Performance Bounds Analysis for Semi-Blind Channel Estimation with Pilot Contamination in Massive MIMO-OFDM Systems

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***Abstract*—Pilot contamination, in a massive Multiple-Input Multiple-Output (MIMO) system, is an undeniable challenging issue severely affecting the performance of the system by including channel estimation errors. The aim of this paper is to investigate the effectiveness of semi-blind channel estimation approaches in massive MIMO-OFDM (Orthogonal-Frequency Division-Multiplexing) systems. For an estimator-independent study, the performance analysis is carried out using the Cramér Rao Bound (CRB) derivation for pilot-based and semi-blind channel estimation strategies. This analysis demonstrates in particular that: (i) when considering the finite alphabet nature of communication signals, it is possible to efficiently solve the pilot contamination problem with semi-blind channel estimation approach; and (ii) the Second Order Statistics (SOS) only are not sufficient to address the full channel identifiability even if the semi-blind approach is considered.**

1. Introduction

Massive Multiple-Input Multiple-Output (MIMO) is a promising technology for the next generation cellular networks [1]. With a higher number of Base Station (BS) antennas (beyond 100 antennas), compared to the classical MIMO systems, massive MIMO technology has proven its ability to improve the spectral and power efficiency [2]. So that, both throughput and reliability will be highly enhanced for the future cellular networks.

In order to fully exploit all of the potentials offered by a massive MIMO system, accurate Channel State Information (CSI) is necessary. It is obtained only during the uplink transmission, thanks to the channel reciprocity property and according to the widely accepted Time Division Duplexing (TDD) protocol. The traditional methods to get the CSI rely on the pilot-based channel estimation (see e.g. [1]). However, due to the non-orthogonality of the pilot sequences, these methods are severely affected by what is called *pilot contamination* [3]. It is one of the major issues of massive MIMO systems that must be addressed because its effect cannot be reduced by increasing the number of BS antennas. Many pilot contamination mitigation strategies have been proposed: creating more orthogonal pilots by slicing the time and frequency resources [4], subspace projection-based interference suppressing [5], data-aided channel estimation [6], or by designing appropriate inter-cell communication protocols and resource allocation. In recent works (e.g. [7], [8]), a particular attention has been drawn to blind and semi-blind methods. The former is fully based on the statistical properties of the transmitted data, whereas the latter depends on the joint use of pilots and data.

The focus of this paper falls into the scope of performance analysis of semi-blind channel estimation with pilot contamination in the context of multi-cell massive MIMO-OFDM systems. For an estimator-independent performance analysis, the Cramér Rao Bound (CRB) is derived for both pilot-based and semi-blind channel estimation. Note that a thorough study has been conducted in [9] where the achievable performance of semi-blind approaches compared to pilot-based ones has been quantified for channel estimation in a MIMO-OFDM system. This study will be extended to a massive MIMO-OFDM system, by taking into account the multi-cell context and the phenomenon of pilot contamination.

The rest of this paper is organized as follows. Section 2 describes the massive MIMO-OFDM system model. Section 3 illustrates the pilot contamination effect. The derivation of the CRB for pilot-based and semi-blind approaches are detailed in section 4. Simulation results are discussed in section 5. Finally, Section 6 concludes the work.

2. Massive MIMO-OFDM system model

This section presents the massive MIMO-OFDM wireless system adopted in this paper. An uplink transmission is considered. The system is composed of N_c cells where each cell contains one BS with N_r antennas and N_t users using each a single antenna. The received signals from the adjacent cells are initially ignored. Therefore the received signal at the r-th BS antenna of the l-th cell, assumed to be a K sub-carriers OFDM signal, is given by [9]:

$$\mathbf{y}_{l,r} = \sum_{i=1}^{N_t} \mathbf{F}\,\boldsymbol{\mathcal{T}}(\mathbf{h}_{l,i,r})\frac{\mathbf{F}^H}{K}\mathbf{x}_{l,i} + \mathbf{v}_{l,r}, \qquad (1)$$

where the superscript $(.)^H$ denotes the Hermitian operator; K is the OFDM symbol length; $\mathbf{F}$ represents a K-point

