

# Optimal Estimation with Extended Battery Life in Wireless Sensor Networks

Liu Yang<sup>1,2,3</sup>, Hongbin Zhu<sup>1,2,3</sup>, Haifeng Wang<sup>1,2</sup>, Kai Kang<sup>4</sup>, and Hua Qian<sup>3,4</sup>

<sup>1</sup> Shanghai Institute of Microsystem and Information Technology, CAS

<sup>2</sup> Shanghai Research Center for Wireless Communications

<sup>3</sup> School of Information Science and Technology, ShanghaiTech University

<sup>4</sup> Shanghai Advanced Research Institute, CAS

**Abstract**—Energy constraint is always a bottleneck in a distributed wireless sensor network (WSN). Online censoring is an effective approach to reduce the overall power consumption by only transmitting statistical informative data. However, individual sensor may still suffer from energy shortage due to frequent transmission of informative data or geographical long distance transmission. In this paper, we consider the parameters estimation problem in WSNs, where the goal is to minimize the estimation error while keeping the network lifetime long. A distributed censoring algorithm is developed, which allows sensor nodes to make autonomous decisions on whether to transmit the sampled data. We show that with the proposed algorithm, the network lifetime extends and approaches to its theoretical limit, and the performance loss in terms of the estimation error is minimal. Simulation results validate its effectiveness.

**Index Terms**—Wireless sensor networks, censoring, network lifetime.

## I. INTRODUCTION

Emerging technologies, such as the Internet of Things, smart appliances, smart grids, and machine-to-machine networks stimulate the deployment of autonomous, self-configuring, large-scale wireless sensor networks (WSNs). Since in many applications, sensor nodes are operated based on battery power. Efficient energy utilization is crucial in order to maintain a fully functional network [1].

There are considerable efforts in the literature dealing with the power consumption reduction and network lifetime extension in WSNs. In [2] [3], authors developed a routing algorithm to find the optimal route from the sensor node to the sink node from the energy perspective. Authors in [4] formulated the network lifetime maximization as a convex optimization problem encompassing routing, scheduling, as well as the transmission rate. Offloading data to other sensors was considered in [5] and [6] to prolong the network lifetime. Network coding has been shown to be able to enhance the energy efficiency, hence improving the network lifetime [7].

The above works take all sampled data into consideration, while reducing the data transmission can be an effective way to reduce the energy consumption [8]. Censoring was recently employed to select only the informative data for

transmission in energy-constrained WSNs. Authors in [9] and [10] confirmed that estimation accuracy of censored data could be comparable to that based on uncensored data. In [11] and [12], authors investigated the distributed measurement censoring method for estimation in WSNs, where each sensor node could make censoring decision individually, but such algorithm ignored the energy cost associated with censoring. In our recent work [13], we considered the adaptive censoring from the energy perspective. With our censoring algorithm, the overall energy consumption of WSNs could be reduced, while the performance loss was negligible. However, this algorithm did not consider the energy status of each individual sensor node, hence could not fully optimize the network lifetime of WSNs.

In this paper, we extend our work to develop the optimal estimation algorithm while keeping the network lifetime long. In this work, we consider the parameters estimation problem in WSNs consisting of a fusion center (FC) and a set of sensor nodes. The sensor nodes are battery powered, thus suffering from the energy constraint. We propose a distributed censoring method, which takes the remaining energy of each sensor node into account. We show that with the proposed algorithm, the network lifetime extends and approaches to the theoretical limit, and the estimation error is minimal.

The rest of this paper is organized as follows. Section II introduces the system model and problem formulation. The proposed censoring approach with network lifetime constraint is presented in Section III. Section IV shows the simulation results, and Section V concludes the paper.

*Notations:* Lower-(upper-) case boldface letters denote column vectors (matrices). Calligraphic symbols are reserved for sets and vector  $\mathbf{I}$  denotes the identity matrix.  $\phi(t) = (1/\sqrt{2\pi}) \exp(-t^2/2)$  denotes the standardized Gaussian probability density function and  $Q(z) = \int_z^{+\infty} \phi(t) dt$  denotes the associated complementary cumulative distribution function.

