On Cyclostationarity-Based Signal Detection

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***Abstract*—A new cyclostationarity-based signal detector is proposed. It is based on (conjugate) cyclic autocorrelation measurements at pairs of cycle frequencies and lags for which the signal-of-interest exhibits cyclostationarity while the disturbance does not. No assumption is made on the noise distribution and/or its stationarity. A comparison is made with a previously proposed statistical test for presence of cyclostationarity. Monte Carlo simulations are carried out for performance analysis.**

***Index Terms*—Cyclostationarity; Detection.**

I. Introduction

Signal detection consists in discriminating the null hypothesis (only noise is present in the data) from the alternative hypothesis (both useful signal and noise are present). The optimum detector in the sense of Neyman-Pearson compares the likelihood ratio with a threshold that depends on the desired false-alarm rate [19, Sec. 2.5]. Its implementation requires the knowledge of the received signal distribution under both hypotheses and in general is a formidable problem if the noise is non Gaussian and/or non stationary.

In this paper, the problem of detecting the presence of an almost-cyclostationary (ACS) process embedded in noise is addressed. Second-order ACS processes have autocorrelation that is a periodic or almost-periodic function of time. That is, it can be expressed as superposition of sinewaves with possibly incommensurate frequencies, referred to as cycle frequencies [4, Chap. 10]. The magnitudes and phases of these sinewaves are referred to as cyclic autocorrelation functions. Cycle frequencies are related to signal parameters such as carrier frequency, symbol rate, sampling frequency, and coding rate. Thus, cyclostationarity-based signal processing algorithms are signal selective.

The problem of detecting an almost-cyclostationary SOI is considered here in the presence of additive nonstationary and/or non Gaussian disturbance with unknown distribution. For this purpose, instead of deriving the detector structure starting from the observed signal, the measurements of the cyclic autocorrelation are adopted as "front-end" data. In fact, properly normalized versions of these measurements are asymptotically complex normal as the data-record length approaches infinity, provided that the SOI and disturbance are processes with finite or practically finite memory. Thus, the log-likelihood ratio test (LLRT) can be derived from these measurements.

The detector performs the test at specific values of the cycle frequency and the lag parameter of the cyclic autocorrelation function. Unlike detectors based on the spectral line regeneration [4, Sec. 14.E], [5], [6], the considered cycle frequencies can be possibly shared with the disturbance signal, and the only requirement is that the disturbance does not exhibits cyclostationarity at the selected pairs of cycle frequencies and lags chosen for the SOI. In addition, the threshold can be analytically derived.

The proposed detector is compared with the statistical test for presence of cyclostationarity presented in [2] and used in [7], [11] in the context of cognitive radio. In [2], the null hypothesis of absence of cyclostationarity in the data is discriminated versus the alternative hypothesis of presence of cyclostationarity in the data. It is clarified that the detector presented in this paper and the test in [2] are intrinsically different. In fact, in the former the observed signal is different under the two hypotheses while in the latter the observed signal is the same under both hypotheses. It is shown that there is a lack of performance in adapting the test [2] to the signal detection problem as done in [9], [13], and [16]. However, in such a case, the detector implementation is simplified.

Theoretical results are corroborated by numerical experiments.

The paper is organized as follows. In Section II, almost-cyclostationary processes are briefly reviewed. In Section III the structure of the proposed detector is derived. A comparison with the test of [2] is made in Section IV and numerical results presented in Section V. Conclusions are drawn in Section VI.

II. Almost-Cyclostationary Signals

Two second-order complex-valued processes $y(t)$ and $x(t)$ are said *wide-sense jointly almost-cyclostationary* if their (conjugate) cross-correlation function is an almost-periodic function of time [4, Chap. 10], [12, Sec. 1.3]. That is,

$$\mathrm{E}\{y(t+\tau)\, x^{(*)}(t)\} = \sum_{\alpha \in A} R^{\alpha}_{yx^{(*)}}(\tau)\, e^{j2\pi\alpha t} . \quad (1)$$

In (1), superscript $(*)$ denotes an optional complex conjugation, A is the countable set possibly depending on $(*)$ of (conjugate) cycle frequencies, and

$$R^{\alpha}_{yx^{(*)}}(\tau) \triangleq \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \mathrm{E}\{y(t+\tau)\, x^{(*)}(t)\}\, e^{-j2\pi\alpha t}\, \mathrm{d}t \quad (2)$$

with $\alpha \in A$, are the (conjugate) cyclic cross-correlation functions. When $y(t) \equiv x(t)$ in (1) and (2), we have the condition of second-order almost-cyclostationarity for a single process.