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Fourier transform matrix; $\mathbf{h}_{l,i,r}$ is a $N \times 1$ vector representing the channel taps between the i-th user and the r-th receive antenna of the l-th cell; $\boldsymbol{\mathcal{T}}(\mathbf{h}_{l,i,r})$ is a circulant matrix; $\mathbf{x}_{l,i}$ is the i-th user OFDM symbol. $\mathbf{v}_{l,r}$ is assumed to be an additive white Circulant Gaussian (CG) noise so that $E[\mathbf{v}_{l,r}(k)\mathbf{v}_{l,r}(i)^H] = \sigma_{v_l}^2 \mathbf{I}_K \delta_{ki}$ where $\sigma_{v_l}^2$ represents the noise variance at the l-th cell; $\mathbf{I}_K$ is a $K \times K$ identity matrix and δ_{ki} being the Dirac operator.

Due to the eigenvalue decomposition of the circulant matrix $\boldsymbol{\mathcal{T}}(\mathbf{h}_{l,i,r})$, the received signal, of dimension $N_r K \times 1$, at the BS is then expressed as follows:

$$\mathbf{y}_l = \boldsymbol{\lambda}_l \mathbf{x}_l + \mathbf{v}_l, \tag{2}$$

where $\mathbf{y}_l = [\mathbf{y}_{l,1}^T ... \mathbf{y}_{l,N_r}^T]^T$; $\mathbf{x}_l = [\mathbf{x}_{l,1}^T ... \mathbf{x}_{l,N_t}^T]^T$; $\mathbf{v}_l = [\mathbf{v}_{l,1}^T ... \mathbf{v}_{l,N_r}^T]^T$; $\boldsymbol{\lambda}_l = [\boldsymbol{\lambda}_{l,1} ... \boldsymbol{\lambda}_{l,N_t}]$ with $\boldsymbol{\lambda}_{l,i} = [\boldsymbol{\lambda}_{l,i,1} ... \boldsymbol{\lambda}_{l,i,N_r}]^T$ where $\boldsymbol{\lambda}_{l,i,r} = diag\{\mathbf{W}\mathbf{h}_{l,i,r}\}$ and $\mathbf{W}$ is the sub-matrix of $\mathbf{F}$ formed by its first N columns.

In order to facilitate the derivation of the CRB, equation (2) is rewritten as follows:

$$\mathbf{y}_l = \tilde{\mathbf{X}}_l \mathbf{h}_l + \mathbf{v}_l, \tag{3}$$

where $\mathbf{h}_l = [\mathbf{h}_{l,1,1}^T ... \mathbf{h}_{l,N_t,1}^T ...... \mathbf{h}_{l,1,N_r}^T ... \mathbf{h}_{l,N_t,N_r}^T]^T$ is a $N_r N_t N \times 1$ vector; $\tilde{\mathbf{X}}_l = \mathbf{I}_{N_r} \otimes \mathbf{X}_l$ is a matrix of size $N_r K \times N_r N_t N$ where $\mathbf{X}_l = [\mathbf{X}_{l,D_1}\mathbf{W} ... \mathbf{X}_{l,D_{N_t}}\mathbf{W}]$ of size $K \times N_t N$, and $\mathbf{X}_{l,D_i}$ is a $K \times K$ diagonal matrix containing the i-th user symbols; and $\otimes$ refers to the Kronecker product.

Now, let's take into account the effect of the neighboring cells on the first cell, considered as the interest cell. With the assumption of perfect synchronization between the N_c cells, equation (2) becomes:

$$\mathbf{y}_1 = \sum_{l=1}^{N_c} \boldsymbol{\lambda}_l \mathbf{x}_l + \mathbf{v}_1 = \boldsymbol{\lambda}_{tot} \mathbf{x}_{tot} + \mathbf{v}_1, \tag{4}$$

where $\boldsymbol{\lambda}_{tot} = [\boldsymbol{\lambda}_1 \dots \boldsymbol{\lambda}_{N_c}]$ and $\mathbf{x}_{tot} = [\mathbf{x}_1^T \dots \mathbf{x}_{Nc}^T]^T$. Similarly, equation (3) becomes:

$$\mathbf{y}_1 = \sum_{l=1}^{N_c} \tilde{\mathbf{X}}_l \mathbf{h}_l + \mathbf{v}_1 = \tilde{\mathbf{X}}_{tot} \mathbf{h}_{tot} + \mathbf{v}_1, \tag{5}$$

where $\tilde{\mathbf{X}}_{tot} = [\tilde{\mathbf{X}}_1 \dots \tilde{\mathbf{X}}_{N_c}]$ and $\mathbf{h}_{tot} = [\mathbf{h}_1^T \dots \mathbf{h}_{N_c}^T]^T$.

3. Uplink pilot contamination effect

This section discusses the impact of the pilot contamination in a massive MIMO-OFDM system. In the uplink data transmission, the BS of a given cell belonging to the system has to learn the transmission channel. The conventional way, adopted in a MIMO scheme, consists of exploiting the known symbols (i.e. pilots) at both ends of the link (users and the BS). To adopt this strategy in a massive MIMO-OFDM system, the pilots used within the same cell and in the neighboring cells should be mutually orthogonal. However, the channel time coherence limits the total number of orthogonal pilots leading to the reuse of the same pilots in many neighboring cells. The worst case occurs when the same set of pilots is exploited in all N_c adjacent cells. In this situation, equation (5) becomes:

$$\mathbf{y}_1 = \sum_{l=1}^{N_c} \tilde{\mathbf{X}}_{1_P} \mathbf{h}_l + \mathbf{v}_1 = \tilde{\mathbf{X}}_{1_P} \sum_{l=1}^{N_c} \mathbf{h}_l + \mathbf{v}_1, \tag{6}$$

where $\tilde{\mathbf{X}}_{1_P}$ stands for the pilots set of the first cell.

To illustrate the pilot contamination effect, let's consider the Least Squares (LS) estimate of the first cell channel vector (i.e. $\mathbf{h}_1$) given by:

$$\hat{\mathbf{h}}_1^{LS} = \tilde{\mathbf{X}}_{1_P}^{\#} \mathbf{y}_1 = \mathbf{h}_1 + \sum_{l=1, l \neq 1}^{N_c} \mathbf{h}_l + \tilde{\mathbf{X}}_{1_P}^{\#} \mathbf{v}_1, \tag{7}$$

with $\tilde{\mathbf{X}}_{1_P}^{\#} = (\tilde{\mathbf{X}}_{1_P}^H \tilde{\mathbf{X}}_{1_P})^{-1} \tilde{\mathbf{X}}_{1_P}^H$, the pseudo inverse of $\tilde{\mathbf{X}}_{1_P}$.

This equation clearly shows that the estimate $\hat{\mathbf{h}}_1^{LS}$ is affected by an additional bias term corresponding to a linear combination of channel components of the users sharing the same pilot sequences in different cells. Indeed this phenomenon, referred to as pilot contamination, severely degrades the channel estimation performance. A solution to this problem would be the use of semi-blind channel estimation approach. In the sequel, the performance and the potential of this approach are analyzed and discussed through the CRB derivation.

4. Cramér Rao Bound derivation

This section derives the CRB for pilot-based and semi-blind channel estimation. First start by recalling some definitions.

The CRB is defined as the inverse of the Fisher Information Matrix (FIM) [10]. The latter is denoted $\mathbf{J}_{\boldsymbol{\theta\theta}}$ where $\boldsymbol{\theta}$ is the unknown parameter vector to be estimated. Using the complex representation, the parameter $\boldsymbol{\theta}$ is defined as follows:

$$\boldsymbol{\theta} = [\mathbf{h}_{tot}^T \ \ (\mathbf{h}_{tot}^*)^T]^T. \tag{8}$$

The FIM, taking into account the pilots and data, is then expressed as follows:

$$\mathbf{J}_{\boldsymbol{\theta\theta}} = \mathbf{J}_{\boldsymbol{\theta\theta}}^p + \mathbf{J}_{\boldsymbol{\theta\theta}}^d, \tag{9}$$

where $\mathbf{J}_{\boldsymbol{\theta\theta}}^p$ is the FIM associated to the known pilots and $\mathbf{J}_{\boldsymbol{\theta\theta}}^d$ is related to the unknown data. For simplicity, the signal and noise powers are assumed to be known.

