Efficient and stable Joint Eigenvalue Decomposition Based on Generalized Givens Rotations

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***Abstract*—In the present paper, a new joint eigenvalue decomposition (JEVD) method is developed by considering generalized Givens rotations. This method deals with a set of square matrices sharing a same eigenstructure. Several Jacobi-like methods exist already for solving the aforementioned problem. The differences reside in the way of estimating the Shear rotation. Herein, we clarify these differences, highlight the weaknesses of the existing solutions and develop a new robust method named Efficient and Stable Joint eigenvalue Decomposition (ESJD). Simulation results are provided to highlight the effectiveness of the proposed technique especially in difficult scenario.**

***Index Terms*—Joint EigenValue Decomposition (JEVD), Efficient and Stable Joint eigenvalue Decomposition algorithm (ESJD), generalized Givens rotations, exact JEVD, approximative JEVD.**

I. Introduction

The Joint EigneValue Decomposition (JEVD) is one of the most challenging problems in multivariate signal processing and finds many applications in different areas like direction finding and MIMO radar [1]–[3], tensors' canonical polyadic decomposition [4]–[6], multidimensional harmonic retrieval [5], biological signal processing [7] and blind sources separation [8]–[10].

This problem is well solved in the literature by considering Jacobi-like techniques [8], [11] based on generalized Givens rotations. According to our best knowledge, a first solution was developed by Tuo Fu et al. in [11] producing the SHRT algorithm. A second one has been proposed by Iferroudjene et al. in [12] referred to as JUST algorithm. Finally, a third method was introduced by Luciani et al. in [13] who developed the JDTM algorithm. In this paper, we summarize these aforementioned approaches in order to reformulate the JEVD problem and propose a new algorithm able to deal with difficult JEVD cases.

Next, section II is dedicated to the problem formulation and the different JEVD criteria including the proposed one. In section III, an efficient and numerically stable JEVD algorithm is developed. In section IV, the developed algorithm is compared with the existing ones by considering both exact and approximate JEVD cases.

II. Problem definition and formulation

Consider K square matrices of dimension N sharing the following joint structure (exact JEVD case):

$$\mathbf{M}_k = \mathbf{A}\mathbf{D}_k\mathbf{A}^{-1} \tag{1}$$

Where $k \in \{1, ..., K\}$, $\mathbf{A}$ is a square full-rank matrix (referred to as mixing matrix in the source separation context) and $\mathbf{D}_k$ is the k^{th} diagonal matrix associated to the k^{th} matrix $\mathbf{M}_k$. The JVED problem consists of obtaining $\{\mathbf{A}, \mathbf{D}_1, \cdots, \mathbf{D}_K\}$ from the set of the K real matrices[1] $\{\mathbf{M}_1, \cdots, \mathbf{M}_K\}$. Another way to define the considered problem consists in obtaining a diagonalizing matrix $\mathbf{V}$ to make the set of matrices $\{\mathbf{V}\mathbf{M}_1\mathbf{V}^{-1}, \cdots, \mathbf{V}\mathbf{M}_K\mathbf{V}^{-1}\}$ diagonal or as diagonal as possible in the approximate JEVD case[2].

In this paper, we are interested in the use of the generalized Givens rotations to solve the JEVD problem. Then matrix $\mathbf{A}$ can be decomposed in a product of generalized Givens rotations, according to:

$$\mathbf{A} = \prod_{\#\text{sweeps}} \prod_{1 \le i < j \le N} \mathbf{H}_{ij}(\theta, y) \tag{2}$$

where $\#$sweeps represents the sweeps number (number of iterations) and $\mathbf{H}_{ij}(\theta, y)$ is the elementary generalized rotation matrix given by

$$\mathbf{H}_{ij}(\theta, y) = \mathbf{S}_{ij}(y)\mathbf{G}_{ij}(\theta) \tag{3}$$

$\mathbf{G}_{ij}(\theta)$ and $\mathbf{S}_{ij}(y)$ being the elementary Givens and Shear rotations. These rotations are equal to the identity matrix except for $(i,i)^{th}$, $(i,j)^{th}$, $(j,i)^{th}$, and $(j,j)^{th}$ elements which are:

$$\begin{bmatrix} G_{ij}(i,i) & G_{ij}(i,j) \\ G_{ij}(j,i) & G_{ij}(j,j) \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \tag{4}$$

$$\begin{bmatrix} S_{ij}(i,i) & S_{ij}(i,j) \\ S_{ij}(j,i) & S_{ij}(j,j) \end{bmatrix} = \begin{bmatrix} \cosh(y) & \sinh(y) \\ \sinh(y) & \cosh(y) \end{bmatrix} \tag{5}$$

[1]The case of complex matrices relies on more elaborate derivations and hence, due to space limitation, it will be presented in a longer version of this paper.

