Sparse Beamforming for mmWave Spectrum Sharing Systems

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***Abstract*—This paper deals with the problem of sparse beamforming for interference mitigation in milimiter wave (mmWave) frequency bands. Multiantenna solutions in mmWave are generally implemented with phase shifters which are known to have good interference rejection capabilities. However, phase shifters are power-demanding components and, depending on their resolution, they require bulky hardware solutions able to accommodate the control lines. On the other hand, the use of switches leads to a cost-efficient alternative able to provide a sufficiently large interference rejection while substantially reducing the hardware cost and power consumption. This work proposes a beamforming scheme able to maximize the signal-to-interference-plus-noise ratio (SINR) assuming that the beamforming weights can only take 0 and 1 values. The resulting optimization problem is a binary quadratic fractional problem which is a difficult non-convex problem. Two optimization approaches are proposed; namely, the semidefinite relaxation and the penalized convex-concave procedure. We show that both techniques behave well in the considered scenarios and their performance is close to the optimization problem upper bound value.**

I. Introduction

A more aggressive frequency reuse among different wireless communication players is not only fostered in sub-6 GHz frequency bands but also in millimiter wave (mmWave) deployments [1]. This is the case of the Ka band which is being investigated by academia and industry for its shared use by satellite and terrestrial services [2].

Bearing this in mind, multiantenna transceivers will be mandatory for reducing the excess of interference from adjacent mmWave transmissions. This is investigated in [3] and [4], assuming an hybrid digital-analog multiantenna solution with phase shifters. Despite the potential of this hardware architecture for delivering large data rates in a cost-effective fashion, alternatives are being proposed [5].

In particular, the use of switches instead of phase shifters can provide a substantial cost reduction. Indeed, even if low resolution phase shifters are adopted, their costs can constitute a large percentage of the overall multiantenna solution. In addition, its power consumption is large and it might involve the construction of ad-hoc power dissipation units. Furthermore, the control lines required to adjust the phase values generally lead to bulky antenna array solutions.

The aim of this paper is to investigate the use of switches in mmWave spectrum sharing scenarios. Instead of directly considering the hybrid analog-digital solution, we target the pure analog solution constructed by a set of N switches with a single radiofrequency chain.

We focus on the maximization of the attainable data rates of the receive beamforming case, which leads to the maximization of the signal-to-interference-plus-noise ratio (SINR). The considered optimization problem is a fractional quadratic problem with non-convex quadratic constraints. In order to solve the problem, we first perform a transformation and, posteriorly, we consider two convex relaxation techniques.

On one hand, we consider the semidefinite relaxation (SDR) [6] followed by a Gaussian randomization. Remarkably, we can easily obtain feasible solutions based on the Gaussian randomization due to the nature of the problem (i.e. binary). On the other hand, we consider the penalized convex-concave procedure (PCCP) [7], [8], which iteratively solves a relaxed convex version of the original problem. This method has been successfully employed for the transmit hybrid analog-digital beamforming case in [9], [10]. The numerical results show that both techniques present a performance close to the SDR upper bound. Concretely, PCCP approach behaves closer to maximum attainable SINR value compared to the SDR and Gaussian randomization yet preserving a low computational complexity.

The outline of the paper is as follows. Section II describes the system model and the optimization problem to be solved. Section III introduces the two considered approaches to tackle the optimization problem. Section IV presents the numerical results and Section V concludes.

Notation: Throughout this paper, the following notations are adopted. Boldface upper-case letters denote matrices and boldface lower-case letters refer to column vectors. $(.)^H$, $(.)^T$, $(.)^*$ denote a Hermitian transpose, transpose and conjugate matrices, respectively. $\mathbf{I}_N$ builds $N \times N$ identity matrix and $\mathbf{0}_{K\times N}$ refers to an all-zero matrix of size $K \times N$. If $\mathbf{X}$ is a $N \times N$ matrix. $[\mathbf{X}]_{ij}$ represents the (i-th, j-th) element of matrix $\mathbf{X}$. $\otimes$, $\circ$ and $||.||$ refer to the Kronecker product, the Hadamard product and the Frobenius norm, respectively. Vector $\mathbf{1}_N$ is a column vector with dimension N whose entries are equal to 1. $\text{vec}(\cdot)$ denotes the vectorization operator.