4.1. CRB for pilot-based channel estimation

As in [9], a block-type pilot arrangement is considered. The noise components are assumed to be independent identically distributed (i.i.d.). The pilot-based FIM is given by:

$$\mathbf{J}_{\boldsymbol{\theta\theta}}^p = \sum_{i=1}^{N_p} \mathbf{J}_{\boldsymbol{\theta\theta}}^{p_i}, \tag{10}$$

with N_p being the number of pilots in each cell, and $\mathbf{J}_{\boldsymbol{\theta\theta}}^{p_i}$ the FIM associated to one pilot symbol.

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Consider now the massive MIMO-OFDM system with N_c cells, the pilot-based FIM associated to the channel vector $\mathbf{h}_{tot}$ is then expressed as follows:

$$\mathbf{J}^{p_i}_{\boldsymbol{h}_{tot}\boldsymbol{h}_{tot}} = \frac{\tilde{\mathbf{X}}^H_{tot}\tilde{\mathbf{X}}_{tot}}{\sigma^2_{v_1}}, \tag{11}$$

which can also be written in a matrix form:

$$\mathbf{J}^{p_i}_{\boldsymbol{h}_{tot}\boldsymbol{h}_{tot}} = \frac{1}{\sigma^2_{v_1}} \begin{bmatrix} \tilde{\mathbf{X}}^H_{1_P}\tilde{\mathbf{X}}_{1_P} & \dots & \tilde{\mathbf{X}}^H_{1_P}\tilde{\mathbf{X}}_{N_{cP}} \\ \vdots & \ddots & \vdots \\ \tilde{\mathbf{X}}^H_{N_{cP}}\tilde{\mathbf{X}}_{1_P} & \dots & \tilde{\mathbf{X}}^H_{N_{cP}}\tilde{\mathbf{X}}_{N_{cP}} \end{bmatrix}. \tag{12}$$

If the pilots of the cells are mutually orthogonal, i.e. $\tilde{\mathbf{X}}^H_{i_P}\tilde{\mathbf{X}}_{j_P} = \mathbf{0} \ \forall \ i \neq j$, then the FIM becomes a bloc diagonal matrix. However if the cells share the same set of pilots, i.e. the worst case of pilot contamination, the FIM is then equivalent to:

$$\mathbf{J}^{p_i}_{\boldsymbol{h}_{tot}\boldsymbol{h}_{tot}} = \frac{1}{\sigma^2_{v_1}} \begin{bmatrix} \tilde{\mathbf{X}}^H_{1_P}\tilde{\mathbf{X}}_{1_P} & \dots & \tilde{\mathbf{X}}^H_{1_P}\tilde{\mathbf{X}}_{1_P} \\ \vdots & \ddots & \\ \tilde{\mathbf{X}}^H_{1_P}\tilde{\mathbf{X}}_{1_P} & \dots & \tilde{\mathbf{X}}^H_{1_P}\tilde{\mathbf{X}}_{1_P} \end{bmatrix}. \tag{13}$$

To deduce the CRB, the FIM has to be inverted. However, according to this last equation, $\mathbf{J}^{p_i}_{\boldsymbol{h}_{tot}\boldsymbol{h}_{tot}}$ is not a full rank matrix. In fact, it can be shown that the kernel of this FIM is of dimension $(N_c-1)N_tN_rN$, corresponding to the number of indeterminacies we need to get rid off. Moreover, equation (7) does not allow the extraction of $\mathbf{h}_1$ from the sum of the channel vectors of adjacent cells. In other words, this translates the *non-identifiability* of the channel vector of the interest cell when pilot contamination occurs.

