A PLUG AND PLAY BAYESIAN ALGORITHM FOR SOLVING MYOPE INVERSE PROBLEMS

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ABSTRACT

The emergence of efficient algorithms in variational and Bayesian frameworks braught significant advances to the field of inverse problems. However, such problems remain challenging when the observation operator is not perfectly known. In this paper we propose a Bayesian Plug-and-Play (PP) algorithm for solving a wide range of inverse problems where the signal/image is sparse in the original domain and the observation operator has to be estimated. The principle consists of plugging the prior considered for the target observation operator and keep using the same algorithm. The proposed method relies on a generic proximal non-smooth sampling scheme. This genericity makes the proposed algorithm novel in the sense that it can be used to solve a wide range or inverse problems. Our method is illustrated on a deblurring problem with unknown blur operator where promising results are obtained.

***Index Terms*—** MCMC, ns-HMC, proximity operator, myope inverse problems

1. INTRODUCTION

The inverse problem literature shows continuous developments in finding the best estimation methods for many image processing applications including denoising, restoration and deconvolution. These developments are mainly due to the emergence of a new generation of variational [1–4] and Bayesian [5–8] optimization algorithms. These algorithms have been used to solve various inverse problems like image deconvolution [7, 9, 10] or reconstruction [11–14]. When the observation operator is not perfectly known, the problem becomes myope and requests specific algorithms that have been investigated for medical imaging [15–17], astronomy [18] or remote sensing [19]. Generally speaking, estimating both the target signal and the observation operator directly from the data, in addition to the regularization hyperparameters is a difficult task. Moreover, convergence issues can limit the interest of some methods like those based on alternating minimization. More recently, the inverse problem literature has faced the emergence of a new generation of regularization algorithm called Plug-and-Play (PP). The concept is to plug any denoiser that can be used within the algorithm steps. This has raised the issue of convergence guarantees, which means that this new research area is still looking for well established algorithms that enjoy solid theoretical guarantees [20, 21].

In this paper, we propose a Bayesian PP algorithm that allows solving inverse problems where the observation operator is also unknown. In order to design a fully automatic method, a Bayesian model is adopted to build a hierarchical Bayesian model. The main contribution of this paper lies in the genericity of the proposed algorithm in the sense that the user has only to plug the prior adopted to model the observation operator. The proposed algorithm also enjoys good convergence properties inherited from the convergence guarantees of Markov chains. Indeed, the properties of the observation operator can dramatically change from an inverse problem to another. This generally implies big differences between Bayesian models developed to solve each problem. For instance, the observation operator in parallel magnetic resonance imaging is complex-valued [12], while the point spread function (PSF) for two photon microscopy is real positive [22]. Using the recently proposed general non-smooth Hamiltonian Monte Carlo (ns-HMC) [23, 24] sampling scheme, the sampling of the observation operator can be done in the same way for a wide range of inverse problems, provided that the adopted prior belongs to the family of exponential probability density functions. The proposed method is therefore applicable even for non-linear inverse problems with possibly high-dimensional observation operators for which standard schemes suffer from a high computational cost and poor convergence speed. Moreover, if other efficient sampling strategies such as standard HMC [6] or Langevin-based [25] are used, they cannot be applied to priors with non-smooth energies, which can be done with the proposed method.

As regards the target signal, we limit our focus (without loss of generality) to inverse problems where the target signal/image is sparse in the original domain. A Bernoulli-Laplace model is used to model this sparsity [26]. We also consider problems for which the acquisition noise is Gaussian for illustration purpose.

The rest of the paper is organized as follows. Section 2 describes the problem formulation. In Section 3 we detail the adopted hierarchical Bayesian model. The PP algorithm investigated in this work is described in Section 4. Section 5

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evaluates the performance of the proposed algorithm for image deconvolution. Conclusions and future work are finally reported in Section 6.

