Effective Network Area for Efficient Simulation of Finite Area Wireless Networks

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***Abstract*—A wide-spread approach for modeling and for the performance evaluation of wireless networks is employing Poisson point processes (PPPs). There, the general assumption is that an infinite number of nodes is distributed in the network environment. This is not problematic in a purely analytic framework, but is unfeasible when Monte-Carlo system level simulations are employed for network evaluation. In order to obtain results in a finite simulation-duration, the simulation area also has to be finite, which leads to a deviation of performance results when compared to analytical approach with infinitely many base stations. This deviation however is only small when the simulation area is sufficiently large. In this paper we discuss which part of the simulation area yields results whose deviation from the analytical results does not exceed a predefined threshold. We present an approximation that is based on the base station geometry and depends on the base station density. Additionally, we discuss the relationship of the minimal simulation area (the smallest area that allows to obtain reliable coverage results) and the reduced simulation overhead by increasing the simulation area.**

***Index Terms*—System level simulations, finite area networks, point processes, wireless cellular networks**

I. Introduction

The global performance of a wireless cellular network is commonly assessed by analytical models and also simulations on system level. Therefore, a large number of network nodes (users and base stations) is considered, while abstracting the actual physical link by suitable propagation models. The average performance is then determined by combining all individual links - desired and interfering - into the signal to interference ratio (SIR) and averaging over all spatial and temporal constellations.

The SIR statistics of a network largely depend on the base station (BS) locations, which are generally abstracted by a baseline model. A recent approach to evaluate the SIR distribution of a cellular network is to apply point process theories, leveraging techniques from stochastic geometry [1]. The Poisson point process (PPP) modeling of networks is the most popular choice for abstracting the BS locations due to its simplicity and tractability [2–4]. The accuracy of the PPP model in abstracting realistic BS deployments has been validated in numerous contributions [5–7]. Applying a PPP for modeling a cellular network is however based on the assumption of an infinite number of BSs in the network environment.

The counterpart to an analytical evaluation of the network performance is using system level simulations. Such simulations are realized as a Monte-Carlo experiment by randomly placing BSs for each spatial realization of the network [8, 9]. Since an infinite number of BSs cannot be simulated in finite time, the number of BSs placed for every spatial realization is finite, and so is correspondingly the area of the considered region of interest (ROI). This of course leads to a deviation with respect to the analytically attained SIR distribution. Due to the decay of interference power over distance, the contribution of the *closest* interfering BS is also dominating the SIR distribution, and therefore, the aforementioned discrepancy between analysis and simulation result becomes negligible, as long as the distance of the user position from the border of the ROI is large enough. When setting up simulations, it has to be known a priori which size of the considered ROI is sufficient to produce reliable results and also for which part of the area this is true. In this work, we investigate this relation between deviation of SIR distribution and network area and dependent on the path loss model and the BS density, attain results for the usable part of the ROI. Since results for the effective network area are not available in closed form we provide approximations that are based on the distance distribution of interferers.

In order to obtain statistically significant results from Monte-Carlo simulations, a sufficient amount of random realizations has to be simulated. Since all user locations within the usable area of the ROI yield reliable results (with respect to the SIR distribution), several simulation points can be obtained from one spatial realization. This corresponds to a trade-off between the size of one spatial realization and the necessary total amount of realizations. We therefore discuss in this paper how an increased ROI can lead to a decrease in simulation overhead and simulation time, as long as the capabilities of the simulation hardware are not exceeded.

II. System Model

In this section, we present the system model to investigate the coverage distribution of a user with respect to its distance to the center of the ROI. We consider downlink transmission in a finite network area. The BSs locations in a ball $B \subset \mathbb{R}^2$ form a Binomial Point Process (BPP) of Φ, when N BSs

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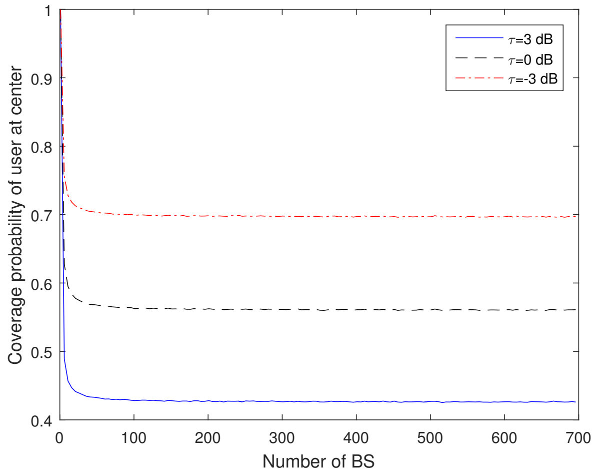