4.2. CRB for Semi-Blind channel estimation

This section derives the CRB for the semi-blind channel estimation of a multi-cell massive MIMO-OFDM system with pilot contamination. Both known pilots and unknown data are taken into account in the derivation of the FIM as given in (9). The pilot arrangement is assumed to be a block-type one. Two data models, Circular Gaussian and BPSK, are considered in what follows.

4.2.1. Circular Gaussian data model. This section considers only the Second Order Statistics (SOS) corresponding to the Gaussian CRB. The unknown OFDM data, in each cell, is assumed to be i.i.d. stochastic Circular Gaussian symbols with zero mean and a covariance matrix composed of the users' transmit powers i.e. $\mathbf{C_{x_l}} = diag(\sigma^2_{x_{l,i}})$ with $l = 1...N_c$ and $i = 1...N_t$.

According to equation (4), the received OFDM signal (i.e. $\mathbf{y}_1$) is Circular Gaussian with zero mean and a covariance matrix given by:

$$\mathbf{C_{y_1}} = \sum_{l=1}^{N_c}\sum_{i=1}^{N_t} \sigma^2_{x_{l,i}} \boldsymbol{\lambda}_{l,i}\boldsymbol{\lambda}^H_{l,i} + \sigma^2_{v_1}\mathbf{I}_{KN_r}. \tag{14}$$

The global data-based FIM can be expressed as follows:

$$\mathbf{J}^d_{\boldsymbol{h}_{tot}\boldsymbol{h}_{tot}} = \begin{bmatrix} \mathbf{J}^d_{\boldsymbol{h}_{tot}\boldsymbol{h}_{tot}} & \mathbf{J}^d_{\boldsymbol{h}_{tot}\boldsymbol{h}^*_{tot}} \\ \mathbf{J}^d_{\boldsymbol{h}^*_{tot}\boldsymbol{h}_{tot}} & \mathbf{J}^d_{\boldsymbol{h}^*_{tot}\boldsymbol{h}^*_{tot}} \end{bmatrix}, \tag{15}$$

where $\mathbf{J}^d_{\boldsymbol{h}_{tot}\boldsymbol{h}_{tot}}$ is a $(N_cN_rN_tN)\times(N_cN_rN_tN)$ matrix with elements $J^d_{h_ih_j}$ given by:

$$\mathbf{J}^d_{h_ih_j} = tr\left\{\mathbf{C}^{-1}_{\mathbf{y_1}}\frac{\partial\mathbf{C}_{\mathbf{y_1}}}{\partial h^*_i}\mathbf{C}^{-1}_{\mathbf{y_1}}\left(\frac{\partial\mathbf{C}_{\mathbf{y_1}}}{\partial h^*_j}\right)^H\right\}. \tag{16}$$

The derivation of $\mathbf{C}_{\mathbf{y}_1}$ with respect to h_i requires to find the indices $\{i_{N_c}, i_{N_t}, i_{N_r}, i_N\}$ corresponding to the cell, the user, the BS antenna and the channel tap associated to h_i. Based on the results provided in [9], $J^d_{h_ih_j}$ is given by:

$$J^d_{h_ih_j} = (J^d_{h^*_ih^*_j})^H = tr\{\mathbf{C}^{-1}_{\mathbf{y_1}}\sigma^2_{i_{N_c},i_{N_t}}\boldsymbol{\lambda}_{i_{N_c},i_{N_t}}\frac{\partial\boldsymbol{\lambda}^H_{i_{N_c},i_{N_t}}}{\partial h^*_i}\times \mathbf{C}^{-1}_{\mathbf{y_1}}\sigma^2_{j_{N_c},j_{N_t}}\frac{\partial\boldsymbol{\lambda}_{j_{N_c},j_{N_t}}}{\partial h_j}\boldsymbol{\lambda}^H_{j_{N_c},j_{N_t}}\} \tag{17}$$

and

$$J^d_{h_ih^*_j} = (J^d_{h^*_ih_j})^H = tr\{\mathbf{C}^{-1}_{\mathbf{y_1}}\sigma^2_{i_{N_c},i_{N_t}}\boldsymbol{\lambda}_{i_{N_c},i_{N_t}}\frac{\partial\boldsymbol{\lambda}^H_{i_{N_c},i_{N_t}}}{\partial h^*_i}\times \mathbf{C}^{-1}_{\mathbf{y_1}}\sigma^2_{j_{N_c},j_{N_t}}\boldsymbol{\lambda}_{j_{N_c},j_{N_t}}\frac{\partial\boldsymbol{\lambda}^H_{j_{N_c},j_{N_t}}}{\partial h^*_j}\} \tag{18}$$