2. PROBLEM FORMULATION

Let $\boldsymbol{x} \in \mathbb{R}_+^N$ be the target signal (or vectorized image) which has to be recovered from an observation $\boldsymbol{y} \in \mathbb{R}^N$. This observation is defined as a perturbation of $\boldsymbol{x}$ by an observation operator $\mathcal{K} \in \mathbb{R}^{k \times k}$ and an additive Gaussian noise $\boldsymbol{n}$ with variance σ_n^2. The observation model can therefore be summarized as

$$\boldsymbol{y} = T(\mathcal{K}, \boldsymbol{x}) + \boldsymbol{n} \tag{1}$$

where $T(\cdot, \cdot)$ denotes the possibly non-linear function that links the observation operator $\mathcal{K}$ to the target signal $\boldsymbol{x}$. For linear inverse problems, this function is simply expressed as $T(\mathcal{K}, \boldsymbol{x}) = \mathcal{K}\boldsymbol{x}$. Since such a problem is generally ill-posed, regularization can help to recover a stable solution through using adequate prior information. In this paper, we handle a particular family of ill-posed inverse problems where the observation operator $\mathcal{K}$ is not perfectly known. Specifically, we handle myope problems where the target signal is estimated from a vague knowledge about the observation operator. Moreover, we adopt a Bayesian framework in order to design a fully automatic approach where the model parameters and hyperparameters are directly estimated from the data.

3. HIERARCHICAL BAYESIAN MODEL

Following a probabilistic approach, the target and observed signals (resp. $\boldsymbol{x}$ and $\boldsymbol{y}$) are assumed to be realizations of random vectors (resp. $\boldsymbol{X}$ and $\boldsymbol{Y}$). The core of our method will be to characterize the probability distribution of $\boldsymbol{X}|\boldsymbol{Y}$, by considering a parametric probabilistic model and by estimating the associated hyperparameters.

3.1. Likelihood

Since the observation noise is additive and Gaussian with variance σ_n^2, the model likelihood can be expressed as

$$f(\boldsymbol{y}|\boldsymbol{x}, \sigma_n^2) = \left(\frac{1}{2\pi\sigma_n^2}\right)^{N/2} \exp\Big(- \frac{\|\boldsymbol{y} - T(\mathcal{K}, \boldsymbol{x})\|_2^2}{2\sigma_n^2}\Big) \tag{2}$$

where $\|.\|_2$ is the Euclidean norm.

3.2. Priors

In our model, the unknown parameters are gathered in the unknown vector $\boldsymbol{\theta} = \{\boldsymbol{x}, \mathcal{K}, \sigma_n^2\}$.

Prior for σ_n^2

In order to use a non-informative prior for σ_n^2 while ensuring positive values, a Jeffrey's prior is considered for σ_n^2 defined as [5]

$$f(\sigma_n^2) \propto \frac{1}{\sigma_n^2} 1_{\mathbb{R}^+}(\sigma_n^2) \tag{3}$$

where $1_{\mathbb{R}^+}$ is the indicator function on $\mathbb{R}_+$, i.e., $1_{\mathbb{R}_+}(\xi) = 1$ if $\xi \in \mathbb{R}_+$ and 0 otherwise.

Prior for $\boldsymbol{x}$

We assume that the signal coefficients x_i are *a priori* independent. The prior distribution for $\boldsymbol{x}$ can be written as

$$f(\boldsymbol{x}|\omega, \lambda) = \prod_{i=1}^{N} f(x_i|\omega, \lambda). \tag{4}$$

In this paper we focus on a category of inverse problems where the target signal/image is sparse in the original space. In order to inforce sparse real valued signals, we consider a Bernoulli-Laplace prior for each x_i as in [26], defined by

$$f(x_i|\omega, \lambda) = (1 - \omega)\delta(x_i) + \frac{\omega}{2\lambda} \exp\left(-\frac{|x_i|}{\lambda}\right) \tag{5}$$

where $\delta(.)$ is the Dirac delta function, $\lambda > 0$ is the parameter of the Laplace distribution, and $w \in [0, 1]$ is a parameter weighting the contribution of the non-zero signal coefficients. Note that for positive real-valued signals, an exponential distribution can be used instead of the Laplace one in (5) akin to [27]. It is worth noticing that this model can still be used for signals that are sparse in some transform domain.

