is sandwiched. Antenna's conductor shape can also be used for further bandwidth improvements but is not considered here since the focus is on material design.

SIMP is very attractive to the engineering community because of its simplicity and efficiency. It basically synthesizes the device starting from any arbitrary topology. A key aspect of the design method is that any device, not known a priori, is represented by specifying the material properties at every point of the fixed design domain. For electromagnetic applications, these are the permittivity and permeability of the dielectric material and conductivity/resistance of the metallic conductors, etc. [19]. In practice, to specify the material properties in the design region, the design space is discretized into material cells/finite elements. Actually, the most straightforward image-based geometry representation is the "0/1" integer choice, where the design domain is represented by either a void or a filled/solid material and this was adopted in [20]. However, this formulation is not well-posed mathematically [21]. It can be well-posed by allowing for the design of materials with intermediate properties; that is, materials having graded properties. This is the essence of the SIMP method in which material grading is achieved by introducing a single density variable,  $\rho$ , and relating it to the actual material property of each finite element thru a continuous functional relationship. A suitable interpolation for the permittivity (and possibly a similar one for the resistance of a metallic patch would be):

$$\rho = (\epsilon_{\text{int}} - \epsilon_{\text{air}}) / (\epsilon_{\text{orig}} - \epsilon_{\text{air}})^{1/n} \quad (2)$$

where  $n$  is a penalization factor;  $\epsilon_{\text{int}}$  and  $\epsilon_{\text{orig}}$  are intermediate and original solid material permittivity, respectively. As  $n$  increases, intermediate values for the permittivity are less likely to occur, hence the term penalization for intermediate material. The on/off nature of the problem has been avoided through the introduction of the normalized density with  $\rho=0$  corresponding to a void (air with  $\epsilon_{\text{air}}$ ),  $\rho=1$  to solid (original material  $\epsilon_{\text{orig}}$ ) and  $0 < \rho < 1$  to a graded intermediate dielectric material ( $\epsilon_{\text{int}}$ ). Moreover, this parameterization allows for the formulation of the problem in a general non-linear optimization framework. The goal is to arrive at the optimum distribution of material (densities) such that a certain performance merit of a device is optimized subject to certain design constraints. The problem formulation and its solution will be discussed in the next section.

### C. The Optimization Model

For our design problem, the specific goal is to determine the substrate material distribution of a spiral SATCOM subject to pre-specified bandwidth and miniaturization requirements. A suitable optimization model for our SATCOM antenna corresponds to the minimization of the objective function:

$$f(\rho) = \min \left[ \max(|s_{11}|_j) \right] \quad j = 1, \dots, N_{\text{freq}} \quad (3)$$

subject to a material volume constraint:

$$\sum_{i=1}^{NFE} \rho_i \cdot V_i \leq V^* \quad (4)$$

and side constraints (0/1) for each density variable.

Minimization of the highest return loss among sampled frequency points  $N_{\text{freq}}$  is known to maximize the return loss ( $S_{11}$ ) bandwidth [22] and the volume constraint basically limits the available amount of material. The design problem is easily recognized as a general non-linear optimization problem with usually several thousand variables/FEs. This makes the use of gradient-based optimization techniques a must for the optimization process. Due to its well-known efficiency and reliability, we specifically chose the Sequential Linear Programming (SLP) method employing the DSPLP package in the SLATEC library [23]. The essence of the SLP routine is to replace the objective function and constraints by their linear approximations at each iteration. Updates of the design variables are pursued thru the use of gradient information obtained thru the adjoint variable method [24], an efficient method that permits full interface with the FE-BI electromagnetic solver. We remark that this is an exact analysis based on the solution of the adjoint problem.

The sensitivity analysis is a crucial part of the design loop since it allows the integration of the solver with the SLP optimizer. The most critical aspect in doing so is to employ the gradients or the derivatives of the mathematical functions in the optimization model with respect to the design variables. Details on the sensitivity analysis are given in [24] and [2]. The linear programming sub-problem is then posed to find the optimal design changes from the current design point. It is of

great importance to impose constraints for the design changes known as move limit bounds to ensure convergence. Typically, during one iteration, the design variables are allowed to change by 5-15% of their original values.

