Renewable Energy Optimization with Centralized and Distributed Generation

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Abstract—We propose optimization strategies for cooperating households with renewable energy generation and storage facilities. We consider two configurations: 1) households with shared access to an energy farm, and 2) households with their own renewable energy generator and storage device. The participants in the second configuration are allowed to exchange energy through the grid. Assuming location and time dependent electricity prices, and parametrized transfer fees, we formulate two optimization problems to minimize the energy cost incurred by the participating households in each configuration. We determine the optimal energy management strategies by solving the corresponding mathematical problems through relaxation and discretization. The proposed energy management strategies are genie-aided, and hence, they can be used to benchmark and devise online algorithms based on forecasting techniques. Finally, numerical results are provided to compare the two configurations.

Index Terms—Renewable energy, optimization, cooperation.

I. INTRODUCTION

The production of solar energy has become cheaper in recent years, and as a result, more users have installed solar panels in their homes [1]. It is therefore interesting to investigate how this locally-generated renewable energy (RE) can be optimized to reduce energy costs, especially when electricity prices are time-varying.

Households are subject to time-varying electricity prices when they subscribe to demand response programs that are meant to reduce peak energy consumption and make distribution networks more efficient [2]. Designing RE management strategies is challenging because the RE generation is characterized by intermittency and geographic variability.

A valid approach to enhance the utility of the RE is to introduce energy storage devices (ESDs) to defer power consumption to periods of low RE production [3]. A different approach is to use cooperative schemes to exploit variations in the RE production across locations [4].

In terms of the RE production and storage configuration, there are two approaches to cooperative energy management: 1) users can share access to a central facility where RE is generated and stored (often referred to as a farm), and 2) users can deploy their own RE generators and share energy through the grid, or dedicated powerlines (e.g., if they belong to the same organization).

In these two scenarios we propose strategies to minimize the energy cost incurred by the participating households over a finite planning horizon. To ensure generality, we assume variations of the electricity prices, loads and RE generation profiles, both across location and time. Moreover, we assume energy transfer fees which range from 0 to the energy prices at the receiving user, thus accounting for all practical scenarios, including connection through dedicated power lines, which do not incur transfer fees.

The main contribution of this paper is a mathematical framework that can be used to devise strategies to minimize the energy cost incurred by a group of grid-tied households with RE assets. The proposed framework accounts for location and time dependent energy prices, and parametrized transfer fees. Moreover, it allows us to compare two approaches to collective RE management, namely cooperation with distributed RE generation (DREG), and cooperation with centralized RE generation (CREG).

Existing works on building/house energy management focus on scheduling deferrable appliances to achieve cost minimization, e.g. [3], [5]–[7]. Some of these works account for comfort constraints, e.g. [5] and [7], others consider intermittent loads (electrical vehicles) [6], and only a few take into account storage devices, such as [3].

Energy management strategies (EMSs) based on evolutionary algorithms have been proposed in [8] and [9]. However, these strategies do not always achieve optimality. Solutions based on game theory have been proposed in [10] and [11], and RE trading systems have been studied in [12]. Cooperative EMS have been proposed in [4], [13]–[22]. However, this is the first time that a comparison between cooperative schemes with CREG and DREG is investigated, especially from the perspective of the users.

The contributions of this work are the following: First, we propose an optimization framework to compare two approaches to cooperative RE management, the first with CREG, and the second with DREG. To do so we solve challenging optimization problems through relaxation and discretization. Second, the proposed strategies guarantee any user comfort requirements throughout the optimization horizon, as the households’ power consumption is assumed to be non-deferrable. Third, through simulations this work illustrates the conditions under which one configuration outperforms the other, thus providing valuable insights for energy planning.

II. SYSTEM MODEL

A. Planning Horizon, Objective, and Decision Variables

We consider $M$ grid-connected households, and propose EMSs to minimize their collective energy cost over the planning horizon $[0, T]$, where $T > 0$. Each household is subject to different energy consumption patterns. The power
consumed by the \(i\)th household is denoted by \(L_i(t) \geq 0\), \(\forall t \in [0, T]\), and is assumed to be non-deferrable.\(^1\) The decision variables are the charging and discharging schedules of the ESDs.

### B. Renewable Energy Production and Storage Configurations

We consider two system configurations, namely a system with CREG, and a system with DREG. These two configurations are illustrated in Fig. 1.