II. System Model and Problem Statement

Let us consider a transmission of the single antenna transmitter and a receiver equipped with N antennas in presence of an interference transmitter equipped with a single antenna. The receive signal can be modeled as

$$y = \mathbf{w}^T\left(\sqrt{P}\mathbf{h}s + \mathbf{g} + z\right), \tag{1}$$

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where $\mathbf{w} \in \mathbb{R}^{N\times 1}$ is the receive beamformer, P is the transmit power, $\mathbf{h} \in \mathbb{C}^{N\times 1}$ is the channel vector of the intended transmitter, s is the unit norm zero-mean Gaussian transmitted symbol, $\mathbf{g} \in \mathbb{C}^{N\times 1}$ is the channel vector of the interference source and z is the zero-mean Gaussian additive noise with variance σ^2.

Let us consider the narrowband mmWave channel model presented in [11], which can be modelled as follows

$$\mathbf{h} = \frac{1}{\sqrt{L}} \sum_{c=1}^{C} \sum_{l=1}^{L} \alpha_{cl} \mathbf{a}_{rx}\left(\theta_{cl}^{rx}\right) a_{tx}\left(\theta_{cl}^{tx}\right), \tag{2}$$

where L denote the number of sub-paths and C the number of clusters. The value α_{cl} is a small scale fading term of the l-th sub-path at the c-th cluster for $c > 1$ and $l > 1$. Vector $\mathbf{a}_{rx}(\cdot)$ is the antenna array responses of the receiver respectively. The function a_{tx} represents the antenna gain of the transmitter. The transmit and receive antenna array responses depend on both the angles of departure (AoD), θ_{cl}^{tx}, and angles of arrival (AoA), θ_{cl}^{rx}, respectively. Note that when $C = L = 1$, the transmission is a pure line-of-sight.

The steering vector $\mathbf{a}$ depends on the antenna array structure and the element spacing. In the following, we consider an uniform linear array (ULA) whose steering vector can be written as

$$\mathbf{a}_{\text{ULA}}(\theta) = \frac{1}{\sqrt{N}}\left(1, e^{j\frac{2\pi}{\lambda}d\sin(\theta)}, \ldots, e^{j\frac{2\pi}{\lambda}(N-1)d\sin(\theta)}\right), \tag{3}$$

where d the element spacing and λ the wavelength. We assume that the interference channel vector, $\mathbf{g}$ has the same distribution as the intended user channel vector $\mathbf{h}$.

The attainable data rates are described by

$$R = \log_2\left(1 + \text{SINR}\right), \tag{4}$$

where

$$\text{SINR} = \frac{|\mathbf{h}^H\mathbf{w}|^2}{|\mathbf{g}^H\mathbf{w}|^2 + \sigma^2\|\mathbf{w}\|^2}. \tag{5}$$

Bearing this in mind, maximizing the data rate is equivalent to the maximization of (5). In this paper, we focus on the case where the entries of $\mathbf{w}$ can only take 0 or 1 values. As a result, the considered optimization problem becomes

$$\begin{aligned} &\underset{\mathbf{w}}{\text{maximize}} \quad \text{SINR} \\ &\text{subject to} \\ &[\mathbf{w}]_i \in \{0,1\} \quad i = 1,\ldots,N. \end{aligned} \tag{6}$$

The optimization problem in (6) is fractional quadratic with non-convex quadratic constraints optimization problem. In the next Section we describe how to obtain efficient solutions of this problem.

III. Sparse Beamforming Optimization

The optimization problem in (6) can be written as

$$\begin{aligned} &\underset{\mathbf{w}}{\text{maximize}} \quad \frac{\mathbf{w}^T\mathbf{H}\mathbf{w}}{\mathbf{w}^T\mathbf{G}\mathbf{w}} \\ &\text{subject to} \\ &\mathbf{w}^T\mathbf{E}_i\mathbf{w} - \mathbf{e}_i^T\mathbf{w} = 0 \quad i = 1,\ldots,N, \end{aligned} \tag{7}$$

where

$$\mathbf{H} = \mathbf{h}\mathbf{h}^H, \tag{8}$$

$$\mathbf{G} = \mathbf{g}\mathbf{g}^H + \sigma^2\mathbf{I}. \tag{9}$$

Moreover, $\mathbf{E}_i$ is a zero matrix whose i-th diagonal element is equal to 1 and $\mathbf{e}_i$ is a zero vector whose i-th entry is equal to 1. The optimization problem in (7) is non-convex due to both the objective function and the quadratical equality constraint.