ISBN 978-90-827970-1-5

Under the assumption of finite or practically finite memory of the processes $y(t)$ and $x(t)$, expressed in terms of summability of second- and fourth-order cumulants (see [12, Sec. 2.4] for details), the (conjugate) cyclic cross-correlogram

$$R_{yx^{(*)}}^{(T)}(\alpha,\tau) \triangleq \frac{1}{T}\int_{-T/2}^{T/2} y(t+\tau)\, x^{(*)}(t)\, e^{-j2\pi\alpha t}\mathrm{d}t \tag{3}$$

is a mean-square consistent estimator of the (conjugate) cyclic cross-correlation $R_{yx^{(*)}}^{\alpha}(\tau)$. In addition, under further conditions of summability of higher-order cumulants, the normalized error

$$\sqrt{T}\,\epsilon_{yx^{(*)}}^{(T)}(\alpha,\tau) \triangleq \sqrt{T}\left[R_{yx^{(*)}}^{(T)}(\alpha,\tau) - R_{yx^{(*)}}^{\alpha}(\tau)\right] \tag{4}$$

is asymptotically ($T \to \infty$) zero-mean complex normal.

III. Cyclostationarity-Based Signal Detection

Let us consider the binary hypothesis test

$$\begin{array}{lll} \mathrm{H}_0 : & r(t) = n(t) & \\ \mathrm{H}_1 : & r(t) = x(t) + n(t) & \quad t \in [-T/2, T/2] \end{array} \tag{5}$$

where $x(t)$ and $n(t)$ are zero-mean statistically independent random processes.

If the joint probability density function of $x(t_i)$ and $n(t_i)$, $i = 1, \ldots, N$, is unknown or complicated, then the likelihood ratio test (LRT) or the generalized likelihood ratio test (GLRT) cannot be derived.

Assume that, for fixed α and τ, $R_{xx^{(*)}}^{\alpha}(\tau) \neq 0$ and $R_{nn^{(*)}}^{\alpha}(\tau) = 0$. Under H_0 and H_1, we have

$$\begin{array}{llcl} \mathrm{H}_0 : & R_{rr^{(*)}}^{\alpha}(\tau) & = & R_{nn^{(*)}}^{\alpha}(\tau) = 0 \\ \mathrm{H}_1 : & R_{rr^{(*)}}^{\alpha}(\tau) & = & R_{xx^{(*)}}^{\alpha}(\tau) + R_{xn^{(*)}}^{\alpha}(\tau) \\ & & & +R_{nx^{(*)}}^{\alpha}(\tau) + R_{nn^{(*)}}^{\alpha}(\tau) \\ & & = & R_{xx^{(*)}}^{\alpha}(\tau)\,. \end{array} \tag{6}$$

Note that the assumptions are made for a specific pair (α, τ). That is, the noise can possibly exhibit cyclostationarity at the same cycle frequency of the SOI, but in correspondence of values of the lag parameter that are not selected for discriminating the two hypotheses.

Equations (5) and (6) suggest the following ad hoc (non-optimum and not equivalent to (5)) binary hypothesis test to discriminate the hypotheses H_0 and H_1 when $r(t)$ is observed for $t \in [-T/2, T/2]$:

$$\begin{array}{llcl} \mathrm{H}_0 : & R_{rr^{(*)}}^{(T)}(\alpha,\tau) & = & \epsilon_0^{(T)}(\alpha,\tau) \\ \mathrm{H}_1 : & R_{rr^{(*)}}^{(T)}(\alpha,\tau) & = & R_{xx^{(*)}}^{\alpha}(\tau) + \epsilon_1^{(T)}(\alpha,\tau)\,. \end{array} \tag{7}$$

In (7),

$$\epsilon_0^{(T)}(\alpha,\tau) \triangleq R_{nn^{(*)}}^{(T)}(\alpha,\tau) \tag{8}$$

$$\epsilon_1^{(T)}(\alpha,\tau) \triangleq \epsilon_{xx^{(*)}}^{(T)}(\alpha,\tau) + R_{xn^{(*)}}^{(T)}(\alpha,\tau) + R_{nx^{(*)}}^{(T)}(\alpha,\tau) + R_{nn^{(*)}}^{(T)}(\alpha,\tau) \tag{9}$$

where quantities are defined according to (3) and (4).

Let z_k denote any of x, x^* or any of n, and n^*, and assume that for any $N \geqslant 2$

$$\mathrm{cum}\{z_1(t+\tau_1), \ldots, z_{N-1}(t+\tau_{N-1}), z_N(t)\} = \sum_{\beta \in \mathbb{A}_N} C_{z_1 \cdots z_N}^{\beta}(\tau_1, \ldots, \tau_{N-1})\, e^{j2\pi\beta t} \tag{10}$$

$$\sum_{\beta \in \mathbb{A}_N} \left|C_{z_1 \cdots z_N}^{\beta}(\tau_1, \ldots, \tau_{N-1})\right| \in L^1(\mathbb{R}^{N-1})\,. \tag{11}$$

where $C_{z_1 \cdots z_N}^{\beta}(\tau_1, \ldots, \tau_{N-1})$ are the reduced-dimension cyclic temporal cross-cumulant functions.