Prior for $\mathcal{K}$

For myope inverse problems, the observation operator is not perfectly known. In this paper, we consider the generic case where no accurate knowledge about the observation operator is available, and where only a prior information can be defined as a member of an exponential family, i.e.,

$$f(\mathcal{K}|\varphi) = C(\varphi) \exp\Big(- g(\mathcal{K}, \varphi)\Big), \tag{6}$$

where φ is the set of involved hyperparameters.

3.3. Hyperpriors

The model hyperparameters are gathered in the vector $\boldsymbol{\Phi} = \{\lambda, \omega, \varphi\}$. As already used in a number of recent works [26, 27], a non-informative version (by setting $\alpha = \beta = 10^{-3}$) of the inverse gamma distribution $IG(\lambda|\alpha, \beta)$ is used for λ.
As regards the weight parameter ω, we simply consider a uniform prior on the interval $[0, 1]$ denoted as $\omega \sim \mathcal{U}_{[0,1]}(\omega)$. A more informative version can also be used if further information is available for ω. Finally, for the hyperparameter φ, an appropriate prior has to be set according to the problem.

4. PLUG-AND-PLAY ALGORITHM

Adopting a maximum a posteriori (MAP) strategy to estimate both the parameter and hyperparameter vectors $\boldsymbol{\theta}$ and $\boldsymbol{\Phi}$, we combine the model likelihood, priors and hyperpriors in order to derive the joint posterior distribution of $\{\boldsymbol{\theta}, \boldsymbol{\Phi}\}$ that can be expressed as

$$f(\boldsymbol{\theta}, \boldsymbol{\Phi}|\boldsymbol{y}, \alpha, \beta) \propto f(\boldsymbol{y}|\boldsymbol{\theta}) f(\boldsymbol{\theta}|\boldsymbol{\Phi}) f(\boldsymbol{\Phi}|\alpha, \beta), \tag{7}$$

which can be reformulated in a detailed version as

$$f(\boldsymbol{\theta}, \boldsymbol{\Phi}|\boldsymbol{y}, \alpha, \beta) \propto \left(\frac{1}{\sigma_n^2}\right)^{N/2} \exp\Big(-\frac{\|\boldsymbol{y} - T(\mathcal{K}, \boldsymbol{x})\|_2^2}{2\sigma_n^2}\Big)$$
$$\times \prod_{i=1}^{N} \left[(1-\omega)\delta(x_i) + \frac{\omega}{2\lambda}\exp\left(-\frac{|x_i|}{\lambda}\right)\right]$$
$$\times \frac{1}{\sigma_n^2} 1_{\mathbb{R}^+}(\sigma_n^2) \times \mathcal{U}_{[0,1]}(\omega) \times C(\varphi)\exp\Big(-g(\mathcal{K}, \varphi)\Big). \quad (8)$$

In order to handle the posterior in (8) which has a complicated form, a Gibbs sampler is built following many recent works [5, 8, 28]. This sampler is based on sequential iterations of sampling according to the conditional distributions $f(\boldsymbol{x}|\boldsymbol{y}, \omega, \lambda, \mathcal{K}, \sigma_n^2)$, $f(\sigma_n^2|\boldsymbol{y}, \boldsymbol{x}, \mathcal{K})$, $f(\mathcal{K}|\varphi, \boldsymbol{y}, \boldsymbol{x})$, $f(\lambda|\boldsymbol{x}, \alpha, \beta)$, $f(\omega|\boldsymbol{x})$ and $f(\varphi|\mathcal{K})$.

The sampling steps that have to be used for $\boldsymbol{x}, \omega, \sigma_n^2$ and λ can be found in many studies including [26,27]. For the hyperparameter vector φ, the conditional distributions to sample from have to be derived based on the likelihood and the adopted priors.