We can re-write this optimization problem by using the Charnes-Cooper transmformation such that

$$\begin{aligned} &\underset{\mathbf{v},t}{\text{maximize}} \quad \mathbf{v}^T\mathbf{H}\mathbf{v} \\ &\text{subject to} \\ &\mathbf{v}^T\mathbf{E}_i\mathbf{v} - t\mathbf{e}_i^T\mathbf{v} = 0 \quad i = 1,\ldots,N, \\ &\mathbf{v}^T\mathbf{G}\mathbf{v} = 1, \\ &t > 0, \end{aligned} \tag{10}$$

where we have considered

$$t = \frac{1}{\sqrt{\mathbf{w}^T\mathbf{G}\mathbf{w}}}, \tag{11}$$

$$\mathbf{v} = t\mathbf{w}. \tag{12}$$

The optimal solution of (10) is related to the one from (7) so that

$$\mathbf{w}^\star = \frac{\mathbf{v}^\star}{t}. \tag{13}$$

Let us define

$$\mathbf{v} = \left[\mathbf{v}^T, t\right]^T. \tag{14}$$

With this, we can re-write the optimization problem in (10) so that

$$\begin{aligned} &\underset{\mathbf{u}}{\text{maximize}} \quad \mathbf{u}^T\overline{\mathbf{H}}\mathbf{u} \\ &\text{subject to} \\ &\mathbf{u}^T\mathbf{J}_i\mathbf{u} = 0 \quad i = 1,\ldots,N, \\ &\mathbf{u}^T\overline{\mathbf{G}}\mathbf{u} = 1, \\ &[\mathbf{u}]_{N+1} > 0, \end{aligned} \tag{15}$$

where

$$\overline{\mathbf{H}} = \text{block-diag}\left\{\mathbf{H}, 0\right\}, \tag{16}$$

$$\overline{\mathbf{G}} = \text{block-diag}\left\{\mathbf{G}, 0\right\}, \tag{17}$$

and

$$\mathbf{J}_i = \begin{pmatrix} \mathbf{E}_i & -0.5\mathbf{e}_i \\ -0.5\mathbf{e}_i^T & 0 \end{pmatrix}. \tag{18}$$

We now consider the optimization problem in (15) which is non-convex quadratically constraint quadratic program (QCQP). Two methods for obtaining efficient solutions of (15) and; thus, from (6) are presented in the following subsections.

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A. SDR and Gaussian Randomization

The SDR of (15) can be written as

$$\begin{aligned}&\underset{\mathbf{U}}{\text{maximize}} \quad \text{Tr}\left\{\overline{\mathbf{H}}\mathbf{U}\right\}\\&\text{subject to}\\&\text{Tr}\left\{\mathbf{J}_i\mathbf{U}\right\}=0 \quad i=1,\ldots,N,\\&\text{Tr}\left\{\overline{\mathbf{G}}\mathbf{U}\right\}=1.\\&[\mathbf{U}]_{N+1,N+1}>0\end{aligned} \tag{19}$$

Considering the optimal solution of (22), $\mathbf{U}^{\star}$, we state that the maximum SINR value that the proposed sparse array solution can attain is

$$\text{SINR}_{\text{upper bound}}=\frac{\text{Tr}\left\{\overline{\mathbf{H}}\mathbf{U}^{\star}\right\}}{\text{Tr}\left\{\overline{\mathbf{G}}\mathbf{U}^{\star}\right\}}. \tag{20}$$

This maximum performance of the original optimization problem in (7) and it is attainable whenever $\mathbf{U}^{\star}$ is rank one. Otherwise, the Gaussian randomization method can attain values close to this upper-bound [6]. For this particular SDR problem, we propose the following Gaussian randomization method.

We first compute a set of M $\mathbf{x}\in\mathbb{R}^{N+1\times 1}$ Gaussian random variables with zero mean and covariance matrix $\mathbf{U}^{\star}$. In all realizations, we perform the following operation

$$\mathbf{d}=\frac{\mathbf{x}_{1:N}}{[\mathbf{x}]_{N+1}}, \tag{21}$$

where $\mathbf{x}_{1:N}$ denotes a vector containing the N first entries of $\mathbf{x}$. All the normalized realizations $\{\mathbf{d}_m\}_{m=1}^{M}$ are rounded to values between 0 and 1. This is, if an entry $[\mathbf{d}_m]_i>0.5$ it is set to 1 and zero otherwise.