Thus, *independently of the distribution of $x(t)$ and $n(t)$*, we have that [12, Section 2.4]

$$\lim_{T\to\infty} \mathrm{E}\left\{\left|\epsilon_i^{(T)}(\alpha,\tau)\right|^2\right\} = 0 \qquad i = 0, 1\,. \tag{12}$$

In addition, the error terms $\sqrt{T}\,\epsilon_0^{(T)}(\alpha,\tau)$ and $\sqrt{T}\,\epsilon_1^{(T)}(\alpha,\tau)$, are both noncircular asymptotically ($T \to \infty$) zero-mean complex normal with covariance and conjugate covariance matrices that are different under the two hypotheses H_0 and H_1 [12, Section 2.4].

Even if test (7) is not optimum, due to (12) it is expected to have asymptotically ($T \to \infty$) good performance. Moreover, it is easily implementable due to the normal distribution of the observations under the two hypotheses H_0 and H_1.

It is worthwhile to underline that the detection problem (7) is different from the statistical test for presence of cyclostationarity proposed in [2] and considered in several applications in cognitive radio (see e.g., [7], [11]). As clarified in Section IV, in [2] the two tested hypotheses are not absence or presence of a SOI and, unlike in (5), the observed signal $r(t)$ is the same under both hypotheses.

In [8], the detection problem (5) is considered when $n(t)$ is colored Gaussian noise. The statistical characterization of the error term $\epsilon_0^{(T)}(\alpha,\tau)$ is derived in this special case.

The test (7) can be considered for several pairs (α_k, τ_k), $k = 1, \ldots, K$, where some of the α_ks or τ_ks can assume the same values. That is, the following test can be considered

$$\begin{array}{llcl} \mathrm{H}_0 : & R_{rr^{(*)_k}}^{(T)}(\alpha_k,\tau_k) & = & \epsilon_0^{(T)}(\alpha_k,\tau_k) \\ \mathrm{H}_1 : & R_{rr^{(*)_k}}^{(T)}(\alpha_k,\tau_k) & = & R_{xx^{(*)_k}}^{\alpha_k}(\tau_k) + \epsilon_1^{(T)}(\alpha_k,\tau_k) \\ & & & k = 1, \ldots, K \end{array} \tag{13}$$

where the optional complex conjugation $(*)_k$ possibly depends on k.

Under assumption (10), (11) and further mild assumptions on the cumulant series expansions [12, Sec. 2.4], the (column) random vector

$$\boldsymbol{Z} \triangleq \{\sqrt{T}\,R_{rr^{(*)_k}}^{(T)}(\alpha_k,\tau_k);\ k = 1, \ldots, K\} \tag{14}$$

for T sufficiently large is complex normal under both hypotheses

$$\boldsymbol{Z} \mid \mathrm{H}_i \sim \mathcal{N}(\boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i, \boldsymbol{\Sigma}_i^{(c)}) \qquad i = 0, 1 \tag{15}$$

with mean vectors

$$\begin{array}{ll} \boldsymbol{\mu}_0 & = \mathrm{E}\{\boldsymbol{Z} \mid \mathrm{H}_0\} = \boldsymbol{0} \\ \boldsymbol{\mu}_1 & = \mathrm{E}\{\boldsymbol{Z} \mid \mathrm{H}_1\} = \{\sqrt{T}\,R_{xx^{(*)_k}}^{\alpha_k}(\tau_k);\ k = 1, \ldots, K\} \end{array} \tag{16}$$

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asymptotic covariance matrix $\boldsymbol{\Sigma}_i$ with entries

$$\begin{aligned}\Sigma_i(k_1,k_2) &= \lim_{T\to\infty} \text{cov}\big\{Z_{k_1}, Z_{k_2} \mid \text{H}_i\big\} \\ &= \lim_{T\to\infty} T\,\text{cov}\Big\{\epsilon_i^{(T)}(\alpha_{k_1},\tau_{k_1}),\ \epsilon_i^{(T)}(\alpha_{k_2},\tau_{k_2})\Big\}\end{aligned} \tag{17}$$

and asymptotic conjugate covariance matrix $\boldsymbol{\Sigma}_i^{(c)}$ with entries $\Sigma_i^{(c)}(k_1,k_2) = \lim_{T\to\infty} \text{cov}\big\{Z_{k_1}, Z_{k_2}^* \mid \text{H}_i\big\}$.

Let us define the *augmented random vector*

$$\boldsymbol{\zeta} \triangleq [\boldsymbol{Z}^{\mathsf{T}} \boldsymbol{Z}^{\mathsf{H}}]^{\mathsf{T}} \tag{18}$$

and let $\boldsymbol{\zeta}_a$ denote a realization. In (18), superscripts T and H denote transpose and Hermitian transpose, respectively. In addition, let us define the *augmented mean vector*

$$\boldsymbol{\mu}_{\zeta|\text{H}_i} \triangleq \begin{bmatrix} \text{E}\{\boldsymbol{Z} \mid \text{H}_i\} \\ \text{E}\{\boldsymbol{Z}^* \mid \text{H}_i\} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\mu}_i \\ \boldsymbol{\mu}_i^* \end{bmatrix} \tag{19}$$

and the *augmented covariance matrix*

$$\boldsymbol{\Gamma}_{\boldsymbol{i}} \triangleq \text{E}\left\{(\boldsymbol{\zeta} - \boldsymbol{\mu}_{\zeta|\text{H}_i})(\boldsymbol{\zeta} - \boldsymbol{\mu}_{\zeta|\text{H}_i})^{\mathsf{H}} \mid \text{H}_i\right\} = \begin{bmatrix} \boldsymbol{\Sigma}_i & \boldsymbol{\Sigma}_i^{(c)} \\ \boldsymbol{\Sigma}_i^{(c)\,*} & \boldsymbol{\Sigma}_i^* \end{bmatrix}. \tag{20}$$