## III. DESIGN RESULTS

### A. Initial Design

The initially chosen antenna for the desired SATCOM band is a 3-by-3 array of the spiral geometry shown in Fig. 1. That is one unit cell of the array corresponds to a  $5.8 \times 5.8 \text{ cm}^2$  horizontally fed slot spiral antenna buried within a 2cm thick cavity backed dielectric substrate. The design region is composed of 4 homogeneous layers each with initially  $\epsilon=36$  and 4mm thickness. The slot spiral antenna is sandwiched between the 2<sup>nd</sup> and 3<sup>rd</sup> dielectric layer.

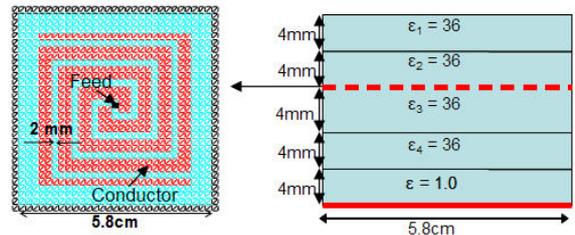


Fig. 1. Initial spiral antenna unit of 3x3 antenna array structure

The optimizer will seek for the material distribution within 4 layers to improve the bandwidth of the initially homogeneous substrate with a resonance around 318 MHz.

### B. Optimized Design

Adopting the above described design algorithm and updating the material distribution via SLP, the design method resulted in a double resonance return loss behavior as shown in Fig. 2a. Convergence was achieved in only 19 iterations as demonstrated by the convergence history in Fig. 2b.

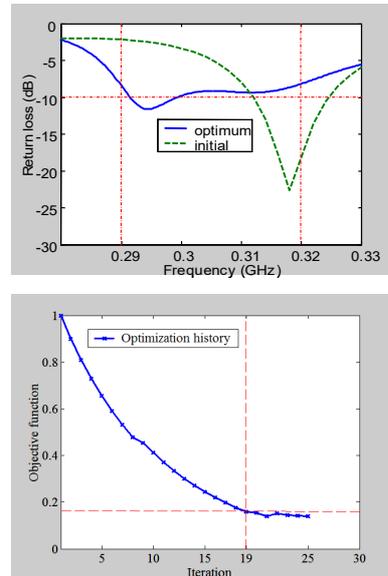


Figure 2: a) (top) Return loss behavior for initial vs. optimum design b) (bottom) Convergence history

The corresponding material distribution across each layer of the final design is plotted in Fig. 3, where each design/FE cell is denoted with a certain color which corresponds to a different permittivity range depicted by the color bar in Fig. 3. For the analysis, the frequency range (288 MHz-322MHz) was sampled with 18 frequency points. It is important to note that the textured metamaterial distribution almost perfectly satisfies the desired SATCOM band with a significant improvement over the initial design. Moreover, filtering/solidification techniques are further employed to transform the design to a simpler manufacturable substrate. The filtering process resulted in the filtered three-shade dielectric distribution as shown in Fig. 4. As a result, the original 4 layer textured substrate is transformed into the filtered 3 layer dielectric substrate on a single homogeneous - fourth layer – with dielectric constant of 15.