## II. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a WSN with  $K$  sensor nodes  $\{\mathcal{S}_k\}_{k=1}^K$  randomly deployed over a geographical area. A scalar measurement of sensor node  $k$  at time slot  $t$  is assumed to obey the linear model [12]

$$y_{tk}^* = \mathbf{h}_{tk}^T \boldsymbol{\theta} + v_{tk}, \quad (1)$$

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where  $k = 1, 2, \dots, K$ ,  $t = 1, 2, \dots, T$ ,  $\theta \in \mathbb{R}^p$  is the vector of unknown parameters, and  $p$  is the length of  $\theta$ . The regressor  $\mathbf{h}_{tk}$  is known at the FC, and  $v_{tk}$  denotes uncorrelated, zero-mean, Gaussian distributed noise. Without loss of generality, we assume that the noise variance is  $\sigma^2$  for all  $K$  sensors in each time slot  $t$ .

Network lifetime is a critical metric in the design of energy-constrained WSNs. We assume that the sensor nodes are battery powered thus suffer from the energy constraint, while the FC has its own power supply and is free of energy limitation. The lifetime  $T_k$  of node  $k$  is defined as the number of time slot until the energy of node  $k$  is exhausted. We define the lifetime  $T$  of the network as the time when the first sensor is drained of its energy, in other words, the system lifetime  $T$  of a sensor network is the minimum lifetime of all nodes in the network [14]

$$T = \min \{T_1, T_2, \dots, T_K\}, \quad (2)$$

Data censoring can be applied to reduce the number of observations adopted for estimation, hence prolong the network lifetime. With  $\mathcal{R}$  denoting the censoring interval, a generic censoring rule to select data in (1) is given by

$$y_{tk} = \begin{cases} \star, & \text{if } y_{tk}^* \in \mathcal{R}, \\ y_{tk}^*, & \text{otherwise.} \end{cases} \quad (3)$$

where  $\star$  denotes an unspecified value. If  $y_{tk}^* \in \mathcal{R}$ , the value of  $y_{tk}^*$  is censored and we only know that  $y_{tk} \in \mathcal{R}$ ; otherwise, the exact measurement  $y_{tk} = y_{tk}^*$  is obtained [10].

Fig. 1 illustrates the data censoring in WSNs. During each time slot, instead of transmitting all observations, only a subset of sensor nodes is selected for transmission.

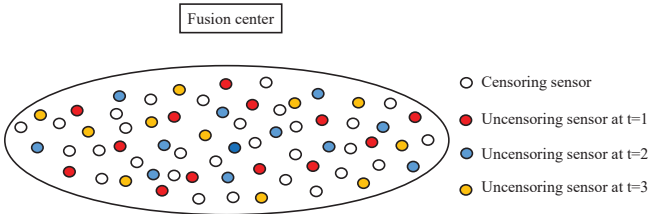


Fig. 1. Data censoring in WSNs.

The rule of selecting sensor nodes is to minimize the estimation error over all possible selections. We assume that  $\mathbf{h}_{tk}$ ,  $\sigma^2$  are available at the FC, since such information can be learnt from the nature of the problem or acquired during a training phase [13].

In WSNs, the energy consumed by the transceiver and the signal processing unit can be considered as a constant. The energy dissipated is approximately  $\epsilon_{\text{elec}} = 400$  nJ/byte to run the transmitter or receiver circuitry. The energy consumption by the power amplifier during the transmission, on the other hand, greatly depends on the Euclidean distance  $d_k$  between the sensor node  $k$  and the FC. A simplified model of energy consumption per byte of the power amplifier is  $\epsilon_{\text{amp}} d_k^2$ , where  $\epsilon_{\text{amp}} = 800$  pJ/byte/m<sup>2</sup> [15]. As the data received and

transmitted are usually short messages, we assume that the data length of the packet is  $m$  bytes for both transmitter and receiver. Thus, the total transmitting energy consumption would be  $E_{mk} = m\epsilon_{\text{elec}} + m\epsilon_{\text{amp}}d_k^2$ , the total receiving energy consumption is  $E_{rk} = m\epsilon_{\text{elec}}$  [14]. To simplify notations, we normalize the power consumption in each reception and transmission by the maximum transmission power, i.e.,