Due to the asymptotic complex normality of $\boldsymbol{Z}$, we have that the joint probability density function of the real and imaginary parts of the components Z_k of $\boldsymbol{Z}$ can be written in the complex form as [14]

$$\begin{aligned} f_{\boldsymbol{\zeta}|\text{H}_i}(\boldsymbol{\zeta}_a) = &\frac{1}{\pi^K |\det \boldsymbol{\Gamma}_i|^{1/2}} \\ &\exp\left[-\frac{1}{2}(\boldsymbol{\zeta}_a - \boldsymbol{\mu}_{\zeta|\text{H}_i})^{\mathsf{H}} \boldsymbol{\Gamma}_i^{-1} (\boldsymbol{\zeta}_a - \boldsymbol{\mu}_{\zeta|\text{H}_i})\right]. \end{aligned} \tag{21}$$

If a detector is designed starting from the (conjugate) cyclic autocorrelation measurements (13) rather than from the observed signal (5), then it consists in comparing the log likelihood ratio (LLR) with a threshold

$$\ln \frac{f_{\boldsymbol{\zeta}|\text{H}_1}(\boldsymbol{\zeta}_a)}{f_{\boldsymbol{\zeta}|\text{H}_0}(\boldsymbol{\zeta}_a)} \underset{\text{H}_0}{\overset{\text{H}_1}{\gtrless}} \lambda \tag{22}$$

where the likelihood functions are given in (21).

If the detector is used in the context of cognitive radio, then $\lambda = \ln(\text{P}[\text{H}_0]/\text{P}[\text{H}_1])$, where $\text{P}[\text{H}_1]$ is the fraction-of-time probability [4, Chap. 5] that the primary user is active. Alternatively, in radar/sonar applications, the threshold is chosen according to the Neyman-Pearson criterion starting from the desired probability of false alarm.

It is worthwhile to underline that such a detector is not optimum. Its adoption is motivated by the fact that, although $x(t)$ and/or $n(t)$ are possibly non Gaussian, the data constituted by the (conjugate) cyclic autocorrelation measurements are Gaussian under the very mild assumption of finite or practically finite memory of the involved processes, provided that the data-record length T is sufficiently large. Note, however, that detector (22) is optimum if we consider as "front-end" data the (conjugate) cyclic correlation measurements $R_{rr^{(*)_k}}^{(T)}(\alpha_k,\tau_k)$ rather than the signal $r(t)$. This point of view was adopted in the context of time-difference-of-arrival estimation in [18].

If the parameters $\boldsymbol{\mu}_{\zeta|\text{H}_1}$, $\boldsymbol{\Gamma}_1$, and $\boldsymbol{\Gamma}_0$ are unknown and are replaced by their maximum likelihood (ML) estimates $\widehat{\boldsymbol{\mu}}_{\zeta|\text{H}_1}$, $\widehat{\boldsymbol{\Gamma}}_1$, and $\widehat{\boldsymbol{\Gamma}}_0$, one obtains the generalized log-likelihood ratio test (GLLRT) [19, Sec. 2.5]

$$\ln \frac{f_{\boldsymbol{\zeta}|\text{H}_1}(\boldsymbol{\zeta}_a; \widehat{\boldsymbol{\mu}}_{\zeta|\text{H}_1}, \widehat{\boldsymbol{\Gamma}}_1)}{f_{\boldsymbol{\zeta}|\text{H}_0}(\boldsymbol{\zeta}_a; \widehat{\boldsymbol{\Gamma}}_0)} \underset{\text{H}_0}{\overset{\text{H}_1}{\gtrless}} \lambda \tag{23}$$

If the estimates $\widehat{\boldsymbol{\mu}}_{\zeta|\text{H}_1}$, $\widehat{\boldsymbol{\Gamma}}_1$, and $\widehat{\boldsymbol{\Gamma}}_0$, are not ML but are consistent, test (23) is not GLLRT, but can be considered as ad hoc test for the considered problem [8, Sec. IV-C-2].

In the following, consistent estimates for $\widehat{\boldsymbol{\mu}}_{\zeta|\text{H}_1}$, $\widehat{\boldsymbol{\Gamma}}_1$, and $\widehat{\boldsymbol{\Gamma}}_0$ are proposed and the resulting test derived.