One can show, that this gaussian FIM is still rank deficient[1] and the SOS-based semi-blind approach allowed only to reduce the number of indeterminacies from $(N_cN_t)^2$, in the blind case, to $((N_c-1)N_t)^2$.

4.2.2. BPSK data model. Consider the finite alphabet nature of the communication signals by using a BPSK data model. The received signal at the k-th sub-carrier is given by [9]:

$$\mathbf{y}_{1(k)} = \boldsymbol{\lambda}_{tot_{(k)}}\mathbf{C}_x{}^{\frac{1}{2}}\mathbf{x}_{(k)} + \mathbf{v}_{1(k)} \text{ for } k = 1, ..., K, \tag{19}$$

where $\boldsymbol{\lambda}_{tot_{(k)}}$ is the k-th FFT component of $\mathbf{h}_{tot}$; $\mathbf{C}_x$ is a block diagonal matrix formed by users' covariance matrices of each cell; $\mathbf{x}_{(k)} = [\mathbf{x}^T_{1,(k)}...\mathbf{x}^T_{N_c,(k)}]^T$ with $\mathbf{x}_{l,(k)} = [x_{l,1,(k)}...x_{l,N_t,(k)}]^T$ so that $x_{l,i,(k)}$ for $k = 1...K$ are i.i.d. symbols taking values ±1 with equal probabilities.

In this case, the likelihood function is a mixture of $2^{N_cN_t}$ Gaussian pdfs:

$$p(\mathbf{y}_{1(k)}, \boldsymbol{\theta}) = \frac{1}{2^{N_cN_t}}\sum_{q=1}^{2^{N_cN_t}}\frac{1}{(\pi\sigma^2_{v_1})^{N_r}}e^{-\left\|\frac{\mathbf{y_{1(k)}}-\boldsymbol{\lambda}_{\mathbf{tot_{(k)}}}\mathbf{C_x}^{\frac{1}{2}}\mathbf{x_q}}{\sigma^2_{v_1}}\right\|}, \tag{20}$$

where $\mathbf{x}_q$ is the q-th realization of $\mathbf{x}_{(k)}$.

1. Proofs are omitted due to space limitation.

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To avoid the high calculation complexity of the FIM, a realistic approximation is proposed in [9]. Consequently, the data-based FIM, at the k-th sub-carrier, is given by:

$$\mathbf{J}^{d}_{\mathbf{h}_{tot}\mathbf{h}_{tot}}(k) = \frac{1}{\sigma^2_{v_1} 2^{N_c N_t}} \sum_{q=1}^{2^{N_c N_t}} \left(\frac{\partial \boldsymbol{\lambda}_{tot_{(k)}} \mathbf{C}_x^{\frac{1}{2}} \mathbf{x}_q}{\partial \mathbf{h}^*_{tot}} \right)^H \times \left(\frac{\partial \boldsymbol{\lambda}_{tot_{(k)}} \mathbf{C}_x^{\frac{1}{2}} \mathbf{x}_q}{\partial \mathbf{h}^*_{tot}} \right). \tag{21}$$

The total data-based FIM is then obtained as follows:

$$\mathbf{J}^{d}_{\mathbf{h}_{tot}\mathbf{h}_{tot}} = N_d \sum_{k=1}^{K} \mathbf{J}^{d}_{\mathbf{h}_{tot}\mathbf{h}_{tot}}(k), \tag{22}$$

where N_d is the total number of data symbols.

The FIM, in this case, can be shown to be invertible and so the top-left $(N_r N_t N) \times (N_r N_t N)$ block of its inverse is considered as the CRB for the semi-blind estimation of the first cell channel vector.