Among all rounded and normalized vectors, we select the one that attains the maximum SINR described in (5).

B. Penalized Convex-Concave Procedure (PCCP)

We can re-write the optimization problem in (15) as follows

$$\begin{aligned}&\underset{\mathbf{u}}{\text{minimize}} \quad -\mathbf{u}^T\overline{\mathbf{H}}\mathbf{u}\\&\text{subject to}\\&\mathbf{u}^T\mathbf{J}_i^{(+)}\mathbf{u}+\mathbf{u}^T\mathbf{J}_i^{(-)}\mathbf{u}\le 0 \quad i=1,\ldots,N,\\&-\mathbf{u}^T\mathbf{J}_i^{(+)}\mathbf{u}-\mathbf{u}^T\mathbf{J}_i^{(-)}\mathbf{u}\le 0 \quad i=1,\ldots,N,\\&\mathbf{u}^T\overline{\mathbf{G}}\mathbf{u}\le 1,\\&-\mathbf{u}^T\overline{\mathbf{G}}\mathbf{u}\le 1,\\&[\mathbf{u}]_{N+1}>0\end{aligned} \tag{22}$$

where $\mathbf{J}_i^{(+)}$ and $\mathbf{J}_i^{(-)}$ are two matrices collecting the eigenvectors associated to the positive and negative eigenvalues of $\mathbf{J}_i$ respectively.

The PCCP method [7] approximates the concave parts of the problem by its first Taylor approximation and iteratively solves the equivalent problem. In addition, in all approximated constraints, a slack variable is added to foster the finding of a feasible point [7].

Considering an arbitrary iteration n, the PCCP version of the optimization problem in (22) can be written as in (23) where s_m $m=1,\ldots,2N+1$ are the slack variables and β is a regularization factor that controls the feasibility of the constraints. For high values of β, the optimization focuses on yielding to a feasible point of (22). On the other hand, for low values of β, the optimization problem targets to maximize the array gain towards the intended user. This regularization factor can be updated over the iterations. For our case, we consider a multiplicative update by a factor $\rho>1$. Note that the optimization problem in (23) is a second order cone program (SOCP) which can be efficiently solved via interior point methods.

The optimization algorithm is summarized in Algorithm 1.

Data: $\mathbf{z}^{(0)}$ and $\beta^{(0)}$
Result: $\mathbf{p}^*$
while $\sum_{m=1}^{2N_T+1} s_m\le\psi$ *and* $\|\mathbf{u}^{(n)}-\mathbf{u}^{(n-1)}\|\ge\omega$ **do**
  if $t<T_{max}$ **then**
    Compute $\mathbf{u}^{(n)}$ according to (23).;
    $\mathbf{z}^{(n+1)}\leftarrow\mathbf{u}^{(n)}$;
    $\beta^{(n+1)}\leftarrow\max\left(\beta^{(n)}\rho,\beta_{\max}\right)$;
    $t\leftarrow n+1$;
  else
    $t\leftarrow 0$;
    Initialize with a new random value $\mathbf{z}^{(0)}$;
    Set up $\beta^{(0)}$ again;
  end
end
Output the final solution;

Algorithm 1: PCCP optimization for sparse beamforming optimization.

As it can be observed, the proposed algorithm includes the stopping criteria $\sum_{m=1}^{2N+1}s_m\le\psi$. This condition guarantees that all the constraints of the original problem (22) are not violated for a sufficiently low ψ. Note that, it is possible to allow different maximum violations of each constraint by weighting the penalty function $\sum_{m=1}^{2N+1}s_m$.

The role of β is to balance the optimization of the array gain to the intended users and the minimization of the constraint violation (i.e. for very high β the optimization problem seeks for a feasible point rather than optimizing the array gain). We variate the value of β over the different iterations. First, we set a relatively low value of $\beta^{(0)}$ and; posteriorly, we sequentially increase this value. In other words, the proposed scheme first focuses on maximizing the array gain to the intended user and, later, it seeks for a feasible solution. To avoid β taking a very large value when the number of iterations becomes large, leading to numerical difficulties, we consider a maximum β value $\beta_{\max}$.