According to (16), an estimate $\widehat{\boldsymbol{\mu}}_1$ of $\boldsymbol{\mu}_1$ is given by

$$\widehat{\boldsymbol{\mu}}_1 = \left\{\sqrt{T}\, R_{rr^{(*)_k}}^{(T)}(\alpha_k,\tau_k);\ k = 1,\dots,K\right\} \tag{24}$$

for T sufficiently large. The (conjugate) cyclic correlograms are consistent under mild assumptions (see (10), (11)).

Let

$$R_{rr^{(*)}}^{(b,u)}(\alpha,\tau) \triangleq \frac{1}{b}\int_{u-b/2}^{u+b/2} r(t+\tau)\, r^{(*)}(t)\, e^{-j2\pi\alpha t} \mathrm{d}t \tag{25}$$

be the (conjugate) cyclic correlogram estimated on the basis of $r(t)$ with $t \in [u - b/2, u + b/2]$, where in the notation the dependence on the central point u of the observation block is evidenced. An estimate of the entries of the augmented covariance matrix $\boldsymbol{\Gamma}_i$ (see (20)) is given by the block-bootstrap or subsampling estimate built by $r(t), t \in [-T/2, T/2]$, under hypothesis H_i. For example, for the entries of the sub-matrix $\boldsymbol{\Sigma}_i$ we have

$$\begin{aligned} &\widehat{\Sigma}_i(k_1,k_2) \\ &= b\left\langle R_{rr^{(*)_{k_1}}}^{(b,u)}(\alpha_{k_1},\tau_{k_1})\, R_{rr^{(*)_{k_2}}}^{(b,u)*}(\alpha_{k_2},\tau_{k_2}) \mid \text{H}_i \right\rangle_u \\ &\quad - b\left\langle R_{rr^{(*)_{k_1}}}^{(b,u)}(\alpha_{k_1},\tau_{k_1}) \mid \text{H}_i\right\rangle_u \left\langle R_{rr^{(*)_{k_2}}}^{(b,u)*}(\alpha_{k_2},\tau_{k_2}) \mid \text{H}_i\right\rangle_u. \end{aligned} \tag{26}$$

In (26), $\langle\cdot\rangle_u$ denotes temporal average for $u \in [-T/2 + b/2, T/2 - b/2]$. Similar estimates can be built for the entries of the other sub-matrices in (20). Under the mild assumption of finite or practically finite memory for the process $r(t)$, the estimate is consistent provided that $b \to \infty$ and $T \to \infty$ with $b/T \to 0$ [3], [10].

If the estimate (24) is taken for $\boldsymbol{\mu}_1$, then the numerator in (23) does not depend on the observation $\boldsymbol{\zeta}_a$ and test (23) reduces to

$$\widehat{Q}(\boldsymbol{\zeta}_a) \triangleq \boldsymbol{\zeta}_a^{\mathsf{H}} \widehat{\boldsymbol{\Gamma}}_0^{-1} \boldsymbol{\zeta}_a \underset{\text{H}_0}{\overset{\text{H}_1}{\gtrless}} \lambda\,. \tag{27}$$

The threshold in (27), different from that in (23), has been renamed λ and can be analytically derived. In fact, for T large, under H_0, the quadratic form $Q(\boldsymbol{\zeta}) \triangleq \boldsymbol{\zeta}^{\mathsf{H}} \boldsymbol{\Gamma}_0^{-1} \boldsymbol{\zeta}$, with $\boldsymbol{\Gamma}_0$ denoting the true augmented covariance matrix of $\boldsymbol{\zeta}$, has central χ^2 distribution with $2K$ degrees of freedom [1, Theorem 3.3.3] (adapted to the complex case). Since $\widehat{\boldsymbol{\Gamma}}_0$

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is a consistent estimate of $\mathbf{\Gamma}_0$, then $\widehat{\mathbf{\Gamma}}_0^{-1}$ approaches $\mathbf{\Gamma}_0^{-1}$ in probability as $T \to \infty$, provided that $\mathbf{\Gamma}_0^{-1}$ exists [15, Sec. 4.1.2, p. 140]. Consequently, under H_0, $Q(\boldsymbol{\zeta})$ and $\widehat{Q}(\boldsymbol{\zeta})$ asymptotically have the same central χ^2_{2K} distribution $F_{\chi^2_{2K}}(\cdot)$ [15, Sec. 4.1.2, p. 140]. Therefore, for a desired false-alarm rate $P_{\mathrm{fa}} = \mathrm{P}[\widehat{Q} > \lambda \mid \mathrm{H}_0] = 1 - F_{\chi^2_{2K}}(\lambda)$ we have

$$\lambda = F^{-1}_{\chi^2_{2K}}(1 - P_{\mathrm{fa}})\,. \tag{28}$$

The implementation of detector (27) requires auxiliary data under H_0 necessary to obtain the estimate $\widehat{\mathbf{\Gamma}}_0$ before performing the test. In cooperative communications, such an estimate can be obtained in time intervals during which it is known that the signal $x(t)$ is absent. In radar/sonar applications, under the assumption of sufficiently spatially homogeneous noise environment, estimate $\widehat{\mathbf{\Gamma}}_0$ can be obtained starting from cells neighboring the cell-of-interest.

