Sequential Polynomial QR Decomposition and Decoding of Frequency Selective MIMO Channels

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***Abstract*—Recently there has been a growing interest in the application of polynomial matrix decomposition methods to the problem of efficient decoding of multiple-input multiple-output (MIMO) communication channels. Essentially, this type of approach decouples the frequency selective MIMO channel into a number of independent, frequency selective, single-input single-output (SISO) sub-channels by way of a polynomial singular-value decomposition (PSVD) or polynomial QR decomposition (PQRD) with successive interference cancellation. In this paper, we investigate a new PQRD algorithm, namely SM-PQRD, which is based on the concept of recently developed sequential matrix diagonalization (SMD). We also propose a new variant of SM-PQRD, namely MESM-PQRD to minimize computational complexity. Simulation results show that the new PQRD converges faster than state of the art algorithms. The applicability of the proposed algorithm is demonstrated for a frequency selective MIMO channel equalization problem.**

I. Introduction

Multiple-Input Multiple-Output (MIMO) is a key technology for recent generation networks by utilizing an array of multiple antennas at both ends [1]. MIMO communications has gained increasing attention over the last decade. Compared to traditional single-input single-output (SISO) systems, MIMO technology offers improvement in the transmission reliabilities, and provide higher channel capacity to wireless communications without requiring additional spectral resources [2]. For a narrowband MIMO systems, the channel is represented as a scalar matrix, and the received signals are instantaneously mixed. In this scenario if accurate channel information is available at receiver side, the QR decomposition (QRD) [3] can be used to convert the channel matrix into upper triangular structure, consequently successive interference cancellation can be used to recover the set of source signals [4].

For a frequency selective (FS) MIMO systems, the receivers have different time delayed versions of the transmitted signals which arrived over multiple paths. For this scenario the channel matrix cannot represent an instantaneous scalar mixture. Instead, polynomial matrices are required to describe FS MIMO channels, where each element is a finite impulse response (FIR) filter [5]. A similar approach in [4] can be taken by extending the scalar QRD to polynomial-matrix QRD (PQRD) as in [6], wherein the PQRD is used to transform FS MIMO systems into SISO subsystems. The proposed method has been implemented for vertical Bell Laboratories layered space time (V-BLAST) and horizontal Bell Laboratories layered space time (H-BLAST) encoding transmitter architectures.

In [7], the authors used an iterative procedure for approximating the PQRD—first proposed in [8]—to factorize the broadband channel matrix into an upper triangular polynomial matrix and a paraunitary (PU) polynomial matrix, or multi-channel all-pass filters. Application of the PU matrix at the receiver, via an H-BLAST encoding architecture, enabled the conversion of the MIMO equalization problem into a set of frequency selective SISO problems. The intersymbol interference (ISI) in each sub-channel is then alleviated using minimum mean square error (MMSE), frequency-domain equalization at the receiver. The authors showed that the PQRD-based approach outperforms a QRD-based, orthogonal frequency division multiplexing (OFDM) system.

There are two main classes of time-domain, iterative algorithms for approximating the polynomial eigenvalue decomposition (PEVD): second-order sequential best rotation (SBR2) [9] and sequential matrix diagonalization (SMD) [10]. The SBR2 algorithm was the first algorithm for calculating the PEVD of parahermitian (PH) polynomial matrices. Continuing research into the operation of the SBR2 algorithm has led to the development of an algorithm for calculating a PQRD [8]. The SMD algorithm, on the other hand, has faster convergence and designs relatively shorter filters, as shown in [10]; and so is more suitable for broadband MIMO communications. Having shorter filters is an important factor for equalization, particularly Turbo equalization. Therefore, in this paper, we develop an SMD-based PQRD algorithm and then demonstrate its applicability to the FS MIMO channel equalization problem.

The rest of this paper is organized as follows. In Section II, the polynomial matrix QRD based SBR2 approach is described. In Section II-B, we introduce the proposed PQRD algorithm based SMD concept, and the complexity of each method is evaluated in Section II-C. The applicability of new PQRD algorithm for MIMO channel equalization problem is explained in Section III. Simulation results are presented in Section IV, which show the potential performances of the new PQRD algorithm. Finally, conclusions are drawn in Section V.

