Estimation of Missing Data in Fetal Heart Rate Signals Using Shift-Invariant Dictionary

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Abstract—**In 2015, an estimated 1.3 million intrapartum stillbirths occurred, meaning that the fetus died during labour. The majority of these stillbirths occurred in low and middle income countries. With the introduction of affordable continuous fetal heart rate (FHR) monitors for use in these settings, the fetal well-being can be better monitored and health care personnel can potentially intervene at an earlier time if abnormalities in the FHR signal are detected. Additional information about the fetal health can be extracted from the fetal heart rate signals through signal processing and analysis. A challenge is, however, the large number of missing samples in the recorded FHR as fetal and maternal movement in addition to sensor displacement can cause data dropouts. Previously proposed methods perform well on estimation of short dropouts, but struggle with data from wearable devices with longer dropouts. Sparse representation and dictionary learning have been shown to be useful in the related problem of image inpainting. The recently proposed dictionary learning algorithm, SI-FSDL, learns shift-invariant dictionaries with long atoms, which could be beneficial for such time series signals with large dropout gaps. In this paper it is shown that using sparse representation with dictionaries learned by SI-FSDL on the FHR signals with missing samples provides a reconstruction with improved properties compared to previously used techniques.**

I. Introduction

Fetal heart rate (FHR) monitoring is a widely used method to assess the fetal well-being during labour. FHR monitoring, used by trained health care professionals, allows for early detection of a fetus at risk and consequently appropriate and timely action to prevent further harm to fetus and mother.

In high income countries, FHR monitoring in labours with high risk are usually measured using continuous Doppler ultrasound in cardiotocography (CTG), while low risk labours are monitored intermittent with handheld Doppler devices. In low resource settings, FHR is often accessed intermittent with a fetal stetoscope or handheld Doppler. In 2015, there were an estimated of 2.6 [uncertainty range 2.4-3.0] million stillbirths [1], [2], with 1.3 [uncertainty range 1.2-1.6] million deaths occuring during labour [1]. The vast majority (98%) of these occur in low and middle income countries [1]. Improved care at birth, including continuous FHR monitoring, is the key to reduce the number of stillbirths. Abnormalities in the FHR signal can be detected earlier with the use of continuous monitoring. If any abnormalities are detected, an alarm can be used to alert qualified health care personnel to assess the situation. Additional information of the fetal well-being can be extracted from the FHR signals through signal processing and analysis. With the introduction of affordable devices for continuous FHR monitoring, such as Moyo Fetal Heart Rate Monitor, used in this study, new opportunities arises as these devices are also obtainable in low resource settings.

A well-known problem measuring FHR using Doppler ultrasound are signal dropouts due to both fetal and maternal movement in addition to sensor displacement. With the introduction of wearable devices for continuous FHR monitoring, allowing the mother to move freely while the device is attached, an increase in both the number and length of signal dropouts are expected. These missing samples are a challenge when determining traditional features used to assess the fetal well-being, such as the short and long time variability of the FHR, as well as when doing time-frequency analysis on the heart rate signal.

Simple methods such as linear interpolation [3] and cubic Hermite spline interpolation [4] and more complex methods such as Gaussian processes [5] and K-SVD [6] have previously been used to estimate the missing samples on FHR recorded by CTG. However, depending on the length of the gaps, these methods affect computation of the traditional heart rate *features*. As more and longer dropouts are expected when using wearable monitors, better estimations are desired.

Dictionary learning and sparse approximation have been shown to produce state of the art results in estimation of missing data [6]–[8]. An important advantage of using dictionary learning over methods such as linear or spline interpolation is that through learning from the signal class, a learned dictionary introduces less artefacts during processing, feature extraction, and time-frequency analysis.

For an inpainting problem with large gaps, unstructured dictionaries, produced by general dictionary learning methods such as MOD [9] or K-SVD [10] require large atom lengths which means increase in number of free variables. This leads

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to slow training and usage as well as the possibility of overfitting.

In this work we propose to use shift-invariant dictionary by utilizing SI-FSDL, a dictionary learning method for shift invariant dictionaries recently proposed by our group [11]. SI-FSDL is capable of handling variable shifts and length of the atoms.