IV. Statistical Test for Presence of Cyclostationarity

In this section, the statistical test for presence of cyclostationarity proposed in [2] and considered in several applications in cognitive radio (see e.g., [7], [11]) is revisited.

The test in [2] consists in checking if the signal $r(t)$ exhibits (H_1') or not (H_0') cyclostationarity at the pair (α, τ). That is,

$$\begin{array}{l} \mathrm{H}_0' \;:\; (\alpha,\tau) \notin \mathrm{supp}\{R^{\alpha}_{rr^{(*)}}(\tau)\} \\ \mathrm{H}_1' \;:\; (\alpha,\tau) \in \mathrm{supp}\{R^{\alpha}_{rr^{(*)}}(\tau)\} \end{array} \tag{29}$$

where $\mathrm{supp}\{\cdot\}$ denotes the support of the argument in the brackets. If the signal $r(t)$ observed for $t \in [-T/2, T/2]$, (29) implies that

$$\begin{array}{lcl} \mathrm{H}_0' \;:\; R^{(T)}_{rr^{(*)}}(\alpha,\tau) & = & \epsilon^{(T)}(\alpha,\tau) \\ \mathrm{H}_1' \;:\; R^{(T)}_{rr^{(*)}}(\alpha,\tau) & = & R_{rr^{(*)}}(\alpha,\tau) + \epsilon^{(T)}(\alpha,\tau)\,. \end{array} \tag{30}$$

In (30), unlike (7), the observed signal $r(t)$ is the same under both hypotheses. Consequently, the error term $\epsilon^{(T)}(\alpha, \tau)$ is the same under both hypotheses H_0' and H_1'.

Similarly to (13), test (30) can be considered for several pairs (α_k, τ_k), $k = 1, \ldots, K$, [11], [16]. In such a case, test (30) reduces to

$$\widehat{Q}'(\boldsymbol{\zeta}_a) \triangleq \boldsymbol{\zeta}_a^{\mathsf{H}} \widehat{\mathbf{\Gamma}}^{-1} \boldsymbol{\zeta}_a \underset{\mathrm{H}_0'}{\overset{\mathrm{H}_1'}{\gtrless}} \lambda' \tag{31}$$

which is presented here in terms of complex-valued augmented vectors and is equivalent to the test originally derived in [2] in terms of real-valued vectors of real and imaginary parts of the (conjugate) cyclic correlograms. In (31), $\boldsymbol{\zeta}_a$ is a realization of the random augmented vector $\boldsymbol{\zeta}$ with augmented covariance matrix $\mathbf{\Gamma}$ whose estimate is denoted by $\widehat{\mathbf{\Gamma}}$. In [2], the estimates of the entries of $\mathbf{\Gamma}$ are obtained by frequency smoothing (conjugate) cyclic correlograms of second-order lag products of the data $r(t)$.

It is worthwhile to underline that test (13) and the vector valued counterpart of test (30) are different. In fact, in the former, data $r(t)$ is different under the two hypotheses H_0 and H_1 and, consequently, $\epsilon_0^{(T)}$ and $\epsilon_1^{(T)}$ in (13) are different and have different statistical characterization. In contrast, in the latter, data $r(t)$ is the same under the two hypotheses H_0' and H_1'. Therefore, in (31), the estimate $\widehat{\mathbf{\Gamma}}$ is obtained by the available data and not from auxiliary data as in (27).

Let us consider now, the situation in which the detection statistic $\widehat{Q}'$ of test (31) is adopted in the context of signal detection (see [9], [13], [16]) that is, to discriminate hypotheses H_0 and H_1 on the basis of the observation $r(t)$ as in (5):

$$\widehat{Q}'(\boldsymbol{\zeta}_a) \triangleq \boldsymbol{\zeta}_a^{\mathsf{H}} \widehat{\mathbf{\Gamma}}^{-1} \boldsymbol{\zeta}_a \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \lambda' \tag{32}$$

We obviously have $\mathrm{H}_0 \;\Rightarrow\; \mathrm{H}_0'$. Consequently, under H_0 we have $\mathbf{\Gamma} = \mathbf{\Gamma}_0$ so that $Q'(\boldsymbol{\zeta}) = \boldsymbol{\zeta}^{\mathsf{H}}\mathbf{\Gamma}^{-1}\boldsymbol{\zeta}$ and $Q(\boldsymbol{\zeta}) = \boldsymbol{\zeta}^{\mathsf{H}}\mathbf{\Gamma}_0^{-1}\boldsymbol{\zeta}$ have the same asymptotic distribution χ^2_{2K}. Thus, for a given P_{fa} we have $\lambda' = \lambda$, with λ given in (28).

Under H_1 we have $\mathbf{\Gamma} = \mathbf{\Gamma}_1$. Consequently, $Q'(\boldsymbol{\zeta}) = \boldsymbol{\zeta}^{\mathsf{H}}\mathbf{\Gamma}^{-1}\boldsymbol{\zeta}$ and $Q(\boldsymbol{\zeta}) = \boldsymbol{\zeta}^{\mathsf{H}}\mathbf{\Gamma}_0^{-1}\boldsymbol{\zeta}$ do not have the same distribution. Hence, $P_{\mathrm{d}}' = \mathrm{P}[Q' > \lambda \mid \mathrm{H}_1] \neq \mathrm{P}[Q > \lambda \mid \mathrm{H}_1] = P_{\mathrm{d}}$.