[2]Often, in practice, matrices $\mathbf{M}_k$ represent some multivariate statistics which are estimated from the data. Hence, due to finite sample size effect, the structure in (1) is only approximately satisfied, leading to an approximate JEVD problem.

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θ and y are Givens and Shear parameters, respectively. For the estimation of the Givens rotation, the optimal solution w.r.t. the criterion detailed in section II-A is given in [8], [11], [12], [14]. Hence in this paper, we consider only the optimal estimation of the Shear parameter.

For that, let's consider $\mathbf{M}'_k$ a k^{th} matrix affected by a Shear transformation $\mathbf{S}_{ij}(y)$.

$$\mathbf{S}_{ij}(y)\mathbf{M}_k\mathbf{S}_{ij}(-y) = \mathbf{M}'_k \qquad k \in \{1, \cdots, K\} \tag{6}$$

Entries of $\mathbf{M}'_k$ can be expressed as follows:

$$\begin{aligned} M'_k(i,j) = & \tfrac{1}{2}\left[M_k(i,j) + M_k(j,i)\right] \\ & +\tfrac{1}{2}\left[M_k(i,j) - M_k(j,i)\right]\cosh(2y) \\ & +\tfrac{1}{2}\left[M_k(j,j) - M_k(i,i)\right]\sinh(2y) \end{aligned} \tag{7}$$

$$\begin{aligned} M'_k(j,i) = & \tfrac{1}{2}\left[M_k(i,j) + M_k(j,i)\right] \\ & -\tfrac{1}{2}\left[M_k(i,j) - M_k(j,i)\right]\cosh(2y) \\ & -\tfrac{1}{2}\left[M_k(j,j) - M_k(i,i)\right]\sinh(2y) \end{aligned} \tag{8}$$

$$\begin{aligned} M'_k(l,i) = & \; M_k(l,i)\cosh(y) + M_k(l,j)\sinh(y) \\ M'_k(l,j) = & \; M_k(l,i)\sinh(y) + M_k(l,j)\cosh(y) \\ M'_k(i,l) = & \; M_k(i,l)\cosh(y) - M_k(j,l)\sinh(y) \\ M'_k(j,l) = & -M_k(i,l)\sinh(y) + M_k(j,l)\cosh(y) \end{aligned} \tag{9}$$

Where indices (i, j, l) are chosen as follows $1 \leq i < j \leq N$, $1 \leq l \leq N$ and $l \notin \{i, j\}$.

Next, these transformed entries will be used within appropriate JEVD criteria to derive the optimal Shear parameter y.

A. Existing JEVD criteria

To deal with the JEVD problem, several criteria are proposed in the literature. These criteria can be summarized as follow

- In [11], the Shear rotation is chosen in such a way it minimizes the departure from normality (symmetry) through the minimization of the Frobinus norm of considered matrices $\mathbf{M}_k$, $k = 1, \cdots, K$. Then, some approximations are introduced to simplify this criterion and obtain an explicit expression of the optimal value of "y".
- In [12], the considered criterion consists of the Frobinus norm of off diagonal matrices, i.e. the Frobinus norm of matrices $\mathbf{M}_k - \text{diag}(\mathbf{M}_k)$, $k = 1, \cdots, K$. Surprisingly, the obtained results are the ones given in equations (17) and (13). An exact solution is provided for the Shear parameter 'y'.
- In [13], the considered criterion is the sum of square norm of the $(i,j)^{th}$ and $(j,i)^{th}$ entries of matrices $\mathbf{M}_k$, $k = 1, \cdots, K$. This criterion is a simplified version of the one given in [12]. However, JDTM outperforms JUST in many scenarios (see [13] for details).

B. Proposed JEVD Criterion

When matrix $\mathbf{A}$ is orthogonal (for which $\mathbf{M}_k$ are symmetric matrices), Givens rotations are enough to achieve the JEVD. However, when $\mathbf{A}$ is non-orthogonal, Shear rotations are needed and used to reduce the deviation of matrices $\mathbf{M}_k$ from symmetry (see [12] for details).