$$E_{mk} = \frac{E_{mk}}{\max(E_{m1}, E_{m2}, \dots, E_{mK})}, \quad (4a)$$

$$E_{rk} = \frac{E_{rk}}{\max(E_{m1}, E_{m2}, \dots, E_{mK})}. \quad (4b)$$

During each time slot  $t = 1, \dots, T$ , each sensor node  $k$  must make a decision  $s_{tk}$  about whether to transmit its current measurement. We set  $s_{tk} = 1$  if the observation is transmitted, while  $s_{tk} = 0$  if discarded. The selection variable  $s_{tk}$  can be obtained by solving the following optimization problem

$$s_{tk} = \arg \min_{s_{tk} \in \{0,1\}} \sum_{t=1}^T \sum_{k=1}^K (y_{tk}^* - s_{tk} \mathbf{h}_{tk}^T \theta)^2 \quad (5a)$$

$$s.t. \quad T E_{rk} + \sum_{t=1}^T s_{tk} E_{mk} \leq E_k, \quad \text{for } k = 1, \dots, K \quad (5b)$$

where  $E_k$  is the total energy of sensor node  $k$ . The goal of optimization problem (5) is to minimize the estimation error under the network lifetime constraint. In the next section, we propose a distributed censoring algorithm for (5).

### III. PROPOSED DISTRIBUTED CENSORING ALGORITHM WITH NETWORK LIFETIME CONSTRAINT

During each time slot  $t$ , all sensors receive  $\hat{\theta}_{t-1}$  broadcast by FC, where  $\hat{\theta}_{t-1}$  denotes the estimation of  $\theta$  after time slot  $t-1$ , which is initialized as

$$\bar{\theta} = \left( \sum_{k=1}^L \mathbf{h}_{tk} \mathbf{h}_{tk}^T \right)^{-1} \sum_{k=1}^L y_{tk}^* \mathbf{h}_{tk}, \quad (6)$$

where  $p < L \ll K$  [9].

To deal with the optimization problem (5), the remaining energy of each sensor node at each time slot should be considered. Sensors deplete their batteries according to the actions  $s_{tk}$  [16]. The available energy  $E(t, k)$  of sensor node  $k$  at time slot  $t$  can be expressed recursively as [8]

$$E(t, k) = E(t-1, k) - s_{tk} E_{mk} - E_{rk}, \quad (7)$$

where  $E(t, k)$  is initialized as  $E(0, k) = E_k$ ,  $E_{rk}$  denotes the energy consumed by receiving  $\hat{\theta}_{t-1}$ , and  $E_{mk}$  denotes energy consumed by transmitting the measurement  $y_{tk}^*$ .

In order to achieve the required network life  $T$ , the probability of transmission  $\rho_{tk}$  for sensor node  $k$  at time slot  $t$  is given by

$$\rho_{tk} = \left\lfloor \frac{E(t, k) - (T-t) E_{rk}}{E_{mk}} \right\rfloor, \quad (8)$$

where  $\lfloor \cdot \rfloor$  denotes the round down operation, and

$$D_k = \begin{cases} t, & \text{if } s_{tk} = 1, \\ D_k, & \text{otherwise.} \end{cases} \quad (9)$$

where  $D_k$  is initialized to 0 for each sensor node. Clearly,  $\rho_{tk}$  changes only if  $s_{tk} = 1$ .