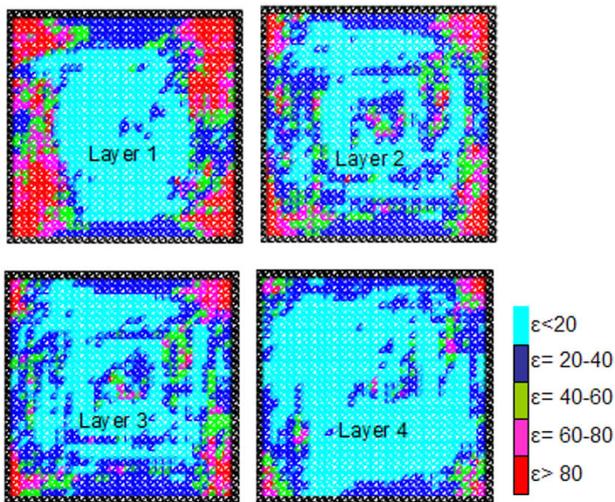


Figure 3: Material permittivity distribution for optimized design

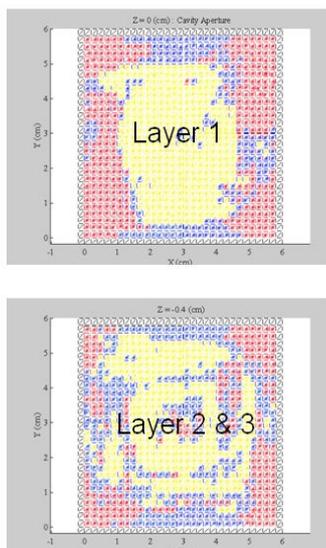


Fig. 4: Layer by layer material permittivity distribution for filtered simplified design with yellow, blue and red indicating  $k=15, 20,$  and  $70$ .

The corresponding return loss performance of the simplified final design is plotted in Fig. 5. It is important to note that during post processing/ intuitive redesigning efforts a natural constraint was to retain the desired SATCOM bandwidth performance (290-320MHz). From the return loss behavior for the simplified design, it is also evident that the use of more discrete yet less number of dielectric shades (3 vs. more than 5) within the dielectric volume resulted in a better matched return loss behavior when compared with the return loss of the previous design (Fig. 2a). When compared with the original homogeneous initial design response (Fig. 2a), the response of the final design presents a significant improvement both in bandwidth and matching properties.

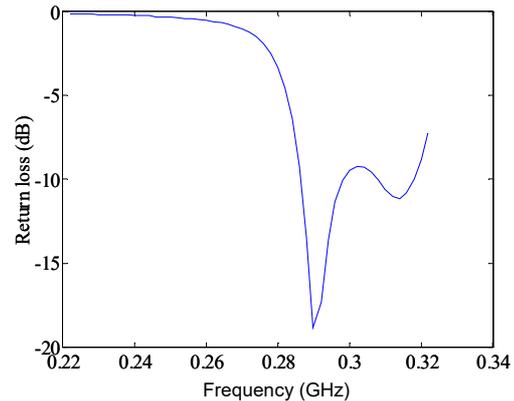


Fig. 5. Return loss behavior for simplified filtered design

#### IV. CONCLUSIONS

In this paper, we extended the SIMP design method along with the SLP optimization method to achieve a pre-specified SATCOM bandwidth for a fixed size of a 3x3 slot spiral array. Critical to the design process was the sensitivity analysis and the use of fast and general analysis tools. As a result, a non-uniform substrate (mix of shades between  $\epsilon_r=1$  and  $\epsilon_r=100$ ) was designed with a significant bandwidth increase. The final design obtained via SIMP demonstrates the ability to design novel material composites with tremendous performance improvement. Initial solidification/filtering efforts indicate manufacturability of the substrate via ceramic laminate tiling. The antenna fabrication efforts along with additional optimization studies for further bandwidth increase will be demonstrated and discussed at the conference. The outcome of these investigations will have tremendous impact on not only microwave devices such as multifunctional novel antennas but also on energy and biomedical applications. It is envisioned that such novel designs are only possible if an attempt is made towards overcoming the challenges of system design from an interdisciplinary engineering perspective, with a focus on using automated design tools such as topology optimization based on Finite Element Analysis and artificially engineered composite materials.

## V. ACKNOWLEDGEMENT

Support of all students in the Kiziltas research group at Sabanci University who have contributed to the metamaterial design and fabrication studies, in particular Isil Berkun, Yasser El-Kahlout and Alparslan E. Bayrak is highly appreciated.

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