Fig. 1: SIR coverage probability of a user at origin under varying number of BSs.

are independently scattered in the finite network. We assume that $B(o, r_{\mathrm{d}})$ forms a circular network centered at o and with radius r_{d}. Since the area of nodal deployment is finite, the arranged point process is non-stationary, which means that the network characteristics in a node, such as the SIR distribution depends on the location of the node. Furthermore, while modeling cellular networks by a PPP, the BS locations are independent of each other, this is not the case when applying a BPP for generating BSs in the network. This means that the distance distribution between a user and its first and second nearest BS are not independent.

The term τ denotes the tier's SIR threshold. More precisely, a mobile user can reliably communicate with a BS, only if the downlink SIR of the BS at the mobile user is greater than τ. The distance based path loss function is $l(x) = \|x\|^{-\alpha}$, where α is the path loss exponent.

Considering N as the number of BSs in the network, x_i as the distance between the user and its i th nearest BS, and h_{x_i} as the fading between the user and its i th nearest BS, the downlink SIR of a user (including all $N-1$ interferers) is defined by

$$S(N) = \frac{h_{x_1} x_1^{-\alpha}}{\sum_{j=2}^{N} h_{x_j} x_j{}^{-\alpha}}. \tag{1}$$

Under a BPP for BS placement, the number of BSs N is a finite value while $N \to \infty$, when using a PPP. Figure 1 reveals the effect of increasing the number of BSs on the coverage probability of a user located at the center point of the network. The coverage probability will decrease for growing numbers of BSs and slowly approaches its limit. In other words for a large value of N we have $S(N) \simeq S(\infty)$.

III. Distance Distribution

When considering a finite network area, the SIR distribution of a mobile user is a function of its distance from the center point. By moving away from the center towards the border of the network area, the average distance from the user to the interfering BSs is increasing as well. Hence, the closer a user is to the border of the network, the smaller the interference it experiences. This is not true however for the analytical evaluation of the network performance using a PPP (which is based on the assumption of infinite network area) [10]. There, the SIR distribution is independent of the user location due to the stationary property of PPP based models. This leads to a divergence between the results of Monte-Carlo simulations (with finite simulation area) and the analytically obtained results. The authors in [10, 11] provide a distance distribution between the user and nearest BS in a finite network area.

To identify the *effective* network area of our simulations, we perform Monte-Carlo simulations with a finite, circular ROI. Due to symmetry, we only consider the distance of the user from the center, not its absolute location. When moving the user from the center point of network towards the border and evaluate the coverage probability, the resulting value changes with distance. This is shown in Figure 2. As expected, only for a portion of the network, the coverage probability of a user located in the center is similar to the coverage probability of a user located at a certain distance. We call this portion *effective* network area. In other words, the *effective* network area is the portion of the network where the difference between the SIR distribution of users located at any point of this area and the SIR distribution of a user located in the center of the network is negligible. The radius of this area corresponds to the flat part of the results-curve in Figure 2.

When moving the user further away from the center, we observe an increase in coverage probability due to the decreasing aggregate interference - the interferer constellation becomes more and more asymmetric and their distance distribution becomes skewed towards larger distances. For even larger distances, another effect comes into play. When the user position is considerably close to the simulation network border, not only will the expected distance to its interferers increase, but also to its associated BS. This manifests in a decrease of the coverage probability, as it can be seen in the figure. In summary, due to the aforementioned effects, the SIR distribution of a user depends hugely on its distance from the center for a finite network area and thus makes the identification of the usable, *effective* network are necessary.

In order to utilize the size of the effective network radius in simulations, it would be advantageous to have access to analytical expression of this size in *closed form*. In general however, a closed form relation for the flat part of the coverage probability is unavailable. Hence, we provide an approximation to estimate the distance of the user from the center, that still does not deviate significantly from the coverage probability in the center. Our approximation is solely based on the geometry of users and BSs and does not depend on, e.g., the path loss model. This is left for future work.