1) **CREG:** In this configuration, households share access to a farm, where RE is generated and stored. The power drawn from the farm by the \(i\)th household is \(D_i(t)\). Hence, the power drawn from the grid by the \(i\)th household is \(L_i(t) - D_i(t)\), where \(D_i(t)\) satisfies:

\[
D_i(t) \leq L_i(t), \quad \forall t. \tag{1}
\]

2) **DREG:** In this configuration, each household has its own RE generator and ESD. Hence, to cooperate, households share their RE through the grid, which may incur transfer fees. The power transferred from household \(i\) to household \(j\) is denoted by \(\Pi_{i,j}(t)\). Similarly, the total power received by household \(i\) from others is \(\Gamma_i(t)\), while the total power transferred from the same household to others is \(\Theta_i(t)\), i.e.,

\[
\Gamma_i(t) = \sum_{j \neq i}^{M} \Pi_{i,j}(t), \quad \Theta_i(t) = \sum_{j \neq i}^{M} \Pi_{i,j}(t), \quad \forall t. \tag{2}
\]

### C. Energy Storage Devices (ESDs)

The ESDs in both configurations are characterized by:

- **Charging/discharging losses.** The charging/discharging efficiency rates of the ESD at the energy farm are respectively denoted by \(\alpha\) and \(\beta\), and satisfy \(0 < \alpha \leq 1\) and \(0 < \beta \leq 1\). The charging/discharging efficiency rates of the ESD at the \(i\)th household are respectively \(\alpha_i\) and \(\beta_i\), and also satisfy \(0 < \alpha_i \leq 1\) and \(0 < \beta_i \leq 1\). A lossless charging (discharging) operation happens when the charging/discharging efficiency rate is 1.
- **Bounded storage capacity.** The sizes of the ESDs are assumed finite in both configurations.
- **Bounded charging/discharging rates.** The amount of power that can be charged to or discharged from the ESDs is upper limited in both configurations.

\(^1\)The load is assumed inflexible to ensure that comfort requirements are satisfied over the entire planning horizon.

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**D. Pricing Scheme**

To maintain generality, we consider location and time dependent electricity prices. The cost of the energy consumed by the \(i\)th household in \([0, T]\) is \(\xi_i = \int_0^T P_i(t) [L_i(t) - D_i(t)] dt\), where \(P_i(t)\) is the pricing function, and \(D_i(t)\) satisfies (1). The cost of the energy consumed by the entire group of households is thus \(\chi = \sum_{i=1}^{M} \xi_i\). This model generalizes pricing schemes in the discrete domain. Throughout this paper, the functions \(P_i(t)\)’s are assumed known to the consumers in advance.\(^2\)

### III. CENTRALIZED RENEWABLE ENERGY GENERATION

#### A. Constraints

Let \(J(t)\) denote the energy available in the ESD over time, i.e.,

\[
J(t) = J(0) + \int_0^t \left[ \alpha C(x) - \frac{1}{\beta} \sum_{i=1}^{M} D_i(x) \right] dx, \tag{3}
\]

where \(C(t)\) is the power charged into the ESD. Given the limited storage capacity, and the causality constraint, \(C(t)\) and the \(D_i(t)\)’s must be such that

\[
0 \leq J(t) \leq \Psi, \quad \forall t \in [0, T], \tag{4}
\]

where \(\Psi\) is the capacity of the ESD at the energy farm. The bound imposed on the charging and discharging rates results in the following constraints:

\[
C(t) \leq \min\{q_C, R(t)\}, \quad \sum_{i=1}^{M} D_i(t) \leq q_D, \quad \forall t \in [0, T], \tag{5}
\]

where \(R(t)\) is the RE generated over time, and \(q_C\) and \(q_D\) are respectively the maximum charging and discharging rates of the ESD at the farm.

#### B. Problem Formulation

With CREG, the decision variables are the \(D_i(t)\)’s and the \(C(t)\). Therefore, the following optimization problem can be formulated to determine the optimal EMS in this configuration:

\[
\text{P0: } \min_{C(t), D_1(t), \ldots, D_M(t)} \chi \quad \text{s.t. } (1), (4), (5). \tag{6}
\]

In P0, \(J(t)\) and the \(D_i(t)\)’s are connected through (3). P0 is not a convex optimization problem because its objective is a sum of functionals (not functions), its decision variables are trajectories (not scalars or vectors), and it has an infinite number of constraints, as stated in (4) and (5). Therefore, we need to introduce some relaxations in order to find an approximate solution. A numerical solution can be obtained by introducing discretization in time. The problem resulting from the discretization can be cast as a linear program, and solved by using existing algorithms. Linear programming problems are solved by using iterative algorithms, and they do not have solutions in closed form. In fact, the dual

\(^2\)The utility designs the functions \(P_i(t)\)’s to influence the users’ grid energy consumption.
of a linear program is another linear program, and the Karush–Kuhn–Tucker conditions are meaningless in linear programming.