The conditional distribution of the observation operator $\mathcal{K}$ can be expressed as

$$f(\mathcal{K}|\varphi, \boldsymbol{y}, \boldsymbol{x}) \propto \exp\left(-E_{\varphi,\sigma_n^2}(\mathcal{K})\right) \quad (9)$$

where $E_{\varphi,\sigma_n^2}(\mathcal{K}) = -g(\mathcal{K}, \varphi) - \frac{\|\boldsymbol{y} - T(\mathcal{K}, \boldsymbol{x})\|_2^2}{2\sigma_n^2}$ is the energy of the conditional posterior in (9).

We propose here a sampling scheme that allows $\mathcal{K}$ to be sampled for all possible exponential priors. Specifically, we propose to use a Metropolis-Hastings (MH) -based move for this sampling. However, to bypass the difficulties due for instance to the large size of the operator, we resort to a non-smooth Hamiltonian Monte Carlo (ns-HMC) scheme recently investigated in [6, 23, 24]. It has been established in the recent literature that ns-HMC allows us to efficiently sample multidimensional distributions with high acceptance ratios in comparison to the standard MH algorithm. We recall here that the standard ns-HMC relies on Hamiltonian dynamics to sample from the target distribution $f(\mathcal{K}|\varphi, \boldsymbol{y}, \boldsymbol{x})$ and extends the standard HMC algorithm by resorting to the concept of proximity operator [29]. This scheme has recently been generalized by performing a Bayesian calculation of the target energy proximity operator [24]. In this paper, we use this principle with the constructed hierarchical Bayesian model to design a generic PP Bayesian regularization algorithm. This algorithm can be configurated according to the used prior for the observation operator by setting the energy function g in E_{φ,σ_n^2}. The resulting Gibbs sampler is summarized in Algorithm 1. After convergence, Algorithm 1 provides chains of coefficients sampled according to the target parameters and hyperparameters. These chains can be used to compute an MMSE (minimum mean square error) estimator (after discarding the samples corresponding to the burn-in period) for $\widehat{\boldsymbol{x}}$ and $\widehat{\mathcal{K}}$, in addition to the hyperparameters $\widehat{\lambda}$, $\widehat{\sigma_n^2}$, $\widehat{\omega}$ and $\widehat{\varphi}$.

5. APPLICATION TO IMAGE DECONVOLUTION

In this section we apply the proposed algorithm to a myope image deconvolution problem. We first set the prior model of the PSF as

$$f(\mathcal{K}|\overline{\mathcal{K}}, \sigma_k^2) \propto \left(\frac{1}{2\pi\sigma_k^2}\right)^{k^2/2} \exp\left(-\frac{||\mathcal{K} - \overline{\mathcal{K}}||_2^2}{2\sigma_k^2}\right) \quad (10)$$

where $\overline{\mathcal{K}}$ is an approximation of the PSF that could be *a priori* estimated or calibrated, and σ_k^2 is the prior variance. The proposed PP algorithm can therefore be configurated by setting $g(\mathcal{K}) = \frac{||\mathcal{K} - \overline{\mathcal{K}}||_2^2}{2\sigma_k^2}$ in (6).

Using this prior, and assuming that $\overline{\mathcal{K}}$ is fixed, the hyperparameter vector φ reduces to the hyperparameter σ_k^2, for which a Jeffrey's prior can be set akin to σ_n^2. The conditional posterior of σ_k^2 can therefore be expressed as

$$\sigma_k^2|\mathcal{K} \sim \mathcal{IG}\Big(\sigma_k^2|N/2, ||\mathcal{K} - \overline{\mathcal{K}}||_2^2\Big). \quad (11)$$

Algorithm 1: Proposed Plug-and-Play algorithm.

- Initialize with some $\boldsymbol{x}^{(0)}$ and $\mathcal{K}^{(0,0)}$;
- Set the iteration number $r = 0$, L_f and ϵ;
- Compute $P_0 = \mathrm{prox}_{E_{\varphi,\sigma_n^2}(\mathcal{K})}(\mathcal{K}^{(0,0)})$ using the Bayesian algorithm in [24, Algorithm 2];