Let us first investigate the effect of interferers on the SIR of a user. When the exact SIR of a user is obtained by computing the value of $S(N)$, the impact of the interferers on the SIR is not the same, i.e., closer interferers have a

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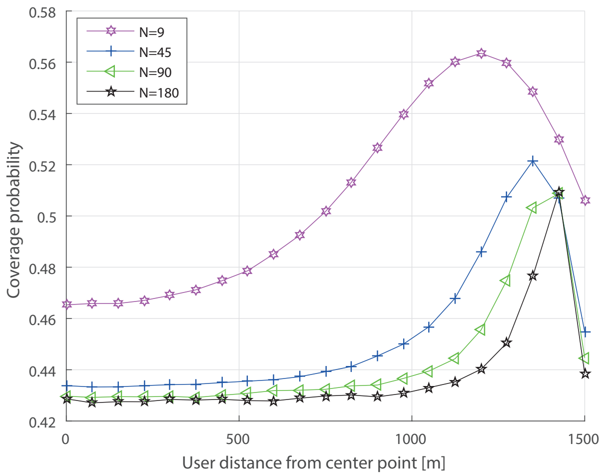

Fig. 2: Coverage probability of user with respect to its distance from origin.

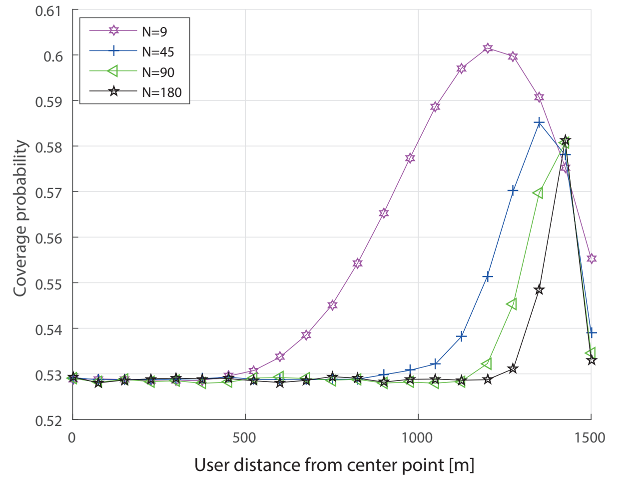

Fig. 4: Coverage probability by considering the effect of one interferer ($\alpha = 4, \tau = 3$ dB).

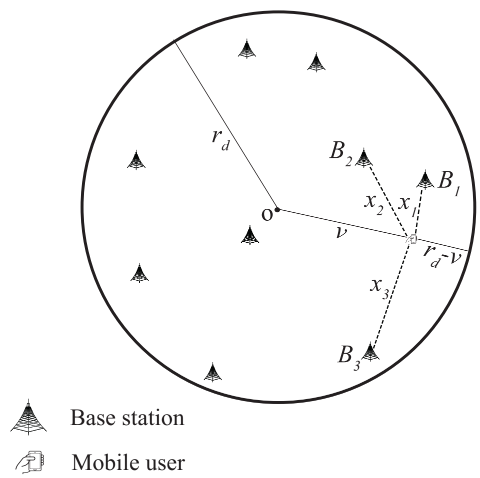

Fig. 3: Coverage probability by considering the effect of one interferer.

higher impact on the SIR. Consider the extreme case when we include the effect of only one interferer, i.e., $S(2)$, into the analysis. In other words, in Figure 3 by associating the user to the nearest BS B_1, we just consider the effect of the closest interferer B_2 in computation of the SIR. The resulting coverage probability with accounting only for the closest interferer is shown in Figure 4. Compared to the results in Figure 2, it can be observed that the absolute value is changed, but the behavior w.r.t. divergence from the result in the center position is preserved. Also the effect of first rising and then receding coverage probability when coming closer to the network boundary is still present. We thus utilize this simplification in order to find the approximate size of the effective network radius.

Figure 3 shows a possible realization of the network when the mobile user is located at distance v of the origin. In Figure 4 the flat part of the coverage probability results is the part, from the center point of network to the point where the distance from the user to its nearest interferer x_2, becomes larger than its distance to the border of the network $r_d - v$.