In order to achieve desired transmission probability  $\rho_{tk}$ , the censoring threshold should be updated accordingly [9]

$$\tau_{tk} = \left[ \frac{p}{(n-1)\rho_{tk}} + 1 \right]^{1/2} Q^{-1} \left( \frac{\rho_{tk}}{2} \right). \quad (10)$$

To simplify calculation, the threshold  $\tau_{tk} = Q^{-1}(\rho_{tk}/2)$  can be used as a rough approximation to achieve the censoring ratio. Then

$$\tau_{tk} = \begin{cases} 0, & \text{if } \rho_{tk} \geq 1, \\ Q^{-1}(\rho_{tk}/2), & \text{if } 0 < \rho_{tk} < 1, \\ \infty, & \text{if } \rho_{tk} \leq 0, \end{cases} \quad (11)$$

where  $\rho_{tk} \geq 1$  means that the remaining energy of sensor node  $k$  is enough to support the transmission in every remaining time slots, and  $\rho_{tk} \leq 0$  indicates that sensor node  $k$  can not complete one transmission with the reserved energy.

Supposing that the threshold  $\tau_{tk}$  and estimation  $\hat{\theta}_{t-1}$  are available at each sensor, censoring can be implemented autonomously at each sensor with the following rule [9]

$$(y_{tk}, s_{tk}) = \begin{cases} (y_{tk}^*, 1), & \text{if } \left| \frac{y_{tk}^* - \mathbf{h}_{tk}^T \hat{\theta}_{t-1}}{\sigma} \right| \geq \tau_{tk}, \\ (*, 0), & \text{otherwise.} \end{cases} \quad (12)$$

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#### Algorithm 1 Censoring with Network Lifetime Constraint

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**Require:** FC knows  $\{\mathbf{h}_{tk}\}_{k=1}^K$ ;  $\mathcal{S}_k$  knows  $\mathbf{h}_{tk}$ ,  $y_{tk}^*$ ,  $E_k$ ,  $T$

- 1: initialize  $n = 1$ ,  $\theta_0 = \theta$ ,  $\mathbf{C}_0 = \epsilon \mathbf{I}$
  - 2: **for**  $t = 1, 2, \dots, T$  **do**
  - 3:   FC: Broadcasts  $\hat{\theta}_{t-1}$
  - 4:   **for**  $k = 1, 2, \dots, K$  **do**
  - 5:      $\mathcal{S}_k$ : Receives  $\hat{\theta}_{t-1}$ , obtains  $\tau_{tk}$ ,  $y_{tk}$  and  $s_{tk}$  from
  - 6:     (11), (12), updates  $E(t, k)$
  - 7:     **if**  $E(t, k) < 0$  **then**
  - 8:       **return**
  - 9:     **end if**
  - 10:   **end for**
  - 11:   **for**  $k = 1, 2, \dots, K$  **do**
  - 12:     FC: Updates  $\mathbf{C}_n$  and  $\hat{\theta}_n$  using (13),  $n \leftarrow n + 1$
  - 13:   **end for**
  - 14:   FC:  $\hat{\theta}_t \leftarrow \hat{\theta}_n$
  - 15: **end for**
  - 16: FC: Set  $\hat{\theta} = \hat{\theta}_t$
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Applying recursive least squares (RLS) algorithm in FC with the censoring rule, yields

$$\mathbf{C}_n = \frac{n}{n-1} \left[ \mathbf{C}_{n-1} - \frac{s_{tk} \mathbf{C}_{n-1} \mathbf{h}_{tk} \mathbf{h}_{tk}^T \mathbf{C}_{n-1}}{n-1 + \mathbf{h}_{tk}^T \mathbf{C}_{n-1} \mathbf{h}_{tk}} \right], \quad (13a)$$

$$\hat{\theta}_n = \hat{\theta}_{n-1} + \frac{s_{tk}}{n} \mathbf{C}_n \mathbf{h}_{tk} (y_{tk} - \mathbf{h}_{tk}^T \hat{\theta}_{n-1}), \quad (13b)$$

where  $\mathbf{C}_n$  is typically initialized as  $\mathbf{C}_0 = \epsilon \mathbf{I}$  for small positive  $\epsilon$  [10]. Note that  $\hat{\theta}_n$  is the estimation result of each iteration, while  $\hat{\theta}_t$  is the estimation result of each time slot.

In the proposed algorithm, the remaining energy of each sensor node is considered in the data censoring process. The censoring threshold  $\tau_{tk}$  increases as the reserved energy  $E(t, k)$  decreases, which means that the transmission probability  $\rho_{tk}$  becomes smaller and only more informative data may be transmitted. The lifetime of each sensor node extends, so does the overall network lifetime.