IV. DISTRIBUTED RENEWABLE ENERGY GENERATION

We formulate a mathematical problem to determine the optimal EMS in the configuration with CREG.

A. Considerations

1) Power Transfer Matrix: To simplify notation, we define the power transfer matrix \( \Pi(t) \) as \( \Pi(t) = [\Pi_{i,j}(t)] \), where \( \Pi_{i,j}(t) \geq 0, \forall i, j, t \). Since the power exchange cannot happen simultaneously, the elements of \( \Pi(t) \) must satisfy:

\[
\Pi_{i,j}(t)\Pi_{j,i}(t) = 0, \forall t, \forall i \neq j. \tag{6}
\]

The diagonal elements of \( \Pi(t) \) can be thought of as the renewable power that the \( i \)th household draws from its own ESD. Hence, we can let \( \Pi_{i,i}(t) = D_i(t) \forall i, \forall t \).

2) Transfer Charges: To maintain generality, we assume that the transfer fees are proportional to the energy rates offered at the receiving household. Therefore, the cost incurred by the set of households in moving \( \sum_{i=1}^{M} \Theta_i(t) \) across the network is:

\[
\epsilon = \sum_{i=1}^{M} \rho \int_{0}^{T} P_i(t)\Theta_i(t) \, dt, \tag{7}
\]

where \( 0 \leq \rho \leq 1 \) is a parameter introduced to ensure generality.

B. Constraints

The energy available in the ESD at the \( i \)th household is \( J_i(t) \), and satisfies

\[
J_i(t) = J_i(0) + \int_{0}^{t} \left[ \alpha_i C_i(x) - \frac{1}{\beta_i} [D_i(x) + \Theta_i(x)] \right] \, dx, \tag{8}
\]

where \( C_i(t) \) and \( D_i(t) \) are, respectively, the power charged to the \( i \)th ESD, and the power used up by the \( i \)th household. Given the limited storage capacity of the ESD, and the causality constraint, the \( C_i(t) \)'s and the \( D_i(t) \)'s must be such that

\[
0 \leq J_i(t) \leq \Psi_i, \forall t \in [0, T], \tag{9}
\]

where \( \Psi_i \) is the storage capacity of the ESD at the \( i \)th house. Following the energy conservation principle, total received and transferred power among users must satisfy:

\[
\sum_{i=1}^{M} \Gamma_i(t) = \sum_{i=1}^{M} \Theta_i(t), \forall t. \tag{10}
\]

Let \( R_i(t) \) be the RE generated at the \( i \)th facility. Then, given the limited charging and discharging rates, the following constraints must be imposed on \( C_i(t) \) and \( D_i(t) \):

\[
C_i(t) \leq \min\{q_{C,i}, R_i(t) + \Gamma_i(t)\}, \forall t, \forall i, \tag{11}
\]

and

\[
D_i(t) + \Theta_i(t) \leq q_{D,i}, \forall i, \forall t \in [0, T], \tag{12}
\]

where \( q_{C,i} \) and \( q_{D,i} \) are respectively the maximum charging and discharging rates of the ESD at the \( i \)th house.

C. Problem Formulation

With the considerations explained in Secs. IV-A1 and IV-A2, the optimization problem can be cast in terms of the decision variables \( \Pi(t) \) and the \( C_i(t) \)'s as follows:

**P1A:** \[
\min_{\Pi(t), C_i(t), \Theta_i(t)} \chi + \epsilon \\
\text{s.t.} \ (1), \ (6), \ (9) \tag{12}
\]

In P1A, the \( J_i(t) \)'s and \( \Pi(t) \) are connected through (8). The quantities \( \Gamma_i(t) \) and \( \Theta_i(t) \) were defined in terms of \( \Pi(t) \) in (2). P1A is not a convex optimization problem because its objective is a functional (not a function), its decision variables are trajectories (not vectors or scalars), and it involves an infinite number of constraints, stated in (9)-(12).