Theorem 1. *Consider V_1 denoting the user position where the distance of the user to the first interferer becomes larger than its distance to the border of the network. The expectation of the distance from the origin to point V_1 is computed as*

$$d_{V_1} = 3(N-1)N2^N r_{\mathrm{d}} \Gamma(N)\, {}_2\bar{F}_1\left[2-N, N, 4+N, \frac{1}{2}\right], \quad (2)$$

where ${}_2\bar{F}_1[.]$ denotes the regularized hypergeometric function.

Proof. Let us first compute the cumulative distribution function (CDF) of the distance from the user to the nearest interferer x_2:

$$F_{x_2}(x) = 1 - \mathbb{P}[x_2 > x] = \left(1 - \frac{x^2}{r_{\mathrm{d}}^2}\right)^N + N\left(\frac{x^2}{r_{\mathrm{d}}^2}\right)\left(1 - \frac{x^2}{r_{\mathrm{d}}^2}\right)^{N-1}.$$

Replacing x with the distance from the network border $r_{\mathrm{d}} - v$ leads to

$$F_{x_2}(r_{\mathrm{d}} - v) = \left(1 - \frac{(r_{\mathrm{d}}-v)^2}{r_{\mathrm{d}}^2}\right)^N + N\left(\frac{(r_{\mathrm{d}}-v)^2}{r_{\mathrm{d}}^2}\right)\left(1 - \frac{(r_{\mathrm{d}}-v)^2}{r_{\mathrm{d}}^2}\right)^{N-1}.$$

Deriving this CDF with respect to v yields

$$f_{x_2}(r_{\mathrm{d}} - v) = \frac{dF_{x_2}(r_{\mathrm{d}} - v)}{dv} = \left(\frac{2N(N-1)(1 - \frac{(r_{\mathrm{d}}-v)^2}{r_{\mathrm{d}}^2})^{N-2}(r_{\mathrm{d}} - v)^3}{r_{\mathrm{d}}^4}\right).$$

The expectation of the distance from the user and the center point where the distance of the user to the first interferer becomes larger than its distance to the border of the network is then given by

$$d_{V_1} = \int_0^{r_{\mathrm{d}}} v f_{x_2}(r_{\mathrm{d}} - v)\, dv,$$

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TABLE I: Percentage of usable radius obtained via analytical results.

r_d	**Theorem 1**	**Theorem 2**	**simulation results**
r_d = 1000 m	39 %	24 %	27 %
r_d = 5000 m	86 %	83 %	78 %
r_d = 10 000 m	93 %	92 %	89 %
r_d = 15 000 m	95 %	94 %	92.5 %

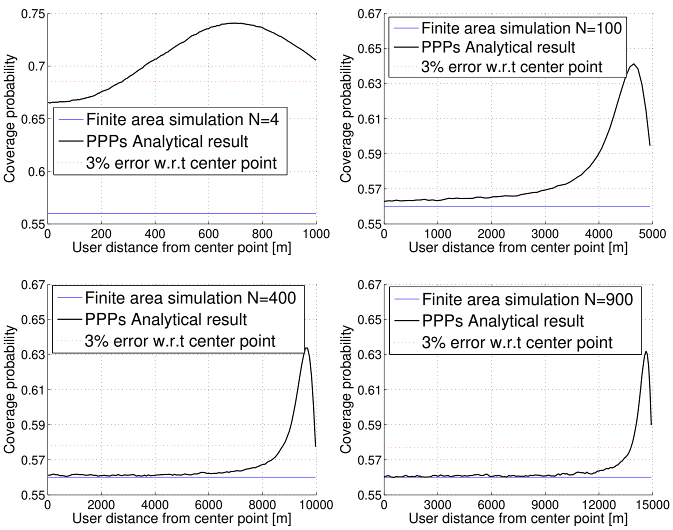

Fig. 5: Coverage probabilities of user under different network radius ($\alpha = 4, \tau = 0$ dB).

by solving the integral, we get (2) □

Theorem 2. *Let V_2 represent a user position for which the distance of the user to the second interferer becomes larger than its distance to the border of network. The expectation of the distance from the origin to point V_2 is computed as*

$$d_{V_2} = 15(N-2)(N-1)N2^N r_\mathrm{d} \cdot \Gamma(N-1)\, {}_2\bar{F}_1\left[3-N, N-1, 5+N, \frac{1}{2}\right]. \quad (3)$$