while *not convergence* **do**
- Sample σ_n^2 according to $f(\sigma_n^2|\boldsymbol{y}, \boldsymbol{x}, \mathcal{K})$;
- Sample φ according to $f(\varphi|\mathcal{K})$;
- Sample λ according to $f(\lambda|\boldsymbol{x}, \alpha, \beta)$.;
- Sample ω according to $f(\omega|\boldsymbol{x})$.;
- Sample $\boldsymbol{x}$ according to $f(\boldsymbol{x}|\boldsymbol{y}, \omega, \lambda, \mathcal{K}, \sigma_n^2)$.;
- Sample $\mathcal{K}$ according to $f(\mathcal{K}|\varphi, \boldsymbol{y}, \boldsymbol{x})$ as follows

begin
* Sample $\boldsymbol{q}^{(r,0)} \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{I}_N)$;
* Compute $\boldsymbol{q}^{(r,\frac{1}{2}\epsilon)} = \boldsymbol{q}^{(r,0)} - \frac{\epsilon}{2}\left[2\mathcal{K}^{(r-1,0)} - \mathcal{K}^{(0,0)} - P_0)\right]$;
* Compute $\mathcal{K}^{(r,\epsilon)} = \mathcal{K}^{(r-1,0)} + \epsilon \boldsymbol{q}^{(r,\frac{1}{2}\epsilon)}$;

for $l_f = 1$ *to* $L_f - 1$ **do**
• Compute $\boldsymbol{q}^{(r,(l_f+\frac{1}{2})\epsilon)} = \boldsymbol{q}^{(r,l_f\epsilon)} - \frac{\epsilon}{2}\left[2\mathcal{K}^{(r,l_f\epsilon)} - \mathcal{K}^{(0,0)} - P_0\right]$;
• Compute $\mathcal{K}^{(r,(l_f+1)\epsilon)} = \mathcal{K}^{(r,l_f\epsilon)} + \epsilon \boldsymbol{q}^{(r,(l_f+\frac{1}{2})\epsilon)}$;

end
* Compute $\boldsymbol{q}^{(r,(L_f+\frac{1}{2})\epsilon)} = \boldsymbol{q}^{(r,L_f\epsilon)} - \frac{\epsilon}{2}\left[2\mathcal{K}^{(r,L_f\epsilon)} - \mathcal{K}^{(0,0)} - P_0\right]$;
* Apply the MH acceptation rule to $(\mathcal{K}^*, \boldsymbol{q}^*)$ with $\boldsymbol{q}^* = \boldsymbol{q}^{(r,\epsilon L_f)}$ and $\mathcal{K}^* = \mathcal{K}^{(r,\epsilon L_f)}$;

end

end

The following sections illustrate the deconvolution results obtained for synthetic and real data.

5.1. Simulated data

In this section, a reference image $\boldsymbol{x}^0 \in \mathbb{R}^{32\times 32}$ is used to simulate a blurred observation using a Gaussian PSF of size 3×3 and a Gaussian noise of variance $\sigma_n^2 = 1$. The ground truth and observed images are displayed in Fig. 1.

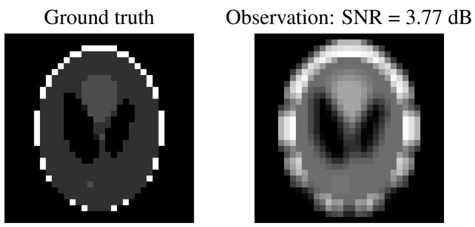

Fig. 1. Ground truth and observed images.

As initialization, the observed image and a uniform PSF have been used for the target image and the observation operator, respectively.
Fig. 2 illustrates the deblurred images using the proposed method (a), the blind maximum likelihood (b) [30], regularized filter (c) [31] and Lucy-Richardson (d) [32] deconvolutions. A visual inspection of the obtained images shows that the proposed method provides the lowest blur level with respect to the other methods. Some quantitative results in terms of signal to noise ratio (SNR) and structural similarity (SSIM) values are reported in Tab. 1. These values demonstrate the good performance of the proposed method providing an accurate image that best fits the ground truth (see Fig. 2(a). The estimated and actual PSFs are illustrated in Fig. 3. The SNR value of the estimated PSF with respect to the ground truth is also provided to indicate the good estimation precision of our method. Regarding the computational time, 100 iterations were enough to reach convergence within 60 seconds.