An alternative formulation can be obtained by casting the problem directly in terms of the \( C_i(t) \)'s, the \( D_i(t) \)'s, the \( \Theta_i(t) \)'s, and the \( \Gamma_i(t) \)'s. Consequently, we can find the power transfer matrix by solving the system of linear equations defined by (2) and (6). Thus, the optimization problem can be formulated in terms of the \( C_i(t) \)'s, the \( D_i(t) \)'s, the \( \Theta_i(t) \)'s, and the \( \Gamma_i(t) \)'s as follows:

**P1B:** \[
\min_{C_i(t), D_i(t), \Theta_i(t), \Gamma_i(t)} \chi + \epsilon \\
\text{s.t.} \ (1), \ (9) \tag{12}
\]

In P1B, the \( J_i(t) \)'s, the \( D_i(t) \)'s, and the \( \Theta_i(t) \)'s are connected through (8). Again, P1B is a non-convex optimization problem, which we relax to find an approximate solution through discretization and linear programming.

V. NUMERICAL RESULTS

We provide simulation results to compare the EMSs proposed in the paper. The proposed strategies can be compared in terms of the achievable cost savings and the RE unused due to ESD overflow. Let \( D^*_i(t), \ldots, D^*_M(t) \) denote the optimal discharging profiles obtained by solving P0 and P1B respectively. Then, the RE unused in the centralized scheme is:

\[
\text{RE}_{UC} = \int_{0}^{T} \left[ R(t) - \sum_{i=1}^{M} D^*_i(t) \right] \, dt. \tag{13}
\]

Similarly, the RE unused in the distributed scheme is:

\[
\text{RE}_{UD} = \int_{0}^{T} \left[ R(t) - \sum_{i=1}^{M} D^*_i(t) \right] \, dt. \tag{14}
\]

Unless otherwise stated, throughout this section we consider the simulation parameters shown in Table I, where \( \minPrice, \maxPrice, \minLoad, \maxLoad, \minGen, \maxGen \) are all real numbers chosen arbitrarily so as to consider various simulation scenarios. Throughout this section storage capacities are stated in generic energy units [EU], and the energy cost is stated in monetary units [MU]. The results presented in this section are obtained by averaging over ten thousand realizations of the random quantities involved in the problem (RE generation and power consumption). Random pricing trajectories are considered to ensure the generality of the results. The uniform distribution is chosen.
for the prices, the RE generation, and the load, because it reflects total uncertainty about a random quantity given that we know its lower and upper limit.\footnote{We assume that prices, loads and RE generation are all upper bounded, and their natural lower bound is 0.}

\begin{table}[h]
\centering
\caption{Simulation Scenarios}
\begin{tabular}{|c|c|}
\hline
Parameter & Value \tabularnewline \hline
$\mathcal{I}(\Delta t, M, f(0), p)$ & $[25, 1, 2, 0, 0]$ \tabularnewline \hline
$P_i(t)$ & $\sim \mathcal{U}(\text{minPrice}, \text{maxPrice})$, $i \in \{1, \ldots, M\}$ \tabularnewline \hline
$L_i(t)$ & $\sim \mathcal{U}(\text{minLoad}, \text{maxLoad})$, $i \in \{1, \ldots, M\}$ \tabularnewline \hline
$R_i(t)$ & $\sim \mathcal{U}(\text{minGen}, \text{maxGen})$, $i \in \{1, \ldots, M\}$ \tabularnewline \hline
$R(t)$ & $\sum_{i=1}^{M} R_i(t)$ \tabularnewline \hline
$q_C$ & $\max \{ \frac{\Psi}{\Delta t}, M\text{maxGen} \}$ \tabularnewline \hline
$q_D$ & $\max \{ \frac{\Psi}{\Delta t}, M\text{maxLoad} \}$ \tabularnewline \hline
$q_{C,i}$ & $\max \{ M \frac{\Psi}{\Delta t}, M\text{maxGen} \}$ \tabularnewline \hline
$q_{D,i}$ & $\max \{ M \frac{\Psi}{\Delta t}, M\text{maxLoad} \}$ \tabularnewline \hline
$\Psi_i$ & $\Psi_{i}, i \in \{1, M\}$ \tabularnewline \hline
\end{tabular}
\end{table}

We consider the simulation scenario shown in Table I, except for $R(t)$, which we choose uniformly distributed between minGen, and $M\text{maxGen}$. Other parameters are set as follows $\text{minLoad} = 1$, $\text{maxLoad} = 1$, $\text{minGen} = 0$, $\text{maxGen} = \{1, 2\}$, $\text{minPrice} = 0$, $\text{maxPrice} = 1$. Then, we plot the average energy cost incurred in $[0, T]$, and the average amount of RE unused, both against the storage size $\Psi$, which ranges from 1 to 10 [EU], in Fig. 2. As observed, in this scenario the configuration with DREG outperforms the configuration with energy farm. This follows because the variability in the RE generation is different in each configuration. In the configuration with DREG, the independence of the random variables $R_1(t)$ and $R_2(t)$ models geographical diversity. Although the average RE generation is the same in both configurations, the variance of $R(t)$ is larger than the variance of $R_1(t) + R_2(t)$. The performance gap between the two configurations is not significantly affected by the change in the RE generation capacity. It is observed, however, that the gap between the two configurations decreases as the storage capacity increases, as there is less RE unused when the storage capacity is large.