The computation and communication steps that constitute censoring are tabulated as Algorithm 1. During each time slot  $t$ , FC broadcasts  $\hat{\theta}_{t-1}$  to all  $K$  sensors. Each sensor  $\mathcal{S}_k$  autonomously decides its threshold  $\tau_{tk}$ , and makes the censoring independently according to (12). The available energy of each sensor node is updated at each time slot. Parameters estimation is performed in FC with RLS algorithm.

#### IV. NUMERICAL RESULTS

In this section, we examine the proposed distributed censoring method with network lifetime constraint. Simulations are done for the model in (1) with  $K = 100$  and SNR=30 dB. The regressors  $\mathbf{h}_{tk}$  and parameters vector  $\theta$  are picked uniformly over  $[-1, 1]$  with dimension  $p = 10$ . During all the simulations in this paper, there are  $K$  sensor nodes randomly deployed in  $1 \text{ km} \times 1 \text{ km}$  area, the FC is located at (0.5 km, 0.5 km), and the energy of each sensor node is initialized as  $E_k = 100$ .

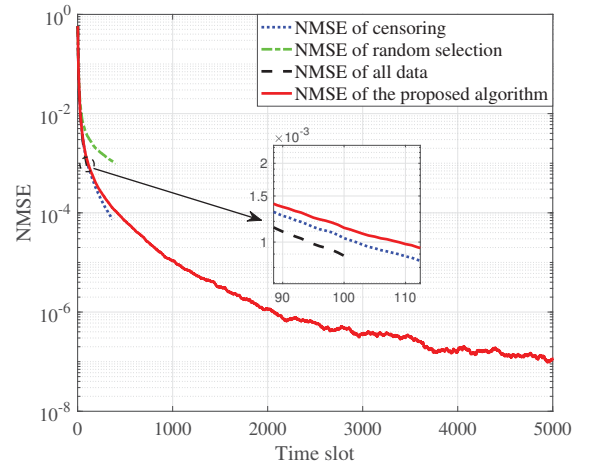


Fig. 2. NMSE of the proposed algorithm compared with different algorithms.

In the first experiment, we would like to check the algorithm performance in terms of modeling accuracy, which is defined as normalized mean-square error ( $\text{NMSE} = \|\hat{\theta}_t - \theta\|^2 / \|\theta\|^2$ ). The NMSE performance of the proposed algorithm compared with different algorithms is shown in Fig. 2, where  $T = 5000$ , the x-axis and y-axis represent time slot and NMSE, respectively; the green dashed line shows the NMSE of randomly selected method; the red solid line reveals the performance of the proposed algorithm; the blue dashed line indicates the

performance of censoring method, which selects the most informative part of all measurements at each time slot; and the black dashed line represents the NMSE performance of all data method.

As shown in Fig. 2, the NMSE of all data method benchmarks the performance of other methods but the network lifetime is only 100, because each sensor node must transmit its measurement at each time slot. The performance of randomly selected method is not satisfactory in both estimation error and expected network lifetime, since the method does not consider both the estimation performance and energy. The proposed method performs slightly worse than censoring method at the beginning, but realizes the highest estimation accuracy in the end, and achieves the network lifetime constraint  $T = 5000$ .

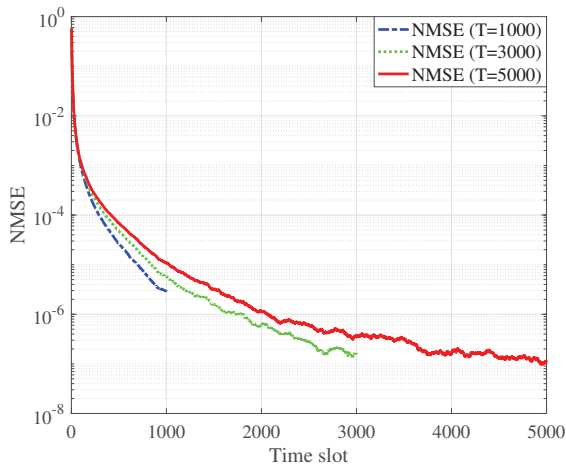


Fig. 3. NMSE of the proposed algorithm with different  $T$ s.