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A. Notation

In this paper polynomial matrices and vectors are represented by bold uppercase and bold lowercase characters, e.g., $\mathbf{A}[\tau]$ and $\mathbf{a}[\tau]$, respectively. Taking the z-transform of $\mathbf{A}[\tau]$ gives

$$\boldsymbol{A}(z) = \sum_{\tau=t_1}^{t_2} \mathbf{A}[\tau] z^{-\tau} \ , \tag{1}$$

which is a polynomial in the indeterminate variable z^{-1}, and has entries $a_{ij}(z) = \sum_{\tau=t_1}^{t_2} a_{jk}[\tau] z^{-\tau}$. A transform pair as in (1) is abbreviated as $\boldsymbol{A}(z) \bullet\!\!-\!\!\circ \mathbf{A}[\tau]$ in the following. Throughout this paper, square brackets represent dependency of the variable on discrete time, whereas parentheses denote dependency on continuous variables.

The polynomial matrix in (1) is parahermitian: $\tilde{\boldsymbol{A}}(z) = \boldsymbol{A}(z)\ \forall z$, where $\tilde{\boldsymbol{A}}(z)$ is the paraconjugate transpose of $\boldsymbol{A}(z)$, i.e. $\tilde{\boldsymbol{A}}(z) = \boldsymbol{A}_*^{\mathrm{T}}(z^{-1})$ and the asterisk denotes complex conjugation of the polynomial coefficients. The matrix $\boldsymbol{Q}(z)$ is a paraunitary (PU) polynomial matrix, if it satisfies

$$\boldsymbol{Q}(z)\tilde{\boldsymbol{Q}}(z) = \tilde{\boldsymbol{Q}}(z)\boldsymbol{Q}(z) = \mathbf{I}_p \ , \tag{2}$$

where $\mathbf{I}_p$ is the $p \times p$ identity matrix. Finally the Frobenius norm, or F-norm, of the polynomial matrix $\boldsymbol{A}(z) \in \mathbb{C}^{p\times q}$ is defined as

$$\|\boldsymbol{A}(z)\|_F = \sqrt{\sum_{\tau=t_1}^{t_2} \sum_{j=1}^{p} \sum_{k=1}^{q} |a_{jk}[\tau]|^2} \quad . \tag{3}$$

II. Polynomial Matrix QR Decomposition

The PQRD in [8] is a time-domain iterative algorithm which applies PU operations in order to factorize a square, or rectangular, polynomial matrix, such as that in (1), into an upper triangular polynomial matrix. The aim of PQRD when applied to a polynomial matrix $\boldsymbol{A}(z) \in \mathbb{C}^{p\times q}$ is to calculate a PU matrix $\boldsymbol{Q}(z) \in \mathbb{C}^{p\times p}$ such that

$$\boldsymbol{Q}(z)\boldsymbol{A}(z) \approx \boldsymbol{R}(z) \ , \tag{4}$$

where $\boldsymbol{R}(z) \in \mathbb{C}^{p\times q}$ is an upper triangular polynomial matrix. In contrast to PEVD, which only operates on PH polynomial matrices, PQRD is not restricted to any structure or requirements of the polynomial matrix $\boldsymbol{A}(z)$.

A. The PQRD algorithm by SBR approach

In [8], the authors generalize the SBR2 concept to obtain an iterative algorithm for calculating the PQRD. At each iteration, the algorithm drives one polynomial coefficient, located below the main diagonal, to zero. The algorithm begins by locating the dominant polynomial coefficient with maximum energy situated beneath the diagonal of $\boldsymbol{A}(z)$, i.e. it finds the coefficient $a_{jk}(\tau)$, for $j > k$. An elementary polynomial Givens rotation (EPGR) is then applied to $\boldsymbol{A}(z)$ such that the dominant coefficient is annihilated. An EPGR takes the form of a Givens rotation preceded by an elementary time shift matrix. For example, a 2×2 EPRG is given as follows:

$$\begin{aligned} \boldsymbol{G}^{(\alpha,\theta,\phi,\tau)}(z) &= \begin{bmatrix} ce^{i\alpha} & se^{i\phi} \\ -se^{-i\phi} & ce^{-i\alpha} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & z^{\tau} \end{bmatrix} \\ &= \begin{bmatrix} ce^{i\alpha} & se^{i\phi} z^{\tau} \\ -se^{-i\phi} & ce^{-i\alpha} z^{\tau} \end{bmatrix} \end{aligned} \ , \tag{5}$$

where c and s define the cosine and sine of the angle θ respectively. The polynomial matrix $\boldsymbol{G}^{(\alpha,\theta,\phi,\tau)}(z)$ in (5) is clearly PU. Applying an appropriate EPGR to the polynomial matrix $\boldsymbol{A}(z)$ as follows

$$\boldsymbol{A}'(z) = \boldsymbol{G}^{(j,k,\alpha,\theta,\phi,\tau)}(z)\boldsymbol{A}(z) \ , \tag{6}$$

where the indices j, k and τ define the position of the dominant polynomial coefficient, which is to be eliminated. Following the application of the EPGR, the dominant coefficient has been driven to zero and so $a'_{jk}(0) = 0$, the diagonal coefficient $a'_{kk}(0)$ is real and will have increased in magnitude squared such that $|a'_{kk}(0)|^2 = |a_{kk}(0)|^2 + |a_{jk}(0)|^2$.

Then an inverse time-shift PU matrix $\boldsymbol{B}^{(j,\tau)}$ is applied to $\boldsymbol{A}'(z)$ to obtain

$$\boldsymbol{A}''(z) = \boldsymbol{B}^{(j,\tau)}\boldsymbol{A}'(z) \ , \tag{7}$$

the matrix $\boldsymbol{B}^{(j,\tau)} \in \mathbb{C}^{p\times p}$ takes the form of an identity matrix with the exception of the jth diagonal element which is $z^{-\tau}$. In ith iteration, the algorithm computes the following $p \times p$ PU matrix:

$$\boldsymbol{Q}_i(z) = \boldsymbol{B}^{(j,\tau)}\boldsymbol{G}^{(j,k,\alpha,\theta,\phi,\tau)}(z) \ , \tag{8}$$

where $i = 1, 2, ..., L$. Here, L represents an unspecified number of iterations.

The algorithm continues with the application of $\boldsymbol{Q}_i(z)$ to the modified matrix $\boldsymbol{A}''(z)$ at the ith iteration. This process repeats until all the coefficients below the main diagonal of $\boldsymbol{A}''(z)$ are sufficiently small in magnitude and satisfy the stopping condition:

$$a_{jk}(\tau) < \epsilon \ , \tag{9}$$

where $\epsilon > 0$ is an arbitrary small value. The PU matrix $\boldsymbol{Q}(z)$ in (4) is given by:

$$\boldsymbol{Q}(z) = \boldsymbol{Q}_L(z)...\boldsymbol{Q}_2(z)\boldsymbol{Q}_1(z) \ . \tag{10}$$

B. The PQRD algorithm by SMD approach (SM-PQRD)

In this paper, a new time-domain iterative algorithm for computing the PQRD is introduced based on the SMD concept, where at each iteration the PU matrix $\boldsymbol{Q}(z)$ calculates a PU matrix of the form as in (10) to satisfy (4). The algorithm begins by factorizing the zero-lag matrix $\mathbf{A}[0]$ into an upper triangular matrix, using the scaler QR decomposition: $\mathbf{A}[0] = \mathbf{Q}\mathbf{A}'[0]$. This transfer all elements beneath the main diagonal in $\mathbf{A}[0]$ onto the diagonal elements of $\mathbf{A}'[0]$. Note that calculation of $\mathbf{Q}$ is only based on the scaler QRD; however, the matrix $\mathbf{Q}$ is applied to $\boldsymbol{A}(z)$ in order to factorize the zero-lag matrix. Thus, letting $\boldsymbol{Q}^{(0)}(z) = \mathbf{Q}$, we have $\boldsymbol{R}^{(0)}(z) = \boldsymbol{Q}^{(0)}(z)\boldsymbol{A}(z)$. Subsequently, at ith iteration, $i = 1, 2, ..., L$, the algorithm has two steps:

ISBN 978-90-827970-1-5

1) Transfer the dominant row situated beneath the diagonal, with index j, at lag τ, on to $\mathbf{A}[0]$. This is achieved with a time-shift PU matrix $\boldsymbol{B}^{(j,\tau)}$ as in (7).

2) Factorize $\mathbf{A}[0]$ via a scalar QRD: $\mathbf{A}[0] = \mathbf{Q}^{(i)}\mathbf{A}'[0]$. Thereafter, at each iteration compute a PU transformation: $\boldsymbol{R}^{(i)}(z) = \boldsymbol{U}^{(i)}(z)\boldsymbol{R}^{(i-1)}(z)$, in which

$$\boldsymbol{U}^{(i)}(z) = \boldsymbol{Q}^{(i)}(z)\boldsymbol{B}^{(i)}(z)\ . \tag{11}$$

The term "dominant row" refers to the row situated beneath the diagonal of an intermediate variable $\boldsymbol{R}^{(i)}(z)$, with the greatest squared L_2-norm. Specifically, a modified row vector $\hat{\mathbf{r}}_j^{(i-1)}[\tau] \in \mathbb{C}^{(p-1)}$ is defined by excluding the upper triangular rows including the diagonal element:

$$\|\hat{\mathbf{r}}_j^{(i-1)}[\tau]\|_2 = \sqrt{\sum_{k=1}^{j-1} |r_{j,k}^{(i-1)}[\tau]|^2}, \quad \text{for} \quad j = 2, 3, ..., p \quad , \tag{12}$$

where $r_{j,k}^{(i-1)}[\tau]$ represents the element in the jth row and kth column of $\mathbf{R}^{(i-1)}[\tau]$ at lag τ.

The SM-PQRD algorithm searches $\mathbf{R}^{(i-1)}[\tau]$, $\forall\tau$, for the modified row $\hat{\mathbf{r}}_j^{(i-1)}[\tau]$ with the maximum L_2-norm, given by the parameter set

$$\{j^{(i)}, \tau^{(i)}\} = \underset{j,\tau}{\operatorname{argmax}} \|\hat{\mathbf{r}}_j^{(i-1)}[\tau]\|_2\ . \tag{13}$$

The iteration continues until $\boldsymbol{R}^{(L)}(z)$ is sufficiently factorized into an upper triangular matrix, such that the dominant row norm satisfies

$$\max_{j,\tau} \|\hat{\mathbf{r}}_j^{(L)}[\tau]\|_2 \le \epsilon\ , \tag{14}$$

where ϵ is a pre-specified small value. After L iterations, SM-PQRD finds an approximate PQRD.

$$\boldsymbol{R}^{(L)}(z) = \boldsymbol{Q}^{(L)}(z)\boldsymbol{A}(z)\ , \tag{15}$$

where $\boldsymbol{Q}^{(L)}(z) = \left(\boldsymbol{U}^{(L-1)}(z)\ ...\ \boldsymbol{U}^{(1)}(z)\mathbf{Q}^{(0)}\right)$ is the resultant PU matrix.

In the following, the maximum element SMD (ME-SMD) idea in [10] is extended to the PQRD problem: the maximum element SM-PQRD (MESM-PQRD). Computationally less intensive than SM-PQRD, searches for the polynomial coefficient with maximum energy situated beneath the diagonal instead of the dominant row, i.e. it maximizes the L_∞ norm instead of the L_2 norm in (12). So the search for the optimum parameter set performed by ME-SMD at every iteration i becomes

$$\{j^{(i)}, \tau^{(i)}\} = \underset{j,\tau}{\operatorname{argmax}} \|\hat{\mathbf{r}}_j^{(i-1)}[\tau]\|_\infty \quad . \tag{16}$$

Proofs of convergence for both SMD and ME-SMD algorithms can be found in [10].