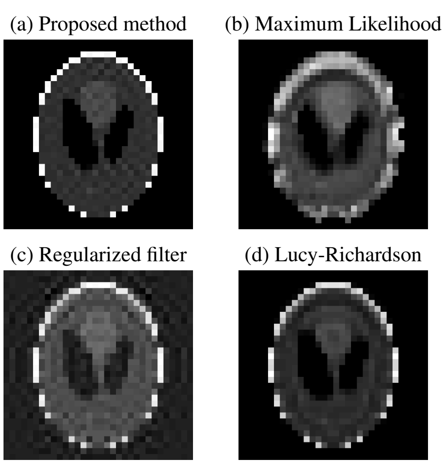

Fig. 2. Deblurred images using (a) our method, (b) blind maximum likelihood, (c) regularized filter and (d) Lucy-Richardson deconvolutions.

Table 1. SNR (dB) and SSIM for the competing methods.

	Prop. meth.	Blind M. L.	Reg. filter	L.-R.
SNR	24.58	4.02	12.45	15.31
SSIM	0.978	0.586	0.914	0.941

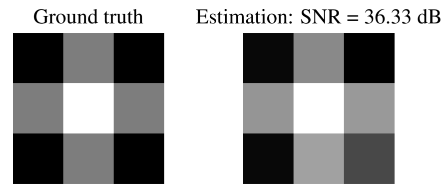

Fig. 3. Ground truth and estimated PSF.

5.2. Two-photon microscopy data

In this section, deblurring of two-photon microscopy data is performed using the proposed method. Two-photon microscopy [22] is among the most recent cell imaging techniques. Due to the deep penetration level, the noise level is generally lower that single photon-based microscopy. However, the collected images still suffer from a high blur level. The observed 3D data is of size $221 \times 247 \times 14$, acquired with 5 channels, 14 slices and 50 frames. Fig. 4(a) displays a 2D 50×50 patch that has been deblurred using the proposed method. The deblurred image is displayed in Fig. 4(b), in addition to the deblurred images using the blind maximum likelihood Fig. 4(c) and Lucy-Richardson Fig. 4(d) methods. The sparsity levels evaluated using the ℓ_0 pseudo-norm are also reported in Fig. 4. These values clearly indicate that the proposed method provides sparser estimation, which is an interesting property.

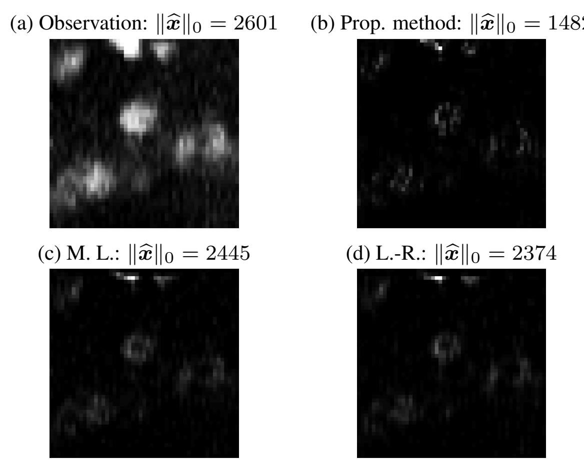

Fig. 4. (a) Observed and deburred images using (b) the proposed method, (c) blind maximum likelihood, and (d) Lucy-Richardson deconvolutions.

6. CONCLUSION

In this paper, we proposed a Bayesian Plug and Play algorithm for solving sparse inverse problems where the target signal/image is sparse in the original domain, and where the observation operator is not perfectly known. The main novelty of the proposed algorithm is that it can be applied to a wide set of inverse problems and has simply to be configurated by setting the energy function of the adopted prior on the observation operator. The efficiency of this algorithm, especially for high-dimensional observation operators is due to the use of the general ns-HMC sampling. Results obtained after applying our algorithm to an image deconvolution problem demonstrate its ability to accurately recover the target image in addition to a clean version of the observation operator.

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