II. Data Material

The data used in this study are collected by the Safer Births Research Project, which is a research collaboration with partners including, but not limited to, University of Stavanger, Laerdal Global Health, and partner hospitals in Tanzania. The data were collected at Haydom Lutheran Hospital (HLH), which is a rural hospital, and Muhimbili National Hospital (MNH), and Temeke Regional Referral Hospital (TRRH) which are both urban hospitals. 85 labours were monitored and recorded at HLH between February 1st. and March 18th. 2016, 227 labours were recorded at MNH between March 15th. and July 13th. 2016, and 1087 births were recorded at TRRH between June 4th. and October 1st. 2016. All data were anonymized prior to transfer to researchers.

The project was ethically approved prior to implementation by the National Institute for Medical Research (NIMR) in Tanzania (NIMR/HQ/R.8a/Vol. IX/1434) and the Regional Committee for Medical and Health Research Ethics (REK) in Norway (2013/110/REK vest) before the start of the study.

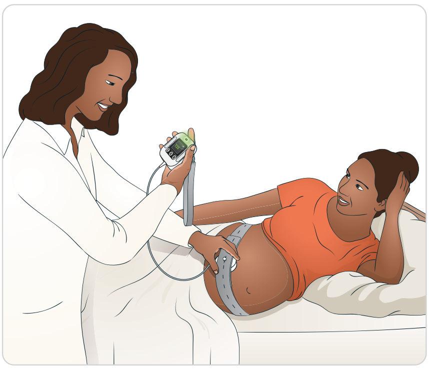

Fig. 1. Moyo Fetal Heart Rate Monitor, Laerdal Global Health AS, Norway. Illustration reproduced with permission [12]

Moyo Fetal Heart Rate Monitor, Fig. 1, is used to record the fetal heart rate and is developed by Laerdal Global Health [13] as an affordable FHR monitoring device for both intermittent and continuous monitoring for use in low resource settings. The device consists of a small handheld unit with display and a sensor unit. For continuous monitoring, the sensor can be applied to the maternal abdomen using an elastic band, shown in Fig. 1. The sensor unit includes a 9-crystal pulsed wave Doppler ultrasound sensor operating at a frequency of 1 MHz and an intensity of less than: $5mW/cm^2$. The detected fetal heart rate is logged at 2Hz. The sensor unit also includes a 3-axes accelerometer sampled at 50Hz used to describe maternal movement, and a temperature sensor, sampled at 2Hz. FHR and maternal HR as well as accelerometer and temperature values are stored to files and can be accessed via USB connection.

III. The Proposed Method

Sparse representation and dictionary learning is based on the idea that it is possible to represent a signal class sparsely in some domain, and that a learned dictionary can represent this domain. Let an $N \times 1$ signal vector be denoted by $\boldsymbol{x}$, and its approximation as $\hat{\boldsymbol{x}} = \boldsymbol{D}\boldsymbol{w}$, where $\boldsymbol{D}$ is the dictionary matrix of size $N \times K$, with the columns $\{\mathbf{d}_i\}_{i=1}^{K}$ forming dictionary *atoms*, and $\boldsymbol{w}$, $K \times 1$, is the vector of *sparse* coefficients. The dictionary learning problem is formulated as follows:

$$\boldsymbol{W}, \boldsymbol{D} = \underset{\boldsymbol{W}, \boldsymbol{D}}{\operatorname{argmin}} \|\boldsymbol{X} - \boldsymbol{D}\boldsymbol{W}\|_F^2 \quad s.t \begin{cases} \boldsymbol{w}_i \text{ is sparse} \\ ||\boldsymbol{d}_i||_2^2 = 1. \end{cases} \tag{1}$$

where $\boldsymbol{W}$ and $\boldsymbol{X}$ are formed from concatenation of coefficient $\boldsymbol{w}_i$ and signal vectors $\boldsymbol{x}_i$ respectively. Since equation 1 is not tractable, it is usually broken into two steps: in the first step, *sparse coding*, one would find $\boldsymbol{W}$ while fixing $\boldsymbol{D}$. In the second step, *dictionary update*, $\boldsymbol{D}$ is found while keeping $\boldsymbol{W}$ constant. MOD [9] and K-SVD [10] are examples of dictionary learning methods using these steps.