We consider the simulation scenario shown in Table I, except for $q_{C,i}$, $q_{D,i}$, $q_C$ and $q_D$, which we set as follows: $q_{C,i} = 0.2 \frac{\Psi}{\Delta t}$, $q_{D,i} = 0.2 \frac{\Psi}{\Delta t}$, $q_C = 0.2 \frac{\Psi}{\Delta t}$ and $q_D = 0.2 \frac{\Psi}{\Delta t}$. Moreover, we let $\text{minLoad} = 1$, $\text{maxLoad} = 1$, $\text{minGen} = 0$, $\text{maxGen} = \{1, 2\}$, $\text{minPrice} = 0$, and $\text{maxPrice} = 1$. Then, we plot the average energy cost incurred in $[0, T]$, and the average amount of RE unused, both against the storage size $\Psi$, which ranges from 1 to 10 [EU], in Fig. 3. As observed, in this scenario the configuration with DREG outperforms the configuration with energy farm. This follows because the maximum charging and discharging rates are higher in the configuration with DREG, i.e. $q_{C,1} + q_{C,2} > q_C$ and $q_{D,1} + q_{D,2} > q_D$. As seen in Fig. 3, lower charging/discharging rates lead to larger RE unused, and lower cost savings. However, the gap between the two strategies reduces as the storage capacity increases. If the generation capacity increases, then a larger ESD is required to achieve similar performance in both configurations. As expected, the amount of RE unused in $[0, T]$ increases with the RE generation capacity.

We consider the simulation scenario shown in Table I, except for $q_{C,i}$, $q_{D,i}$, $q_C$ and $q_D$, which we set as follows: $q_{C,i} = 0.2 \frac{\Psi}{\Delta t}$, $q_{D,i} = 0.2 \frac{\Psi}{\Delta t}$, $q_C = 0.2 \frac{\Psi}{\Delta t}$ and $q_D = 0.2 \frac{\Psi}{\Delta t}$. Other parameters are set as follows: $\text{minLoad} = 1$, $\text{maxLoad} = 1$, $\text{minGen} = 0$, $\text{maxGen} = \{1, 2\}$, $\text{minPrice} = 0$, and $\text{maxPrice} = 1$. Then, we plot the average energy cost incurred in $[0, T]$, and the average amount of RE unused,
both against the storage size $\Psi_i$, which ranges from 1 to 10 [EU], in Fig. 4. As observed, in this scenario the configuration with energy farm outperforms the configuration with DREG. This follows because the maximum charging and discharging rates are the same in both configurations, i.e., $q_{C,1} + q_{C,2} = q_C$ and $q_{D,1} + q_{D,2} = q_D$, and the configuration with energy farm has a higher level of flexibility to move RE to the location with the highest prices, while the configuration with DREG is constrained by the limit in the charging/discharging rates ($q_{C,i} = 0.2 \frac{\Psi_i}{2T}$, $q_{D,i} = 0.2 \frac{\Psi_i}{2T}$).

**Fig. 4.** CREG outperforms DREG due to larger discharging rates. Parameters: maxGen = 1 (top) and maxGen = 2 (bottom).

**VI. CONCLUSIONS**

We have proposed cooperation schemes with different energy production and storage configurations. In the first configuration we have considered a set of households with shared access to a farm, where renewable energy is generated and stored. In the second configuration we have assumed households with their own renewable energy generator and storage device. We have then proposed strategies to minimize the energy cost incurred by the participating households in each configuration. To devise our strategies, we have solved constrained optimization problems through relaxation and discretization. Simulation results have shown that the proposed strategies can lead to significant cost savings in both schemes. We have also illustrated the conditions under which one configuration outperforms the other. All else unchanged, the configuration with distributed generation was able to outperform the configuration with energy farm due to statistical differentiation across generators. In general, the configuration with higher energy management flexibility (in terms of charging/discharging rates) was shown to have the greatest savings potential. The strategies proposed can be used to devise online energy management algorithms by incorporating forecasting techniques to estimate future energy production and power consumption.

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