A New Beamformer Design Method for Multi-Group Multicasting by Enforcing Constructive Interference

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***Abstract*—In this paper, we propose a new multi-group multicast beamforming design method for phase shift keying (PSK) modulated signals. Quality of service (QoS)-aware optimization is considered where the aim is to minimize transmission power of multiple-antenna base station under the QoS constraints of single-antenna users. In this paper, we show that symbol-level beamforming scheme proposed in the literature is not an effective design method for multi-group multicasting and modify it using rotated constellation approach in order to reduce transmission power. Proposed method enforces the known interference in a constructive manner such that the received symbol at each user is inside the correct decision region for any set of symbols. Hence, designed beamformers can be utilized throughout a transmission-frame rather than symbol-by-symbol basis. An alternating direction method of multipliers (ADMM) algorithm is presented for the proposed design problem and closed-form update equations are derived for the steps of the ADMM algorithm. Simulation results show that the proposed method decreases the transmission power significantly compared to the conventional and symbol-level beamforming.**

***Index Terms*—Multi-group multicast beamforming, ADMM, constructive interference.**

I. Introduction

Multicasting has gained great popularity as a key technology in wireless applications such as venue casting, video streaming, emergency alerts where common message is delivered to multiple users simultaneously [1], [2]. Physical layer multicasting by taking advantage of beamforming at the multiple-antenna transmitter is an energy and spectral efficient method and studied extensively in the literature [3], [4]. The design of multicast beamformer requires solving a non-convex quadratically-constrained quadratic programming (QCQP) problem for multi-group multicasting. Recently in [3], a consensus ADMM based computationally inexpensive algorithm is presented for general QCQP and it is applied for single-group multicast beamforming. Later in [4], a more efficient ADMM formulation is proposed in order to reduce the computational complexity further. This is the algorithm we will take as a benchmark for conventional beamforming.

The above conventional approach, [4], aims at satisfying QoS constraint of each user by suppressing interference from the signals of other multicast groups. Hence, interference constitutes a major roadblock to power saving. In [5], the concept of interference exploitation is introduced. In this work and later in [6]-[8] beamformers are designed such that interference contributing to the symbol of interest for each user is treated as a useful signal in order to increase effective signal-to-noise ratio (SNR). For this, both data knowledge is used in addition to channel state information (CSI). Different from conventional beamforming which tries to eliminate all the interference, this paradigm benefits from the constructive part of interference by optimizing beamformers on symbol-by-symbol basis. Symbol-level beamforming is usually considered for unicast systems where each message signal is sent to a single user [6]-[8]. It is shown that it is a more effective design method in satisfying QoS of subscribers with a significantly reduced transmission power. However, to the best of authors' knowledge, it is not yet elaborated for multi-group multicast beamforming.

In this paper, we first formulate symbol-level beamforming problem for multi-group multicasting. Different from unicast case, this optimization enforces the same phase alignment condition for every user in a multicast group, restricting the feasibility region. As we show in the simulations, symbol-level approach is not power efficient due to the nature of the problem formulation. Nonetheless, we propose a new scheme which takes advantage of the constructive interference idea for PSK modulated message symbols. In order to loosen up the difficult phase-alignment constraints, we rotate the constellation points for each user appropriately in order to reduce the transmit power. Rather than symbol-level beamforming, we consider block-level approach where the designed beamformers are used during a whole transmission frame. The proposed method enforces the interference to be constructive at each user for any set of symbols. We impose the constraint that the total interference does not move the received symbol out of the correct decision region determined by SNR and minimum distance of PSK constellation. The addressed optimization problem is formulated as a non-convex problem. In order to solve it, we use ADMM approach which has superior convergence [9], [10]. After reformulating the original problem in a proper ADMM form, the closed-form optimum solutions are derived for each step of the proposed ADMM algorithm. Simulation results show that the proposed method is significantly power efficient compared to conventional and symbol-level beamforming.

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II. System Model

Consider a multi-group multicasting system where an N-antenna base station (BS) serves K single-antenna users. Assume that there are M multicast groups, $\{\mathcal{G}_1, ..., \mathcal{G}_M\}$, where $\mathcal{G}_m$ denotes the m^{th} multicast group of users for $m \in \mathcal{M} = \{1, ..., M\}$. The users in each multicast group are interested in a common message signal and each user is a member of single multicast group, i.e., $\mathcal{G}_m \cap \mathcal{G}_{m'} = \emptyset$ for $m \neq m'$, $\forall m, m' \in \mathcal{M}$. Narrowband block fading channel and PSK modulation for the message signals are considered. The signal transmitted from the antenna array is $\mathbf{x} = \sum_{m=1}^{M} \mathbf{t}_m e^{j\phi_m}$ where $e^{j\phi_m}$ is the PSK modulated symbol for the users in $\mathcal{G}_m$ and $\mathbf{t}_m$ is the corresponding $N \times 1$ complex beamformer weight vector. Assuming that message symbol for each multicast group is independent of others, the average transmit power is given by $P_{tot} = \sum_{m=1}^{M} ||\mathbf{t}_m||^2$. Let $\mathbf{h}_k \in \mathbb{C}^N$ denote the channel vector for the k^{th} user. The received signal at the k^{th} user is then given by