Algorithm 1 is not a descent algorithm [7]. With the aim of fostering the convergence, a maximum number of iterations $T_{\max}$ is imposed and, in case it is reached, we start with a new random initial point.

IV. Numerical Results

We now proceed with the numerical evaluation of the mentioned beamforming techniques. We first describe the channel model paramaters considering a backhaul scenario

$$\begin{aligned}
&\underset{\mathbf{u},\{s_m\}_{m=1}^{2N+1}}{\text{maximize}} \quad 2\mathcal{R}\left\{\mathbf{z}^{T,(n)}\overline{\mathbf{H}}\mathbf{u}\right\} - \beta^{(n)}\sum_{m=1}^{2N+1} s_m \\
&\text{subject to} \\
&\mathbf{u}^T\mathbf{J}_i^{(+)}\mathbf{u} - 2\mathcal{R}\left\{\mathbf{z}^{T,(n)}\mathbf{J}_i^{(-)}\mathbf{u}\right\} \leq \mathbf{z}^{T,(n)}\mathbf{J}_i^{(-)}\mathbf{z} + s_i \quad i = 1,\ldots,N, \\
&\mathbf{u}^T\mathbf{J}_i^{(-)}\mathbf{u} - 2\mathcal{R}\left\{\mathbf{z}^{T,(n)}\mathbf{J}_i^{(+)}\mathbf{u}\right\} \leq \mathbf{z}^{T,(n)}\mathbf{J}_i^{(+)}\mathbf{z} + s_{i+N} \quad i = 1,\ldots,N, \\
&\mathbf{u}^T\overline{\mathbf{G}}\mathbf{u} \leq 1, \\
&-2\mathcal{R}\left\{\mathbf{z}^{T,(n)}\overline{\mathbf{G}}\mathbf{u}\right\} \leq 1 + \mathbf{z}^{T,(n)}\overline{\mathbf{G}}\mathbf{z} + s_{N+1}, \\
&[\mathbf{u}]_{N+1} > 0 \\
&s_m \geq 0 \quad m = 1,\ldots,2N+1,
\end{aligned} \tag{23}$$

described in [12]. The amplitude variations are modeled so that

$$\alpha_{cl} = A_{cl}e^{\psi_{cl}j}, \tag{24}$$

where A_{cl} is Rayleigh distributed with mean 0.1 and ψ_{cl} is uniformly distributed from 0 to 2π. We assume that $\theta_{11}^{tx}, \theta_{11}^{rx}$ are deterministic and they can be computed by known the relative positions between the transmitter and the receiver. Otherwise, for $c > 1$ and $l > 1$, we assume that

$$\theta_{cl}^{tx} = \theta_{11}^{tx} + \chi^{tx}, \tag{25}$$

$$\theta_{cl}^{rx} = \theta_{11}^{rx} + \chi^{rx}, \tag{26}$$

where χ^{tx} and χ^{rx} are zero mean Gaussian distributed random variables with standard deviation equal to 5. All the simulation results have been obtained over 500 Monte Carlo runs.

We first identify the potential of the use of sparse arrays in mmWave spectrum sharing systems. With this aim, we compute the upper bound performance (i.e. solution of the optimization problem in (22)) and the performance of a pure digital beamforming solution. For the latter case, the optimal design is known to be

$$\mathbf{u}_{\text{digital}} = \mathbf{G}^{-1}\mathbf{h}. \tag{27}$$

Figure 1 and 2 depict these results for $N = 20, 40, 60$ and 80 antennas and for different P values assuming $\sigma^2 = 1$. In all cases it can be observed that the sparse array solution loses between 3 and 4 dBs of SINR with respect to the ideal fully-digital beamforming alternative. This indicates the huge potential of sparse arrays in spectrum sharing scenarios.

For instance, it can be noted that the solution with $N = 40$ switches presents a slightly better SINR values with respect to the case of a fully digital solution with $N = 20$. Performing a cost and power consumption comparison of the different alternatives is out of the scope of the current paper and it is left for further works. Finally, it is important to remark that in all cases the variation of P minimally impact the resulting SINR.