5. Performance analysis and discussions

This section discusses the semi-blind channel estimation performance when pilot contamination occurs in a massive MIMO-OFDM system. The pilots are generated according to Zadoff-Chu sequences [11]. Simulation results are based on the CRBs derived in section 4.2, whereas the simulation parameters are summarized in Table 1.

Figure 1 presents the normalized CRB $\left(\frac{tr\{CRB\}}{\|\mathbf{h}_1\|^2}\right)$ for semi-blind channel estimation (SB) versus the SNR for BPSK and Gaussian (G) data models. These CRBs are compared to the CRB_{OP} for pilot-based channel estimation using orthogonal (O) intra and inter-cell pilots. As mentioned in section 4.1, when pilot contamination occurs, the channel vector of the interest cell cannot be identified with pilot-based methods. However, simulation results show that such an ambiguity is overcome by semi-blind channel estimation. Indeed, with a finite alphabet signal, corresponding to the BPSK case, one can observe that the pilot contamination impact has disappeared and the CRB_{SB}^{BPSKNO}, that stands for the CRB of a BPSK signal when pilots in different cells are not orthogonal (NO), is almost superposed with CRB_{SB}^{BPSKO}, which stands for the case of orthogonal pilots. On the other hand, if all pilots, in all cells, are orthogonal to one another, the CRBs illustrated by CRB_{OP}, CRB_{SB}^{BPSKO} and CRB_{SB}^{GO} match perfectly with the results already given in [9] when considering a single cell system.

To further investigate the effect of pilots orthogonality level, when considering only the SOS, we adopted the following metric:

$$\rho = \frac{\left\| \tilde{\mathbf{X}}^H_{i_P} \tilde{\mathbf{X}}_{j_P} \right\|}{\left\| \tilde{\mathbf{X}}_{i_P} \right\| \left\| \tilde{\mathbf{X}}_{j_P} \right\|}, \tag{23}$$

where $\|.\|$ is the 2-norm.

Note that $0 \leq \rho \leq 1$, so that $\rho = 0$ corresponds to the perfect orthogonality, whereas $\rho = 1$ stands for the worst case of pilot contamination. For such a case, simulations (not given for lack of space) and results given in section 4.2.1 have shown that, when using only SOS, the channel components of the interest cell cannot be identified. However, as illustrated in Figure 2, it is possible to identify the channel components with the SOS in the case of non-perfect pilot orthogonality.

Considering the worst case of pilot contamination, Figure 3 illustrates the CRB_{SB} versus the number of OFDM data symbols for a given SNR$= 10dB$. One can observe that, starting by one OFDM symbol, the BS can successfully identify and estimate the channel components of the interest cell. Besides, the CRB is significantly lowered with just few tens of OFDM data symbols, which matches perfectly with the limited coherence time of massive MIMO systems.

Figure 4 illustrates the behavior of the CRBs with respect to the number of BS antennas, i.e. N_r, taking into account the impact of pilot contamination. One can observe that when N_r increases, which increases also the number of channel components to be estimated, the CRB_{SB} is significantly lowered. This result supports the effectiveness of semi-blind techniques for pilot contamination mitigation in massive MIMO-OFDM systems.

6. Conclusion

This paper focused on the performance analysis of semi-blind channel estimation approaches, with pilot contamination, in the context of multi-cell massive MIMO-OFDM systems. The analysis has been conducted on the basis of the CRB to provide an estimator-independent tool. Analytical CRB expressions have been derived by considering the worst case of pilot contamination for different data models. The main concluding remarks of this paper are as follows:

- For the pilot-based case, pilot contamination introduces a non-identifiability of channel components of the interest cell corresponding to an $(N_c - 1)N_t N_r N$-dimensional kernel of the FIM.
- For the semi-blind case, it has been shown that by considering the finite alphabet nature of communication signals, one is able to solve efficiently the pilot contamination problem. Whereas, when considering only SOS, one could not sufficiently address the full channel identifiability. Nevertheless, with lower orthogonality levels, it is possible to overcome the ambiguity introduced by the pilot contamination.
- Simulations showed that there is no need for high number of data symbols which matches with the short coherence time constraint in massive MIMO-OFDM systems. This result is supported by those given in [9], where an important pilot reduction is possible allowing the use of more data for a semi-blind channel estimation.
- It has been noticed that increasing the number of BS antennas, implies furthermore extra channel components, makes however the CRB_{SB} decreasing, which is a very

interesting result for massive MIMO systems.