In our work, the Shear rotation is explicitly introduced to minimize the departure from symmetry of matrices $\mathbf{M}_k$. Hence, for each elementary rotation $\mathbf{S}_{ij}(y)$, the minimized criterion can be expressed as:

$$\begin{aligned} C_{T,ij}(y) & = \textstyle\sum_{k=1}^{K} \sum_{1 \leq p < q \leq N} \left|M'_k(p,q) - M'_k(q,p)\right|^2 \\ & = C_{s,ij}(y) + C_{c,ij}(y) \end{aligned} \tag{10}$$

The considered criterion is formed of two terms. The first term, computed by using only the $(i,j)^{th}$ and $(j,i)^{th}$ entries which are affected twice by the Shear rotation, is the simplified criterion denoted $C_{s,ij}(y)$ and the second term $C_{c,ij}(y)$ is the complementary one.

$C_{s,ij}(y)$ can be written as

$$C_{s,ij}(y) = \sum_{k=1}^{K} \left|M'_k(i,j) - M'_k(j,i)\right|^2 \tag{11}$$

The rest of entries are introduced in $C_{c,ij}(y)$. This criterion is computed using all i^{th} and j^{th} rows and columns except $(i,i)^{th}$, $(i,j)^{th}$, $(j,i)^{th}$ and $(j,j)^{th}$ entries.

$$\begin{aligned} C_{c,ij}(y) = \sum_{k=1}^{K} \sum_{l=1, l \notin \{i,j\}}^{N} & \left|M'_k(i,l) - M'_k(l,i)\right|^2 \\ & + \left|M'_k(j,l) - M'_k(l,j)\right|^2 \end{aligned} \tag{12}$$

We have introduced the expressions given in equations (7) and (8) in the simplified criterion $C_{s,ij}$ given in (11). After some workouts, the obtained result is expressed as

$$C_{s,ij}(y) = \boldsymbol{v}^T \mathbf{Q} \boldsymbol{v} \tag{13}$$

where

$$\boldsymbol{v} = \left[\cosh(2y) \;\; \sinh(2y)\right]^T \tag{14}$$

and $\mathbf{Q} = \mathbf{C}^T\mathbf{C}$ and the matrix $\mathbf{C}$ is expressed by

$$\mathbf{C} = \begin{bmatrix} M_1(i,j) - M_1(j,i) & M_1(j,j) - M_1(i,i) \\ \vdots & \vdots \\ M_K(i,j) - M_K(j,i) & M_K(j,j) - M_K(i,i) \end{bmatrix} \tag{15}$$

Remark: Note that the criterion $C_1(y)$ given in [13] coincides with $C_{s,ij}$. Indeed the former criterion is expressed by

$$\begin{aligned} C_1(y) & = \textstyle\sum_{k=1}^{K} M'_k(i,j)^2 + M'_k(j,i)^2 \\ & = \tfrac{1}{2}\textstyle\sum_{k=1}^{K} \left(M'_k(i,j) + M'_k(j,i)\right)^2 \\ & \quad + \\ & \quad \tfrac{1}{2}\textstyle\sum_{k=1}^{K} \left(M'_k(i,j) - M'_k(j,i)\right)^2 \end{aligned} \tag{16}$$

It contains two terms, the first one is the square of summing $(i,j)^{th}$ and $(j,i)^{th}$ entries. As given in (7) and (8), the sum of these entries is independent from the Shear rotation parameter. Then, minimizing $C_1(y)$ is equivalent to minimizing $C_{s,ij}(y)$ which is another way to show that Shear rotations minimize the departure of matrix $\mathbf{A}$ from orthogonality (or equivalently, the departure of matrices $\mathbf{M}_k$ from symmetry).

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The complementary criterion $C_{c,ij}(y)$, given in (13), can also be simplified by using the expressions of equation (9) which leads to

$$C_{c,ij} = \boldsymbol{v}^T \boldsymbol{g} - \alpha \tag{17}$$

where

$$\boldsymbol{g} = \sum_{k=1}^{K} \sum_{l=1, l \notin \{i,j\}}^{N} \left[\begin{array}{c} M_k(l,i)^2 + M_k(i,l)^2 + M_k(j,l)^2 + M_k(l,j)^2 \\ 2\left(M_k(l,i)M_k(l,j) - M_k(i,l)M_k(j,l)\right) \end{array} \right] \tag{18}$$

and α is a scalar independent from the Shear rotation parameter.