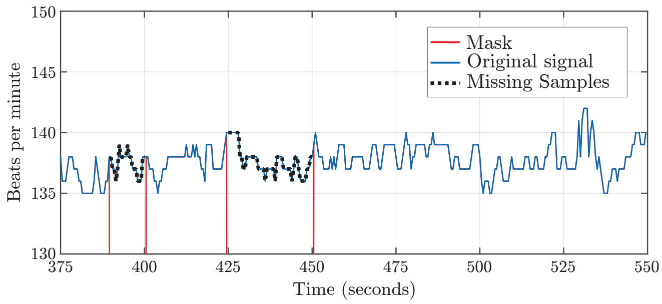

Fig. 2. A sample FHR signal, its masked version and simulated missing samples. The masked signal has zeros as value where there are missing samples and is equal to the original signal elsewhere.

In this paper we are dealing with recovering of missing data or inpainting where the location of missing data is known beforehand. Since signals and the location of missing samples differ from patient to patient, and these analyses are done in retrospect, we wish to learn a dictionary on the signals with missing data to tailor the dictionary to the person, before performing the reconstruction. For this reason, the information about the missing samples is built into a mask matrix, $\boldsymbol{M}$. The mask matrix used in this paper, removes the corrupted samples in the signal. This corresponds to removing rows in a vector. So when there are p corrupted samples in a vector, the mask matrix is made from an identity matrix with its p rows removed which makes it $(N-p) \times N$. Applying this matrix to a dictionary (another matrix) leads to removal of p rows.

When having multiple vectors such as in $\boldsymbol{X}$, there is a mask matrix, $\boldsymbol{M}_i$, for every vector $\boldsymbol{x}_i$. Fig. 2 shows a sample FHR signal along with the masked signal and the missing samples.

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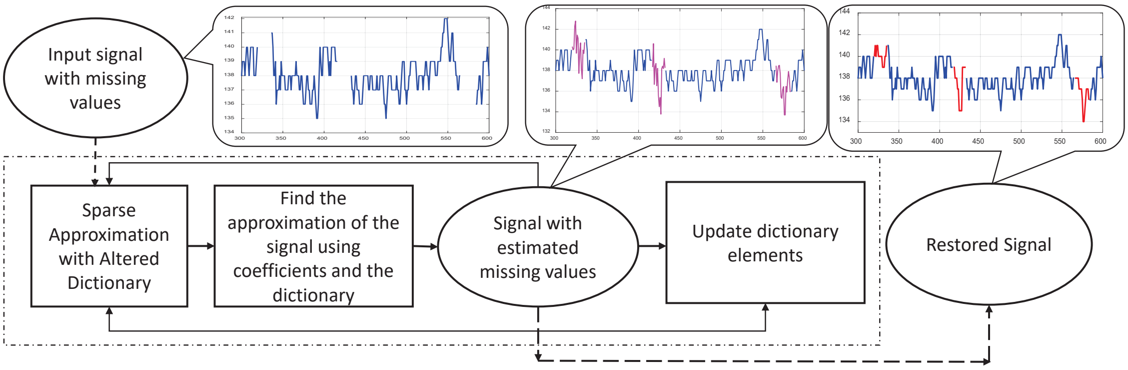

Fig. 3. Different steps of inpaiting. The input contains missing samples. Each iteration of this method replaces the missing values with their approximated version. This is done until some criteria is satisfied.

The literature describes two ways to incorporate the mask information into the dictionary learning steps. One method was used in [8], [14], [15] which alters both dictionary learning steps. An alternative approach was briefly discussed in [6] which requires only changes to the sparse coding step. Meaning that the mask matrices are not used directly in dictionary update stage. In this paper, we apply the second method for inpainting using shift-invariant dictionary.

The altered steps required for the second method are as follows:

1) While the dictionary entries are fixed, remove the rows of the dictionary and the signal vectors corresponding to the missing samples for each signal vectors and normalize the dictionary columns to 1.
2) Find the coefficients using their own respective dictionaries and scale them proportional to masked dictionary scale.
3) Find the approximation of the signal vectors by multiplying the coefficients with the full dictionary.
4) Reconstruct the signal by replacing the missing samples with their approximated resulted from above.
5) Update the dictionary elements using any desired method and normalize the columns to 1.