The same results hold asymptotically, when the true covariance matrices are replaced by their consistent estimates.

If we consider the detection problem starting from "front-end" data $R^{(T)}_{rr^{(*)}}(\alpha,\tau)$ (rather than from the original data $r(t)$), we have that test (22) is optimum (among those derived from data $R^{(T)}_{rr^{(*)}}(\alpha,\tau)$). In addition, accounting for the Slusky theorem [15, p. 19], $\widehat{Q}$ approaches Q in distribution as $T \to \infty$ also under hypothesis H_1. Thus, test (23) is asymptotically optimum provided that estimates are consistent. For a fixed P_{fa} the threshold (calculated under H_0) is the same and we have $P_{\mathrm{d}}' \leqslant P_{\mathrm{d}}$.

From the above considerations, it follows that there is a lack of performance in using test (32) to discriminate between hypothesis H_0 versus H_1. However, there is the advantage that no side data are necessary for the estimation of the covariance matrix.

V. Numerical Results

In this section, simulation experiments are presented to evaluate the performance of the proposed detector (27) in the presence of severe noise and interference environment. In addition, the detector performance is compared with that of test of [2] when used for the purpose of signal detection as in [9], [13], [16] (see (32)).

The SOI $x(t)$ is a filtered pulse-amplitude modulated (PAM) signal with stationary white binary modulating sequence, rectangular pulse with 50% duty cycle, and symbol period $T_p = 8T_s$, where $T_s = 1/f_s$ is the sampling period. The linear time-invariant (LTI) filter is a one-pole system with bandwidth $f_s/8$. The SOI exhibits cyclostationarity at the pairs $(\alpha, \tau) = (k/T_p, 0)$ with k integer.

The disturbance $n(t)$ is constituted by additive Gaussian noise with slowly varying power spectral level flat within the bandwidth $(-f_s/2, f_s/2)$ and an interfering signal. The interference is a PAM signal with stationary white binary modulating sequence, full duty-cycle rectangular pulse, and symbol period $T_i = 7T_s$. The signal-to-interference ratio (SIR)

ISBN 978-90-827970-1-5

is fixed at – 5 dB. The signal-to-noise ratio (SNR) ranges from – 12 dB to 4 dB.

Only one pair $(\alpha, \tau) = (1/T_p, 0)$ is considered for the proposed detector (27) and test (32). $N_b = 2^9$ bits are processed for each Monte Carlo run to compute the cyclic correlograms. Thus, the data-record length is $N = N_b T_p / T_s = 2^{12}$. The estimate $\widehat{\mathbf{\Gamma}}_0$ of the augmented covariance matrix is obtained from data under H_0 adopting $b = 32 T_s$ in (26). The estimate $\widehat{\mathbf{\Gamma}}$ of the covariance matrix in test (31) is made according to the guidelines in [2] using a Kaiser frequency-smoothing window with parameter $\beta = 10$ and bandwidth $\Delta f = f_s/8$.

For the proposed detector, the probability of missed detection P_{md} as a function of SNR for two values of the false-alarm rate P_{fa} is estimated via 10^5 Monte Carlo trials (Fig. 1). For comparison purpose, the performance of the proposed detector with covariance matrix estimated from the available data (i.e., assuming that no auxiliary data under H_0 are available), of test [2] used for signal detection, and of the energy detector is also determined.

According to the results of Sections III and IV, the proposed detector (27) outperforms all other detectors based on (conjugate) cyclic autocorrelation measurements. The poor performance of the energy detector is due to the variability of the noise power spectral density [17].

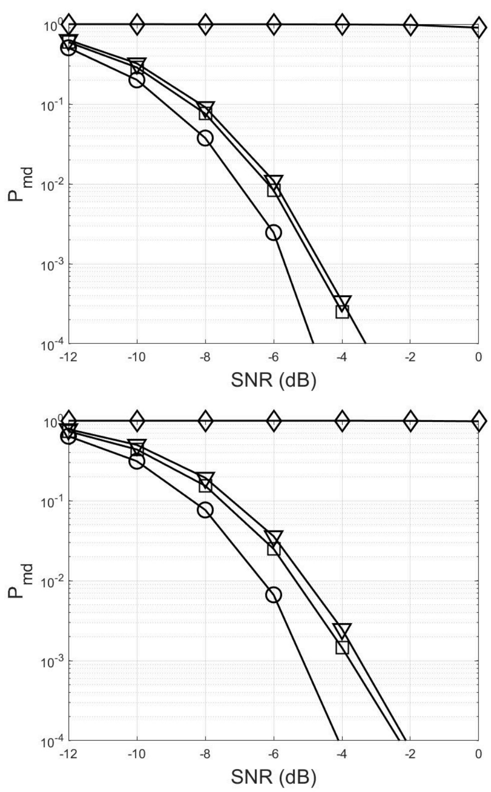

Fig. 1. P_{md} as function of SNR for (top) $P_{\mathrm{fa}} = 10^{-3}$ and (bottom) $P_{\mathrm{fa}} = 10^{-4}$. (○) proposed detector (27) with analytically derived threshold; (□) proposed detector (27) without using auxiliary data; (▽) test (32) of [2] with analytically derived threshold; (◇) energy detector.