To get optimal value of Shear rotation 'y', an optimization scheme should be applied to $C_{T,ij}$.

III. Proposed ESJD method

TABLE I
ITERATIVE SCHEME TO GET $\boldsymbol{v}_{\text{OPT}}$

Step 1: Initialization, $\boldsymbol{v}_1$ is the solution given in [13]
$\boldsymbol{v}_n \leftarrow \boldsymbol{v}_1$.

Step 2: Compute $\mathbf{g}$ using (18). If $\|g\| > \mu$,
update the value of λ.
$\lambda \leftarrow \boldsymbol{v}_n^T \mathbf{Q} \boldsymbol{v}_n + \boldsymbol{v}_n^T \boldsymbol{g}$.

Step 3: Update the value of $\boldsymbol{v}_n$
$\boldsymbol{v}_n \leftarrow -\frac{1}{2}\left(\mathbf{Q} + \lambda \mathbf{J}\right)^{-1} \boldsymbol{g}$.
($n \leftarrow n+1$ starting with $n = 1$)

Step 4: Repeat **Step 2** and **Step 3** according
to the stopping criterion.

Getting the optimal solution for 'y' is similar to look for the vector $\boldsymbol{v}$ given in (14) under the following constraint

$$\boldsymbol{v}^T \mathbf{J} \boldsymbol{v} = 1 \text{ and } v(1) > 0 \tag{19}$$

Where $\mathbf{J} = \left[\begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array} \right]$.

Then, the JEVD can be summarized as a constrained minimization problem with the following Lagrangian[3].

$$L(\boldsymbol{v}, \lambda) = \boldsymbol{v}^T \mathbf{Q} \boldsymbol{v} + \lambda \left(\boldsymbol{v}^T \mathbf{J} \boldsymbol{v} - 1 \right) + \boldsymbol{v}^T \boldsymbol{g} \tag{20}$$

where λ is the Lagrangian factor.

Note that the Lagrangian given in (20) is the same as the one in [12]. The solution proposed in [12] consists of computing λ_m which minimizes (20) using a polynomial rooting (see [12] for details), then estimate the optimal vector as follows

$$\boldsymbol{v}_{\text{opt}} = -\frac{1}{2}\left(\mathbf{Q} + \lambda_m \mathbf{J}\right)^{-1} \boldsymbol{g} \tag{21}$$

The problem with this formula appears when the algorithm converges. The vector $\boldsymbol{g}$ given in (18) converges to $[0\ 0]^T$ and $\mathbf{Q}$ becomes degenerate, what gives the zero solution to $\boldsymbol{v}$ and leads to numerical instability especially in the large dimensional case.

[3]For the inequality relation $v(1) > 0$, we just check the first entry of the obtained vector in (21) and retain the solution when the latter is positive valued.

However, in [13], the complementary criterion $C_{c,ij}$ is neglected and only the simplified criterion $C_{s,ij}$ is kept. Then, the proposed solution is the generalized eigenvector of the least positive eigenvalue associated to $(\mathbf{Q}, \mathbf{J})$. In the sequel, we note this solution $\boldsymbol{v}_1$. The problem with this solution appears when the matrix dimension N is much larger than the number of matrices K. In this case, the complementary criterion $C_{c,ij}$ becomes more important than the simplified one $C_{s,ij}$ which makes the simplification of $C_{c,ij}$ not justified. This remark is confirmed and illustrated by simulation experiments in section IV.

To deal with this problem, we have proposed an alternative solution that overcomes the previous shortcomings. More precisely, we propose to use an iterative scheme to minimize our proposed cost function. To avoid the issue faced by JUST when vector $\mathbf{g}$ converges to zero, we use a threshold on the norm value of $\mathbf{g}$. The value of $\boldsymbol{v}$ is initialized by $\boldsymbol{v}_1$ (the solution of JDTM) and $\boldsymbol{g}$ is computed by (18). If the Frobinus norm of $\boldsymbol{g}$ is greater than the chosen threshold μ, then iterative corrections are introduced to $\boldsymbol{v}$, else $\boldsymbol{v}$ is kept with no modification. The iterative correction is summarized in Table I where we used relation (18) to get:

$$\lambda = -\mathbf{v}^T \mathbf{Q} \mathbf{v} - \frac{1}{2} \mathbf{v}^T \mathbf{g}$$

In our simulations, we have used a threshold value equal to $\mu = 1$ (chosen in an ad-hoc way) and we observed that a maximum of 4 iterations is enough to achieve the algorithm's potential gain. Therefore, we used a maximum of 4 iterations as stopping criterion. The overall proposed algorithm is summarized in Table II.