The first two steps describe the sparse coding stage for inpainting and steps three and four are for keeping the dictionary update stage in step 5 unchanged. The proposed method is summarized in Alg. 1 using pseudocode and MATLAB syntax, and it is visualized in Fig. 3.

For imposing the shift-invariant structure onto the dictionary we utilized our previously proposed method, SI-FSDL [11] which is a special form of FSDL [16]. One of the benefits of shift-invariant dictionaries is that we can address larger gaps (by using larger shift-invariant atoms) than a general dictionary while keeping the number of free variables fixed.

An example of a small shift-invariant dictionary with three shift-invariant atoms or SIAs, is depicted in Fig. 4. This example does not have circular shifts, i.e. the the shifting ends as the last non-zero element of a SIA reaches the bottom row of the dictionary matrix. SI-FSDL handles variable length and variable shift atoms as Fig. 4 illustrates.

Algorithm 1: Inpainting using Dictionary learning

Inputs : $\boldsymbol{X}^{(0)}$ and $M_i, i \in 1$ .. *Number of vectors*
for n ***in*** *Number of iterations* **do**
  Sparse Coding:
  for i ***in*** *Number of vectors* **do**
    $\boldsymbol{D_{M}}_i = \boldsymbol{M}_i * \boldsymbol{D}$
    $\boldsymbol{D_{M}}_i = \boldsymbol{D_{M}}_i * diag(1./sqrt(sum(\boldsymbol{D_{M}}_i . * \boldsymbol{D_{M}}_i)))$
    $\boldsymbol{x}_r = \boldsymbol{M}_i * \boldsymbol{x}_i$
    $\boldsymbol{w}_i = \underset{\boldsymbol{w}}{\operatorname{argmin}} \|\boldsymbol{x}_r - \boldsymbol{D_{M}}_i * \boldsymbol{w}\|_2^2$ s.t $\boldsymbol{w}$ is sparse
    $\boldsymbol{w}_i = \boldsymbol{w}_i * diag(sqrt(sum(\boldsymbol{D_{M}}_i . * \boldsymbol{D_{M}}_i)))$
  end
  Dictionary Update:
  $\boldsymbol{X}_c =$ signal matrix with corrupt samples zeroed out
  $\hat{\boldsymbol{X}} = \boldsymbol{D} * \boldsymbol{W}$
  $\hat{\boldsymbol{X}}_a = \hat{\boldsymbol{X}}$ with clean samples zeroed out
  $\boldsymbol{X}^{(n)} = \boldsymbol{X}_c + \hat{\boldsymbol{X}}_a$
  Update $\boldsymbol{D}$ using the desired dictionary learning method
  $\boldsymbol{D} = \boldsymbol{D} * diag(1./sqrt(sum(\boldsymbol{D} . * \boldsymbol{D})))$
end
Output: Inpainted $\boldsymbol{X}^{(final)}$

$$\begin{bmatrix} \star_0 & 0 & *_0 & 0 & 0 & \alpha_0 & 0 \\ \star_1 & \star_0 & *_1 & *_0 & 0 & \alpha_1 & 0 \\ \star_2 & \star_1 & 0 & *_1 & *_0 & 0 & \alpha_0 \\ 0 & \star_2 & 0 & 0 & *_1 & 0 & \alpha_1 \end{bmatrix}$$

Fig. 4. A simple shift-invariant dictionary with 3 shift-invariant atoms(SIAs). The first two SIAs have 1 shift while the last one has 2 shifts.

IV. Experiments

A total of 691400 segments of missing samples are found in the 1399 recordings in the dataset, with an average of 494 missing segments in each recording. In total, the missing percentage of data is 36.4%. However, 96.9% of the missing data gaps are less than 50 samples in length. The distribution of the length in these gaps from 1 to 50 samples is shown in Fig. 5.