Proof. Let us first compute the CDF of the distance from the user to the second nearest interferer x_3:

$$F_{x_3}(x) = 1 - \mathbb{P}[x_3 > x] = \left(1 - \frac{x^2}{r_\mathrm{d}^2}\right)^N + N\left(\frac{x^2}{r_\mathrm{d}^2}\right)\left(1 - \frac{x^2}{r_\mathrm{d}^2}\right)^{N-1} + \frac{N(N-1)}{2}\left(\frac{x^2}{r_\mathrm{d}^2}\right)^2\left(1 - \frac{x^2}{r_\mathrm{d}^2}\right)^{N-2},$$

by following the same steps as in Theorem 1 we reach (3). □

IV. Efficiency Trade-off

In order to obtain reliable results by Monte-Carlo simulations, enough samples have to be acquired to provide statistical significance. Often, a user-centric approach is chosen and for each spatial realization, only the resulting SIR in the center will be stored. A way to reduce the overhead is to evaluate the performance at several user positions. Thus, the generated BSs serve two purposes - once as serving BS and otherwise as interferers. Additionally, several users have to be considered when not only the SIR at user locations is to be investigated, but also further network functionalities such as scheduling, feedback and throughput performance. This larger complexity will also lead to a larger overhead reduction when BSs are not only dummies for generating interference.

For our simulations with finite network area, only results from users whose position is *inside* the effective network area will be taken into account. Results obtained for user locations outside of this area are not reliable due to the aforementioned effects of skewed distance distributions for associated BS and interferers.

Following this argumentation, we conclude that there is a minimum size of ROI, which corresponds to the smallest radius where the deviation of the coverage probability in the center from the analytical results does not exceed a predefined value. Only then we obtain reliable simulation results. For radii exceeding this minimum value, the ratio of the effective network area w.r.t. the complete simulation area and thus the efficiency of the simulation is increased, i.e., less spatial realizations are necessary to obtain the same amount of result samples. In the extreme case of $r_d \to \infty$ only one spatial realization would be sufficient since it would include all possible BS and user constellations. As already discussed before, an infinite network cannot be simulated and thus the upper limit in this case for increasing the size of the ROI is given by the simulation hardware, especially the available memory/RAM.

To demonstrate the relation of the effective network area and the simulation area, we fix the BSs distribution density and change the network radius from $r_\mathrm{d} = 1000\,\mathrm{m}$ to $r_\mathrm{d} = 15\,000\,\mathrm{m}$. Figure 5 shows that the coverage probability stays rather constant for larger distances from the center. As a reference, the theoretical result for a network with infinite area is also displayed there, which is constant over distance from the center. We also observe, that the first simulated network is not large enough for the given parameters such that the simulation result in the center would overlap with the theoretical result. For $r_d = 5000\,\mathrm{m}$, this is almost the case, while for larger radii, there is almost no deviation visible.

The increase of the proportion of the radius of the effective network area is shown in Table I. When at most $3\,\%$ error of the simulated average coverage probability w.r.t. the value at the center point is acceptable, then the proportion grows from $27\,\%$ for $r_\mathrm{d} = 1000\,\mathrm{m}$ up to $92.5\,\%$ for $r_\mathrm{d} = 15\,000\,\mathrm{m}$ in our simulations. This results in a significant overhead reduction that is constituted by BSs close to the border of the simulation area, that only serve as interferers. We also observe that for the computation of the radius of the *effective* network, the analytical evaluations in Theorem 1 and Theorem 2 provide a good approximation for the simulation results. The approximation is tighter when the effect of two interferers is considered, i.e., Theorem 2.

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V. Conclusion

In this paper, we discussed the relation between the system level simulation of a finite area network with a fixed radius and the analytical evaluation of a network via the theory of point processes. We show that the SIR values of users located close to the border of the network deviate drastically from the SIR of users located closer to the center of the network. In this paper, we provide an approximation based on the scenario geometry (w.r.t. interferer distances) in order to identify the *effective* network area, i.e., the part of the simulation area where the SIR distribution does not deviate more than a predefined fraction from the one in the center. We also show that the relative size of the effective network area grows with the absolute network radius. Furthermore, we discuss the efficiency of simulations in terms of overhead reduction for an increased size of effective network area and also point out minimum network radius (for reliable results, identical to the analytical ones) and maximum radius (given by the simulation hardware).

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