$$y_k = \mathbf{h}_k^H \sum_{m=1}^{M} \mathbf{t}_m e^{j\phi_m} + n_k, \quad \forall k \in \mathcal{K} \tag{1}$$

where $\mathcal{K} = \{1, ..., K\}$ and n_k is the additive zero mean Gaussian noise at the k^{th} user's receiver with variance σ_k^2. Received noise is assumed to be uncorrelated with the message signals for each user. In this paper, we consider quality-of-service (QoS)-aware beamformer design where the aim is to satisfy the individual target SINR constraint for each user with minimum transmit power at BS. In the following section, we first visit the conventional beamformer design method for the addressed problem.

III. Conventional Beamformer Design for QoS Problem

The conventional QoS-aware beamformer design treats all the interference coming from other multicast message signals as destructive. In this case, the receive signal-to-interference-plus-noise ratio (SINR) at the k^{th} user is given as

$$\text{SINR}_k = \frac{|\mathbf{h}_k^H \mathbf{t}_{m_k}|^2}{\sum_{m \neq m_k} |\mathbf{h}_k^H \mathbf{t}_m|^2 + \sigma_k^2}, \quad \forall k \in \mathcal{K} \tag{2}$$

where m_k denotes the index of the multicast group to which the k^{th} user belongs. The QoS-aware optimization problem is formulated for the conventional design as follows,

$$\min_{\{\mathbf{t}_m\}_{m=1}^{M}} \quad \sum_{m=1}^{M} ||\mathbf{t}_m||^2 \tag{3a}$$

$$\text{s.t.} \quad \text{SINR}_k \geq \gamma_k, \quad \forall k \in \mathcal{K} \tag{3b}$$

where γ_k is the target SINR for the k^{th} user. In [6], it is shown that the above beamformer design is suboptimal from an instantaneous viewpoint for unicast systems where there is a single user for each message signal. This is due to the fact that interference can be constructive, i.e. keeps the received symbol inside the decision region of the symbol of interest. In [6], a symbol-level beamforming design is considered for PSK modulation where the instantaneous interference is aligned to the message symbol for each user. In the following section, we will formulate this problem for multi-group multicasting systems.

IV. Symbol-Level Beamforming Based on Constructive Interference

Symbol-level beamforming is shown to be a power efficient method for unicast systems in [6] since it takes the constructive effect of interference into account in the QoS-aware optimization problem. Symbol-level QoS-aware problem considered in [6] can be reformulated for multi-group multicasting as follows,

$$\min_{\{\mathbf{t}_m\}_{m=1}^{M}} \quad \left|\left| \sum_{m=1}^{M} \mathbf{t}_m e^{j\phi_m} \right|\right|^2 \tag{4a}$$

$$\text{s.t.} \quad \left| \text{Im}\Big(\mathbf{h}_k^H \sum_{m=1}^{M} \mathbf{t}_m e^{j(\phi_m - \phi_{m_k})} \Big) \right| \leq \left(\text{Re}\Big(\mathbf{h}_k^H \sum_{m=1}^{M} \mathbf{t}_m e^{j(\phi_m - \phi_{m_k})} \Big) - \sqrt{\gamma_k \sigma_k^2} \right) \tan\left(\frac{\pi}{P}\right), \quad \forall k \in \mathcal{K} \tag{4b}$$

where P denotes the modulation order for PSK signaling. Note that the constraint in (4b) ensures that the angle of interference falls within the constructive area of the constellation for each user's symbol of interest. Although an efficient gradient projection algorithm is proposed for unicast symbol-level beamforming in [6], that algorithm is not compatible with multi-group multicasting given in (4a-b). Still, (4a-b) can be solved using a standard second-order cone program optimally. The main disadvantage of the problem in (4a-b) is that the number of constraints, K, is larger than the number of vector variables, M, unlike unicast formulation where $M = K$. Hence, the vector $\sum_{m=1}^{M} \mathbf{t}_m e^{j(\phi_m - \phi_{m_k})}$ should satisfy constructive phase alignment condition for each user in the ${m_k}^{th}$ multicast group. This is expected to reduce the effectiveness of symbol-level beamforming for multi-group multicasting as we will show in the simulations.