We have chosen a signal *without* missing samples from our

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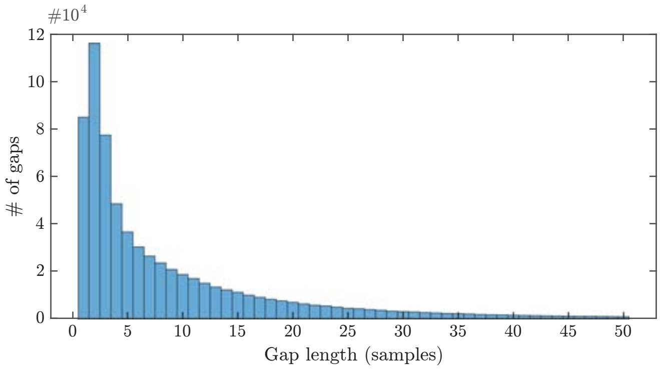

Fig. 5. Distribution of gap lengths

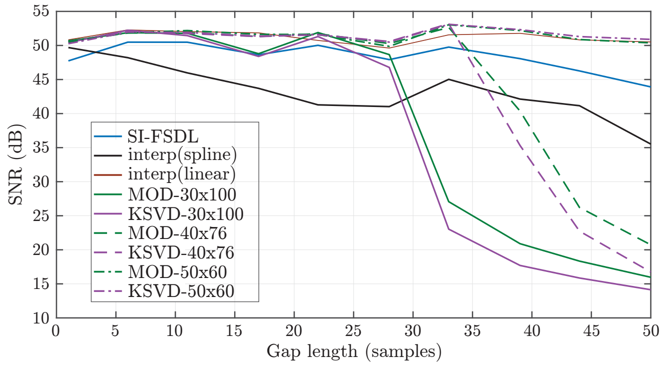

Fig. 6. Recovery performance for different methods when the missing interval changes but the missing percentage stays the same (10%)

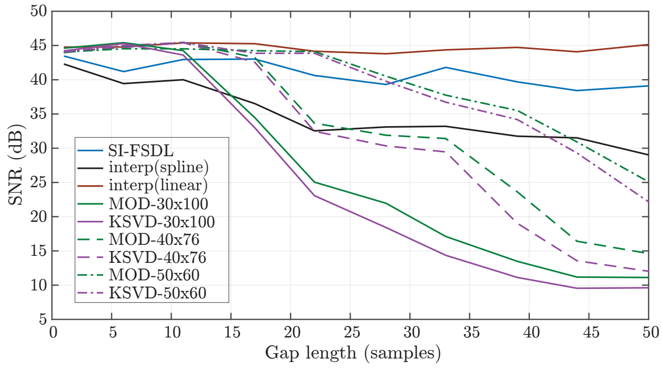

Fig. 7. Recovery performance for different methods when the missing interval changes but the missing percentage stays the same (30%)

database and randomly removed parts of it so that the true signal is available to evaluate the recovery results. The used recovery methods are SI-FSDL, MOD, K-SVD, spline and linear interpolation. During the experiments the number of free variables in the dictionaries are kept constant at approximately 3000. The ratio of non-zeros coefficients to the number of elements in the signal is 0.1. Signal blocks are chosen in an overlapping fashion for the dictionary learning methods. Block lengths, N, of 30, 40 and 50 are used for MOD and K-SVD, and 500 for SI-FSDL. Orthogonal Matching Pursuit (OMP) was used for sparse coding.

A. Experiment 1

The first experiment is designed to evaluate the average performance of each method when the missing percentage is fixed, but the gap lengths change. In order to have a realistic scenario, the fixed missing percentages are set to 10 and 30. The length of the gaps ranges from 1 sample to 50 samples. To find the average performance for each gap length, different randomly created masks are used.

Performance of the tested methods for 10% missing data are shown in Fig. 6. All methods achieves similar performance for short interval lengths, with the exception of spline interpolation. As the gap lengths increases, the performance of MOD and K-SVD decreases. The exception to this is when the segment length is 50. Their performance is at least as good as linear interpolation and always better than SI-FSDL.

Performance of the tested methods for 30% missing data are shown in Fig. 7. All methods achieves similar performance for short interval lengths. With higher ratio of missing data, the performance for MOD and K-SVD for all segment lengths decrease faster than the case of 10%. The performance of linear interpolation and SI-FSDL, remain almost the same regardless of length of missing sample interval.

B. Experiment 2

The experiment is devised to have a closer look at the best performing methods of last experiment when the missing percentage is 30%, which is close to the percentage for our dataset. These methods are linear interpolation and SI-FSDL. The intent is to look at the continuous wavelet transform of their reconstructed signals and see how similar the time-frequency distribution of the reconstructed signal is to the original signal.