The propose techniques evaluation are presented in Figure 3 for $N = 20, 60$ and 80 and $P = 0$ dBW. For the PCCP method, the following parameters have been used

$$\beta^{(0)} = 0.01, \rho = 2.5, \beta_{\max} = 10^9, \tag{28}$$

$$\psi = \omega = 10^{-3}. \tag{29}$$

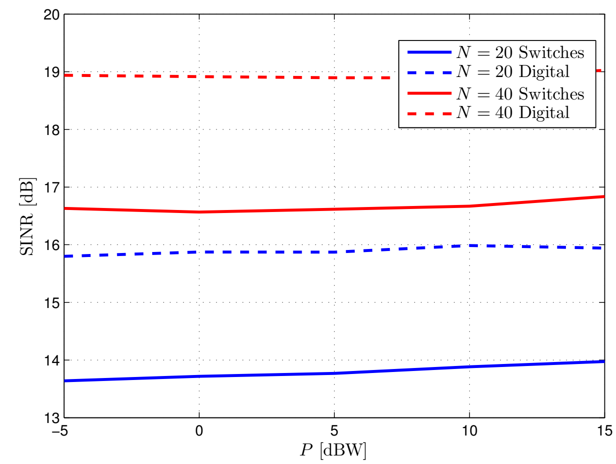

Fig. 1. Digital beamforming versus sparse antenna array for $N = 20$ and 40.

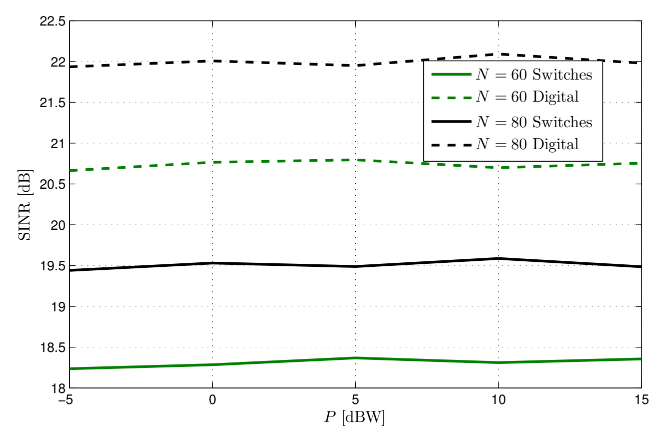

Fig. 2. Digital beamforming versus sparse antenna array for $N = 60$ and 80.

Moreover, as initial point $\mathbf{z}^{(0)}$ we have used a random vector from a Gaussian distribution with zero mean and uncorrelated entries.

In all cases it can be observed that PCCP yields to a

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performance higher to the SDR and Gaussian randomization. In this latter technique we have assumed a number of $M = 10^3$ randomizations.

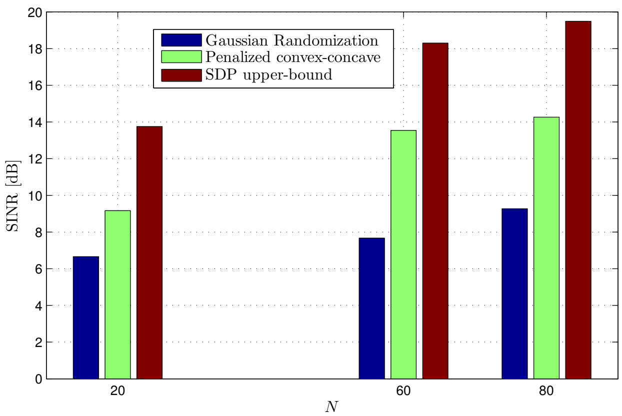

Fig. 3. Performance results for $N = 20, 60$ and 80.

For the different number of antennas employed we observe that the difference between the PCCP method and its performance upper bound is approximately 4 dB in all cases.

V. Conclusions

This paper has explored the use of switches instead of phase shifters in spectrum sharing mmWave systems. This was motivated by the fact that the use of switches offers a large reduction of the cost and power consumption of the multiantenna solution. The resulting optimization problem results in a non-convex fractional quadratic problem with was efficiently soled via two alternatives. The SDR followed by a Gaussian randomization shows a lower performance compared to the PCCP technique which behaves close to the SDR upper bound. In any case, the comparison of sparse beamforming compared to the pure digital alternative, suggests a huge potential of this hardware implementation in next generation mmWave deployments.

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