VI. Conclusion

A cyclostationarity-based signal detector is proposed, provided that pairs of cycle frequencies and lags exist for which the SOI exhibits cyclostationarity while the disturbance does not. The detector is based on (conjugate) cyclic autocorrelation measurements and is asymptotically optimum assuming these measurements as "front-end" data. It requires the availability of auxiliary measurements under the null hypothesis. In the absence of these measurements a sub-optimum structure can be adopted that reduces to a statistic originally proposed to test the presence of cyclostationarity in the data.

References

[1] T. Anderson, *An Introduction to Multivariate Statistical Analysis*, 3rd ed. Hoboken, New Jersey: John Wiley & Sons, Inc., 2003.

[2] A. V. Dandawaté and G. B. Giannakis, "Statistical tests for presence of cyclostationarity," *IEEE Transactions on Signal Processing*, vol. 42, no. 9, pp. 2355–2369, 1994.

[3] D. Dehay, A. Dudek, and J. Leskow, "Subsampling for continuous-time almost periodically correlated processes," *Journal of Statistical Planning and Inference*, vol. 150, pp. 142–158, 2014.

[4] W. A. Gardner, *Statistical Spectral Analysis: A Nonprobabilistic Theory*. Englewood Cliffs, NJ: Prentice-Hall, 1987.

[5] ——, "Signal interception: A unifying theoretical framework for feature detection," *IEEE Transactions on Communications*, vol. COM-36, pp. 897–906, 1988.

[6] W. A. Gardner and C. M. Spooner, "Signal interception: Performance advantages of cyclic feature detectors," *IEEE Transactions on Communications*, vol. 40, no. 1, pp. 149–159, January 1992.

[7] S. Haykin, D. J. Thomson, and J. H. Reed, "Spectrum sensing for cognitive radio," *Proceedings of the IEEE*, vol. 97, no. 5, pp. 849 – 877, May 2009.

[8] G. Huang and J. K. Tugnait, "On cyclostationarity based spectrum sensing under uncertain Gaussian noise," *IEEE Transactions on Signal Processing*, vol. 61, no. 8, pp. 2042–2054, 15 April 2013.

[9] W. A. Jerjawi, Y. A. Eldemerdash, and O. A. Dobre, "Second-order cyclostationarity-based detection of LTE SC-FDMA signals for cognitive radio systems," *IEEE Transactions on Instrumentation and Measurement*, vol. 64, no. 3, pp. 823–833, March 2015.

[10] L. Lenart, J. Leśkow, and R. Synowiecki, "Subsampling in testing autocovariance for periodically correlated time series," *Journal of Time Series Analysis*, vol. 29, no. 6, pp. 995–1018, 2008.

[11] J. Lundén, V. Koivunen, A. Huttunen, and H. V. Poor, "Collaborative cyclostationary spectrum sensing for cognitive radio systems," *IEEE Transactions on Signal Processing*, vol. 57, no. 11, pp. 4182–4195, November 2009.

[12] A. Napolitano, *Generalizations of Cyclostationary Signal Processing: Spectral Analysis and Applications*. John Wiley & Sons Ltd - IEEE Press, 2012.

[13] M. Oner and F. Jondral, "On the extraction of the channel allocation information in spectrum pooling systems," *IEEE Journal on Selected Areas in Communications*, vol. 25, no. 3, pp. 558–565, April 2007.

[14] B. Picinbono, "Second-order complex random vectors and normal distributions," *IEEE Transactions on Signal Processing*, vol. 44, pp. 2637–2640, October 1996.

[15] R. J. Serfling, *Approximation Theorems of Mathematical Statistics*. USA: John Wiley & Sons, Inc., 1980.

[16] J.-C. Shen and E. Alsusa, "Joint cycle frequencies and lags utilization in cyclostationary feature spectrum sensing," *IEEE Transactions on Signal Processing*, vol. 61, no. 21, pp. 5337–5346, November 1 2013.

[17] R. Tandra and A. Sahai, "SNR walls for signal detection," *IEEE Journal of Selected Topics in Signal Processing*, vol. 2, no. 1, pp. 4–17, February 2008.

[18] M. Teplitsky and A. Yeredor, "TDOA estimation for cyclostationary sources: New correlations-based bounds and estimators," in *IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP 2009)*, April 2009, pp. 3309–3312.

[19] H. L. Van Trees, *Detection, Estimation, and Modulation Theory. Part I*. New York: John Wiley & Sons, Inc., 1971.

ISBN 978-90-827970-1-5 © EURASIP 2018