Parameters	Specifications
Number of cells	N_c = 3
Number of receive antennas	N_r = 10
Number of users per cell	N_t = 2
Channel taps	N = 4
Number of OFDM sub-carriers	K = 64
Number of OFDM pilot symbols	N_p = 4
Number of OFDM data symbols	N_d = 40
N_c pilot signal powers (dBm)	P_{x_p} = [23 13 10.5]
$N_c N_t$ data signal powers (dBm)	P_{x_d} = [20 18.8 10.7 10.4 7.2 8]

Table 1. SIMULATION PARAMETERS.

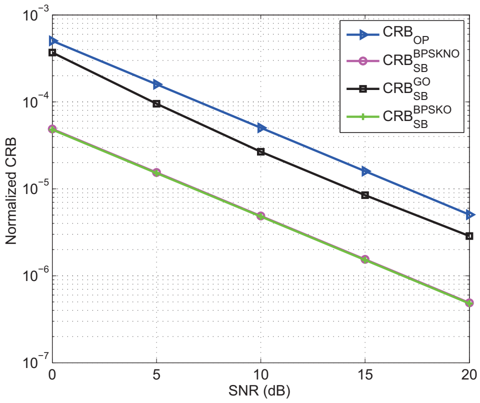

Figure 1. Normalized CRB versus SNR.

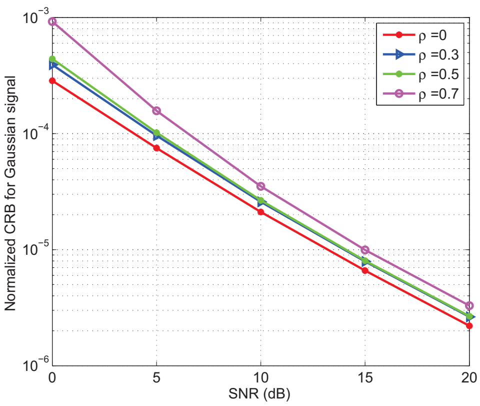

Figure 2. Gaussian CRB versus SNR with different orthogonality levels.

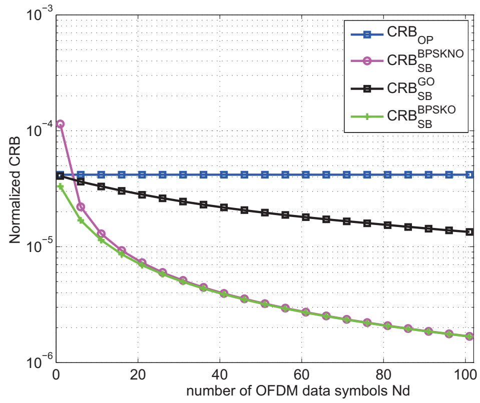

Figure 3. Normalized CRB versus number of OFDM data symbols N_d.

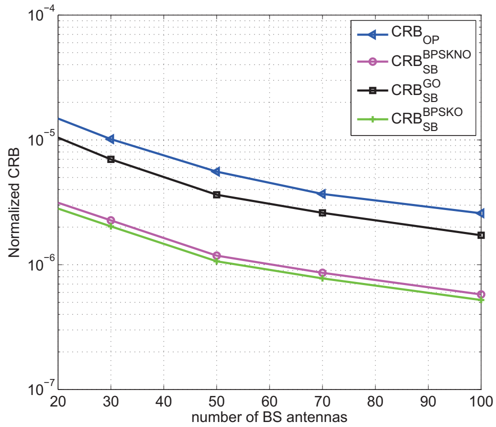

Figure 4. Normalized CRB versus number of BS antennas N_r.

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