TABLE II
PROPOSED ESJD ALGORITHM

Require : $\mathbf{M}_k,\ k = 1, \cdots, K$, fixed thresholds τ and μ
and maximum sweep number M_{it}.
Initialization: $\mathbf{V} = \mathbf{I}_N$ and $\mathbf{A} = \mathbf{I}_N$.
$\mathbf{J} = \left[\begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array} \right]$.
while $\max_{i,j}\left(\left|\sinh(y)\right|, \left|\sin(\theta)\right|\right) > \tau$ or (#sweeps $< M_{it}$)
for all $1 \le i < j \le N$
Compute $\mathbf{Q}$ using (15).
Estimate $\boldsymbol{v}_1$ using the solution given in [13].
Compute $\boldsymbol{g}$ as given in (18).
If $\|\boldsymbol{g}\|_F^2 > \mu$
Apply the correction given in Table I.
End if
Up date matrices $\{\mathbf{M}_k | k = 1, \cdots, K\}$ as in (6).
$\mathbf{V} \leftarrow \mathbf{S}_{ij}(y)\mathbf{V}$
$\mathbf{A} \leftarrow \tilde{\mathbf{A}}\mathbf{S}_{ij}(-y)$

Estimate θ using the solution given in [8].
Update matrices $\{\mathbf{M}_k | k = 1, \cdots, K\}$ as
$\mathbf{M}_k \leftarrow \mathbf{G}_{ij}(\theta)\mathbf{M}_k\mathbf{G}_{ij}(-\theta)$
$\mathbf{V} \leftarrow \mathbf{G}_{ij}(\theta)\mathbf{V}$
$\mathbf{A} \leftarrow \mathbf{A}\mathbf{G}_{ij}(-\theta)$
end for
end while.

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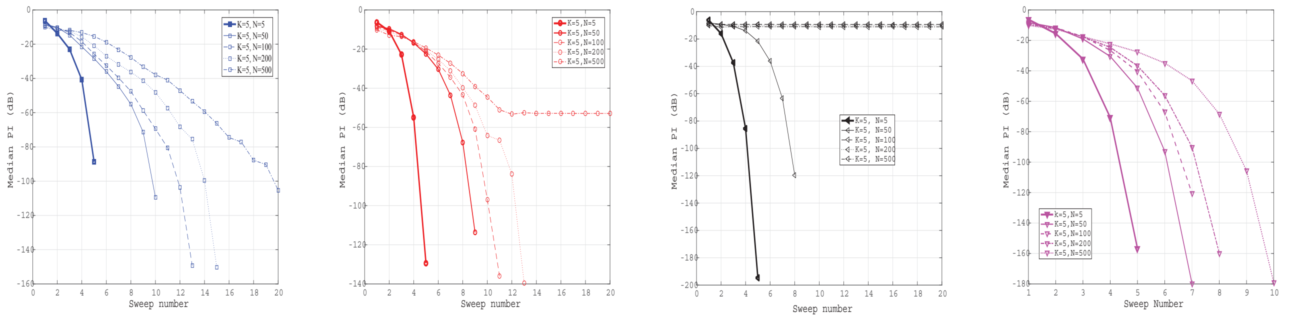

(a) SHRT algorithm results (b) JUST algorithm results (c) JDTM algorithm results (d) Proposed ESJD algorithm results

Fig. 1. Median PI versus sweep number in exact JEVD for different scenarios

IV. Simulation, Results and Discussions

A. Exact JEVD case

In this subsection, matrices $\mathbf{A}$ and $\{\mathbf{D}_k\}_{k=1,\ldots,K}$ are generated by considering independent variables and normal distribution for all entries. Then, target matrices $\{\mathbf{M}_k\}_{k=1,\ldots,K}$ can be obtained using equation (1). Once the target matrices are obtained, the different algorithms are applied to these matrices to estimate the JEVD. The Performance Index used here is the same as in [15] evaluated over 100 Monte-Carlo realisations.