C. Algorithm Complexity

The computational complexity of the two proposed PQRD algorithms is investigated by calculating the number of arithmetic operations in each iteration. The complexities in Table I are derived for polynomial matrices $\boldsymbol{A}(z) \in \mathbb{C}^{p\times q}$, where, at the ith iteration, is of order T_i. Note the value of T_i grows at each iteration. Here, $\mathcal{O}(.)$ denotes the big-O notation for the measure of computational complexity.

TABLE I: Order comparison of PQRD search methods.

method	norm calc.	comparisons	total
SBR-PQRD	$\mathcal{O}(0)$	$\mathcal{O}\left(\frac{p(p-1)}{2}T_i\right)$	$\mathcal{O}\left(\frac{p(p-1)}{2}T_i\right)$
SM-PQRD	$\mathcal{O}\left(\frac{p(p-1)}{2}T_i\right)$	$\mathcal{O}((p-1)T_i)$	$\mathcal{O}\left(\frac{p(p-1)}{2}T_i\right)$
MESM-PQRD	$\mathcal{O}(0)$	$\mathcal{O}\left(\frac{p(p-1)}{2}T_i\right)$	$\mathcal{O}\left(\frac{p(p-1)}{2}T_i\right)$

III. Application of PQRD to MIMO Channel Equalization

The system block diagram for MIMO PQRD communication system is shown in Fig. 1. We consider a FS MIMO system where, without loss of generality, the number of transmit and receive antennas is M. A set of source signals $\boldsymbol{x}(z) \in \mathbb{C}^M$ is transmitted over a multipath channel with channel matrix $\boldsymbol{A}(z) \in \mathbb{C}^{M\times M}$, which takes the form of polynomial matrix. The received broadband signals $\boldsymbol{v}(z) \in \mathbb{C}^M$ are give by

$$\boldsymbol{v}(z) = \boldsymbol{A}(z)\boldsymbol{x}(z) + \boldsymbol{n}(z)\ , \tag{17}$$

where $\boldsymbol{n}(z) \in \mathbb{C}^M$ are spatially and temporally uncorrelated Gaussian noise signals with zero mean and covariance matrix $\sigma_n^2\boldsymbol{I}_M$. The PQRD of the channel matrix can be calculated at the receiver to obtain the FIR PU matrix $\tilde{\boldsymbol{Q}}(z)$ – decoder matrix – which is applied to the received signals thus:

$$\boldsymbol{y}(z) = \tilde{\boldsymbol{Q}}(z)\boldsymbol{v}(z) = \tilde{\boldsymbol{Q}}(z)\boldsymbol{A}(z)\boldsymbol{x}(z) + \tilde{\boldsymbol{Q}}(z)\boldsymbol{n}(z)\ . \tag{18}$$

The convolutive mixing model can be rewritten as

$$\boldsymbol{y}(z) = \boldsymbol{R}(z)\boldsymbol{x}(z) + \boldsymbol{n}'(z)\ , \tag{19}$$

where $\boldsymbol{n}'(z) = \tilde{\boldsymbol{Q}}(z)\boldsymbol{n}(z)$ and $\boldsymbol{R}(z)$ is an upper triangular polynomial matrix. The MIMO channel problem is converted into equalization of a set of M SISO channels with successive interference cancellation.

IV. Simulation Results

In this section, simulation results are presented to illustrate the performance of the proposed SMD-based PQRD algorithms. The first set of simulations presented in Section IV-C investigates the evolution of the energy minimization beneath the main diagonal of $\boldsymbol{A}(z)$, for each iteration of the proposed algorithm. The second set of simulations, analyzes the effectiveness of proposed PQRD algorithm when applied to MIMO-channel equalization problem in terms of bit error rate (BER) performance. Results are averaged across an ensemble of 1000 random realizations of the channels.