Next, we study the NMSE performances of the proposed algorithm with different expected lifetimes. As shown in Fig. 3, the blue dashed line, the green dashed line and the red solid line indicate the convergence of the proposed algorithm with  $T = 1000$ ,  $T = 3000$  and  $T = 5000$ , respectively. We observe from Fig. 3 that the larger  $T$  is, the slower convergence speed achieves. The result is expected as larger  $T$  suggests that smaller number of sensors is selected in each round. On the other hand, longer battery life is achieved with larger  $T$ . The estimation error of the blue curve is large comparing to the other two cases as the estimation has not converged yet when the network dies.

In Fig. 4, we show the remaining energy of each sensor node when the target network life is achieved, where the x-axis and y-axis represent the time slot and the remaining energy, respectively; the blue mark “+”, the green mark “o” and the red mark “x” represent the remaining energy of each sensor node with  $T = 1000$ ,  $T = 3000$  and  $T = 5000$ , respectively. From Fig. 4, we observe that the remaining energy is nearly 0 for most sensor nodes in all cases. The larger  $T$  is, the more nodes nearly deplete their energy. The results indicate that the proposed algorithm indeed can extend the network life.

In the third simulation, we would like to verify the conver-

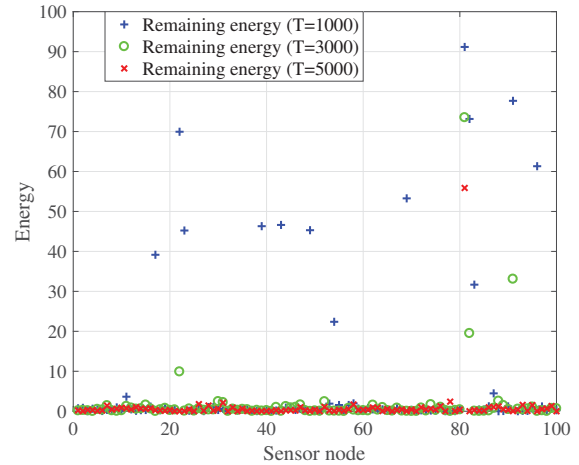


Fig. 4. Remaining energy of the proposed algorithm with different  $T$ s.

gence performance of the proposed algorithm with another distribution, where  $h_{tk}$  is a standard Gaussian random variable. The legends of Fig. 5 are the same as Fig. 3's. We observe that the proposed method achieves the theoretical network lifetime, and has similar convergence performance compared with Fig. 3. Therefore, the proposed algorithm is robust and can be applied to other distributions.

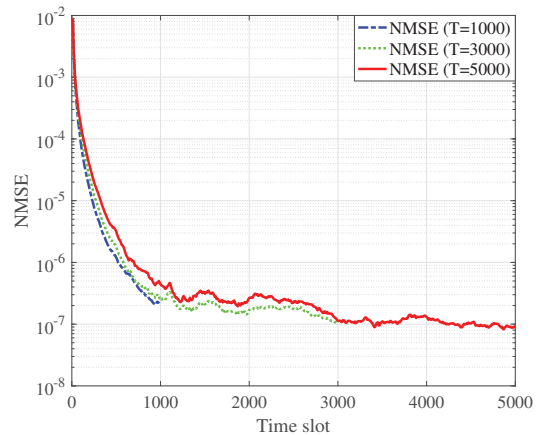


Fig. 5. NMSE of the proposed algorithm with standard Gaussian distribution.

### V. CONCLUSIONS

In this paper, a distributed censoring method with network lifetime constraint for parameters estimation in WSNs is explored. The remaining energy is considered in the data censoring process, where the transmission probability becomes smaller and only more informative data may be transmitted as the reserved energy of each sensor node decreases. The proposed algorithm extends the network lifetime with little estimation performance loss. Simulations show that the proposed algorithm achieves a good trade-off between estimation accuracy and network lifetime.

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