$$\mathrm{PI}(\mathbf{G}) = \frac{1}{2N(N-1)}\sum_{n=1}^{N}\left(\sum_{m=1}^{N}\frac{|G(n,m)|^2}{\max_k|G(n,k)|^2}-1\right) + \frac{1}{2N(N-1)}\sum_{n=1}^{N}\left(\sum_{m=1}^{N}\frac{|G(m,n)|^2}{\max_k|G(k,n)|^2}-1\right) \tag{22}$$

where $\mathbf{G} = \hat{\mathbf{V}}\mathbf{A}$ is the global matrix. The closer the PI is to zero, the better is the JEVD quality.

Obtained results are given in Figure 1. It's emphasizing that as N increases, the convergence rate of different algorithms decreases. Hence, the considered algorithms are differently affected by increasing N and our solution is the less affected one and presents the best convergence rates. The JDTM algorithm diverges completely when N is greater than 100 due to the simplified criterion considered by this algorithm which neglects the complementary criterion given in (17). When N increases, the complementary criterion becomes important and discarding it causes divergence problem to JDTM as we can see in Figure (1d).

B. Approximative JEVD case

Herein, target matrices are supposed to be noise corrupted according to

$$\mathbf{M}_k \approx \mathbf{A}\mathbf{D}_k\mathbf{A}^{-1} + \Xi_k \quad, k = 1, \cdots, K \tag{23}$$

where Ξ_k are perturbation (noise) matrices generated here randomly with gaussian i.i.d. entries. The performance are evaluated in term of the perturbation level (PL) (a kind of SNR) expressed by:

$$\mathrm{PL(dB)} = 10\log_{10}\left(\frac{\|\mathbf{A}\mathbf{D}_k\mathbf{A}^{-1}\|_F^2}{\|\Xi_k\|_F^2}\right) \tag{24}$$

The simulation results (i.e performance index) are averaged over 100 Monte Carlo runs. Two cases are considered, the first one corresponds to small matrix sizes where $N = 5$ and $K = 5$. The second one is the difficult case where K is kept equal to five and N is increased to 100.

The obtained results are shown in Figure 2, where for each PL value, estimated performance are taken after twenty sweeps. As it can be seen in Figure 2a, the proposed algorithm and JDTM have the same performance in the case of small matrix dimension and both perform better than JUST and SHRT. In this case, the simplified criterion is more important than the complementary one which justifies the negligible impact of the latter.

Otherwise, in the difficult case, the proposed algorithm is continuing to reach the best results as compared to the other ones. The JDTM algorithm diverges in this case due to the simplification introduced in its JEVD criterion.

V. Conclusion

In this paper, we have addressed the JEVD problem by reformulating the minimization criterion and proposed a new efficient and stable algorithm. Indeed, after a critical analysis of the existing methods, JUST, SHRT and JDTM, the proposed solution is introduced in such a way it overcomes their shortcomings and improves the desired decomposition in certain difficult scenarios. Simulation results show that our algorithm presents good performance in the case of large matrix dimension where others diverge.

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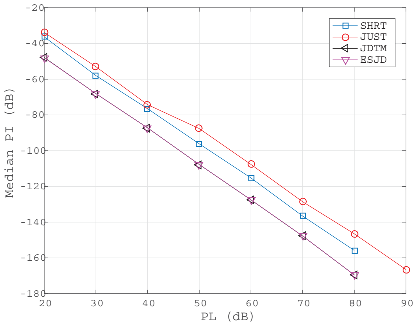
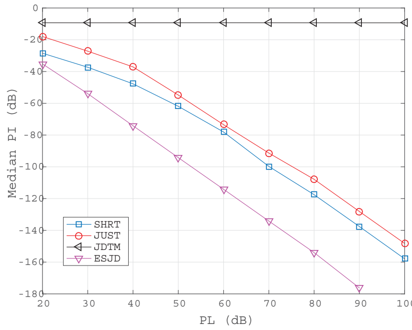


(a) Simple case of JEVD ($K = 5$ and $N = 5$)



(b) Difficult case of JEVD ($K = 5$ and $N = 100$)

Fig. 2. Median PI versus perturbation level in approximate JEVD

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