The time-frequency response for a short section of FHR with 3 missing gaps are shown in Fig. 8. In close-ups of the signal around each mask, the original signal is shown in blue, estimations using linear interpolation in dotted red, and SI-FSDL in dashed black.

It can be seen by visual inspection that SI-FSDL restores the time-frequency properties better than linear interpolation even though the later has higher SNR.

V. Discussion

It is worthwhile to note that while inpainting methods can be utilized to reconstruct the gaps, they might miss some details if the gaps become too large. In the case of FHR signals temporary increases or decreases, known as accelerations and decelerations in the heart rate, are important details when determining the fetal well-being. In order to recover such information, we need to know the duration of these patterns. In an abrupt accelerations and decelerations the FHR has a change of 15 beats per minute with a time from onset to extremum of $\leq$ 30 seconds and total duration of less than 2 minutes. Based on this, it is safe to reconstruct segments with a maximum length of 25 seconds, corresponding to 50

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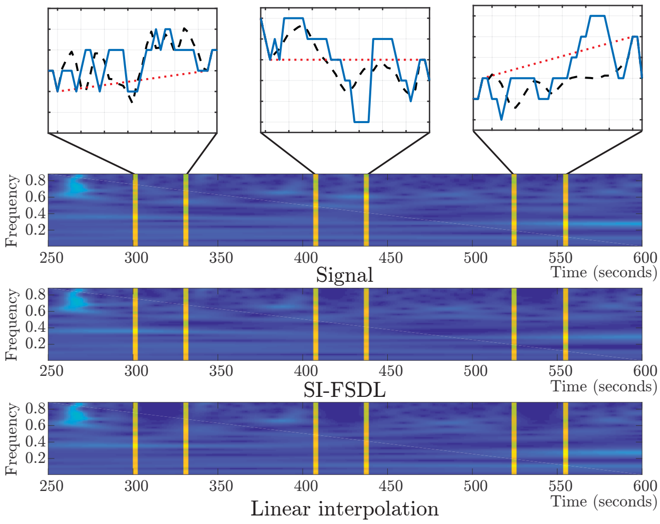

Fig. 8. Continuous wavelet transform over a short signal with 3 gaps missing samples. Original signal shown in blue, linear interpolation shown as dotted red line, SI-FSDL shown as dashed black line.

samples. Fig. 5, shows that most of the gaps are short in length, with 96.9% of the gaps below our upper limit.

On data where 10% of the samples are missing, Fig. 6, linear interpolation and all dictionary learning techniques achieve similar SNR for gaps ≤ 28 samples in length. MOD and K-SVD with block length 50 show the best performance for all the missing gaps. Increasing the missing percentage to 30%, Fig. 7, a large drop in SNR is seen in both MOD and K-SVD. Depending on the gap size, this dropout occurs for gaps larger than 10, 17 and 23 samples. Since MOD and K-SVD have block lengths of 30 to 50, they cannot restore gaps close or larger than their size. A possible solution to this is to increase the block length in MOD and K-SVD. However, this usually means increasing the overall number of free variables as well and learning a larger dictionary which requires more data and processing time. Due to its structure, however, SI-FSD can reconstruct larger gaps by adjusting the length and number of SIAs.

A high SNR is seen for linear interpolation and SI-FSDL for all gaps. The challenge of using linear interpolation, however, is that it introduces artefacts, as seen in Fig 8. In the first and third gaps, samples 300-330 and 525-555, linear interpolation introduces artefacts by removing high frequencies. In the same gaps, SI-FSDL shows more fidelity to the original signal. Both methods perform similarly in the second gap, samples 405-435, and introduce artefacts.

VI. Conclusion

The results presented in this work indicate that for dictionary learning based methods, gap interval and missing percentage are important parameters when attempting to recover missing data in the signal. When the missing percentage is low, MOD and K-SVD achieve the highest performance, while SI-FSDL outperforms the other methods when the missing percentage is increased. A high SNR is also observed for linear interpolation. Reconstruction based on dictionary learning methods, however, are shown to be closer to the true signal in terms of the spectral content of the signal. In order to have reliable information, having less artefacts is crucial when performing further analysis on the data.

VII. Conflict of Interest

The study was supported by the Laerdal Foundation and the Research Council of Norway through the Global Health and Vaccination Program (GLOBVAC), project no. 228203

Sara Brunner is an employee of Laerdal Medical AS.

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