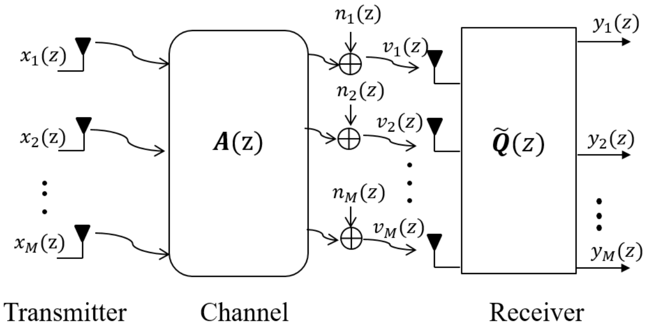

Fig. 1: PQRD-based MIMO communication system.

A. Performance Metric

The PQRD algorithms proceed by minimizing the energy under the main diagonal of $\boldsymbol{A}(z)$ in an iterative fashion. The energy that remains below the diagonal at the ith iteration is given by

$$E^{(i)} = \sum_{j=2}^{q} \sum_{\tau} \|\hat{\mathbf{r}}_j^{(i)}[\tau]\|_2 \,, \tag{20}$$

where $|\hat{\mathbf{r}}_j^{(i)}[\tau]\|$ is the modified norm vector defined in (12). This can be normalized by the total energy, or the F-norm of $\boldsymbol{A}(z)$, i.e.

$$E_{norm}^{(i)} = \frac{E^{(i)}}{\|\boldsymbol{A}(z)\|_F} \,. \tag{21}$$

B. Simulation Scenario

In our simulation scenario, the coefficients of $\boldsymbol{A}(z) \in \mathbb{C}^{5\times5}$ with order 100, are drawn from the randomized source model that have zero mean and unit variance. The PQRD algorithms are allowed to run for $L = 250$ iterations, a truncation parameter of $\mu = 10^{-6}$ and a stopping threshold of $\epsilon = 10^{-6}$ were used. The metric in (21) is computed at each iteration, along with the elapsed execution time.

C. Algorithm Convergence

The ensemble average of the measure in (21) was calculated for the new PQRD methods SM-PQRD and MESM-PQRD and the prior art, SBR-PQRD. In Fig. 2, we see that the SMD-based PQRD algorithm converge significantly faster than SBR-PQRD. This is because our SMD-like approach transfers more energy to the main diagonal of the zero-lag matrix, per iteration, than the SBR2-based method, which only moves the energy of one coefficient situate below diagonal.

Fig. 3, shows normalized energy beneath the diagonal, $E_{norm}^{(i)}$, versus elapsed system time T_i as a function of the iteration index i. Note that we show the logarithm of (21), which takes into account the quadratic term. It is clear that the PQRD-based SMD type algorithms have a considerably higher computational complexity than the SBR-PQRD algorithm, which is due to the necessity to apply a full unitary matrix for every lag τ of $\boldsymbol{R}^{(i)}(z)$ rather than just a simple Givens rotation. The extra cost of the SMD based PQRD algorithms goes towards unlocking performance regions in

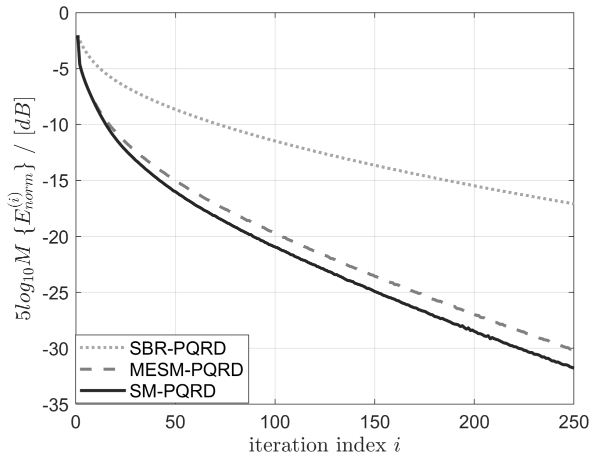

Fig. 2: Ensemble average for normalized lower triangular energy $E_{norm}^{(i)}$ vs. algorithm iterations.

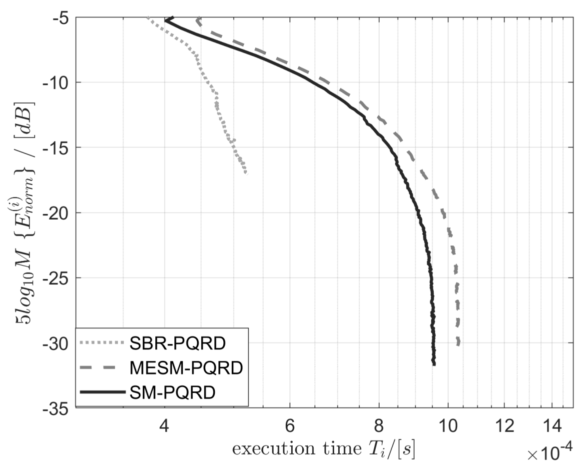

Fig. 3: Ensemble average for normalized lower triangular energy $E_{norm}^{(i)}$ vs. algorithm execution time.

terms of reduction of $E_{norm}^{(i)}$ that are inaccessible to SBR-PQRD algorithm

D. BER Performance

In this section, the BER performance for the MIMO PQRD-based communication system is studied. The channel matrix considered to be a constant power delay profile with equal average gain for each tap and the coefficients of the channel matrix, are drawn from a zero-mean, complex Gaussian, wide sense stationary (WSS) distribution [7]. The FS transmission channel $\boldsymbol{A}(z) \in \mathbb{C}^{3\times3}$, are chosen to be order five; thus can be represented by 3×3 frequency selective systems, which has five-path fading components from the jth input to the kth output. We assume receiver has perfect channel knowledge.

A turbo equalizer is utilized at the receiver in order to minimize the multipath delay spread caused by FS channel. Turbo equalization [11] is known as an iterative interference cancellation and decoding technique for coded data transmission

ISBN 978-90-827970-1-5

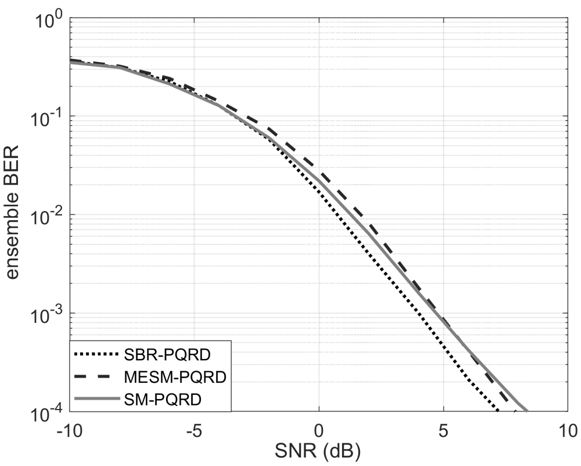

Fig. 4: Average coded BER performance for the MIMO PQRD-based communication system, using the Turbo encoding and decoding process.

over ISI channels. Furthermore, the binary phase shift keying (BPSK) modulation scheme was used for evaluation purposes, extension to complex constellation is straightforward.

Fig. 4 shows BER performance for 3×3 MIMO-PQRD system as a function of signal to noise ratio (SNR). Also, identical performances are observed for all PQRD methods. The reason for this is that PU decoder matrices produced by the PQRD algorithms have similar order. The performance of a Turbo equalizer is strongly dependent on the filter lengths.

V. Conclusion

This paper introduces two algorithms for calculating the QRD of polynomial matrices (or PQRD). These new PQRD algorithms can be seen as extensions of the SMD idea, originally proposed for computing the PEVD in [10]. Simulation results show that the proposed PQRD methods converge significantly faster than state-of-the-art algorithms, although exhibiting comparable execution times. Applicability of our PQRD method to frequency selective (FS) MIMO-channel equalization has also been demonstrated, where similar BER-performance results to the prior art was observed.

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