V. Proposed Beamformer Design

In this paper, we propose a new beamformer design enforcing known interference as constructive. In order to solve the problem of phase alignment for each user in the same multicast group in (4b), we assume that the constellation diagram for each user is rotated appropriately without the need for the strict phase alignment constraints for the users in a multicast group. This means that for P-PSK modulation, the constellation points are $\{\mathbf{h}_k^H \mathbf{t}_{m_k} e^{j0}, \mathbf{h}_k^H \mathbf{t}_{m_k} e^{j2\pi/P},\ \mathbf{h}_k^H \mathbf{t}_{m_k} e^{j2\times 2\pi/P}, ..., \mathbf{h}_k^H \mathbf{t}_{m_k} e^{j(P-1)\times 2\pi/P}\}$, $\forall k \in \mathcal{K}$. In addition, since the phase-rotation of the constellation for each user is determined after the optimization of beamformer vectors, our proposed approach is based on the worst-case scenario and the beamformer vectors are designed such that target SNRs of the users are satisfied for any multicast symbol set. This design problem can be formulated as follows,

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$$\min_{\{\mathbf{t}_m\}_{m=1}^M} \quad \sum_{m=1}^{M} ||\mathbf{t}_m||^2 \tag{5a}$$

$$\text{s.t.} \min_{\substack{\{\theta_m^1,\theta_m^2 \in \mathcal{C}_P\}_{m=1}^M \\ \theta_{m_k}^1 \neq \theta_{m_k}^2}} \Bigg| \Bigg(\mathbf{h}_k^H \mathbf{t}_{m_k} e^{j\theta_{m_k}^1} + \mathbf{h}_k^H \sum_{m \neq m_k} \mathbf{t}_m e^{j\theta_m^1} \Bigg) - \Bigg(\mathbf{h}_k^H \mathbf{t}_{m_k} e^{j\theta_{m_k}^2} + \mathbf{h}_k^H \sum_{m \neq m_k} \mathbf{t}_m e^{j\theta_m^2} \Bigg) \Bigg| \geq d_k, \quad \forall k \in \mathcal{K} \tag{5b}$$

$$\max_{\{\theta_m \in \mathcal{C}_P\}_{m \neq m_k}} \Bigg| \Bigg(\mathbf{h}_k^H \sum_{m \neq m_k} \mathbf{t}_m e^{j\theta_m} \Bigg) \Bigg| < \big|\mathbf{h}_k^H \mathbf{t}_{m_k}\big| \sin\left(\frac{\pi}{P}\right) \quad \forall k \in \mathcal{K} \tag{5c}$$

where $\mathcal{C}_P \triangleq \{0,\ 2\pi/P,\ 2 \times 2\pi/P,\ ...,\ (P-1) \times 2\pi/P\}$ is the set of phase angles for P-PSK message symbols. (5b) guarantees that the minimum distance between two different received constellation points is always greater than d_k which is the threshold determined by the SNR requirement of the k^{th} user. It is given by $d_k = 2\sqrt{\gamma_k \sigma_k^2} \sin(\frac{\pi}{P})$ for P-PSK signaling. In (5b), $\mathbf{h}_k^H \mathbf{t}_{m_k} e^{j\theta_{m_k}^1}$ and $\mathbf{h}_k^H \mathbf{t}_{m_k} e^{j\theta_{m_k}^2}$ represent two nominal constellation points rotated by $\mathbf{h}_k^H \mathbf{t}_{m_k}$ for the k^{th} user. Furthermore, (5c) guarantees that the added interference do not move the received symbol away from the correct decision region determined by the rotated constellation points. In order to simplify the constraint (5b), consider the following lower bound of the left side of (5b), i.e.,

$$\min_{\substack{\{\theta_m^1,\theta_m^2 \in \mathcal{C}_P\}_{m=1}^M \\ \theta_{m_k}^1 \neq \theta_{m_k}^2}} \Bigg| \Bigg(\mathbf{h}_k^H \mathbf{t}_{m_k} e^{j\theta_{m_k}^1} + \mathbf{h}_k^H \sum_{m \neq m_k} \mathbf{t}_m e^{j\theta_m^1} \Bigg) - \Bigg(\mathbf{h}_k^H \mathbf{t}_{m_k} e^{j\theta_{m_k}^2} + \mathbf{h}_k^H \sum_{m \neq m_k} \mathbf{t}_m e^{j\theta_m^2} \Bigg) \Bigg| \tag{6a}$$

$$\geq \min_{\theta_{m_k}^1,\theta_{m_k}^2 \in \mathcal{C}_P, \theta_{m_k}^1 \neq \theta_{m_k}^2} \Big| \mathbf{h}_k^H \mathbf{t}_{m_k} (e^{j\theta_{m_k}^1} - e^{j\theta_{m_k}^2}) \Big| - \max_{\{\theta_m^1,\theta_m^2 \in \mathcal{C}_P\}_{m \neq m_k}} \Big| \mathbf{h}_k^H \sum_{m \neq m_k} \mathbf{t}_m (e^{j\theta_m^1} - e^{j\theta_m^2}) \Big|$$

$$\geq 2\big|\mathbf{h}_k^H \mathbf{t}_{m_k}\big| \sin\left(\frac{\pi}{P}\right) - \sum_{m \neq m_k} \max_{\theta_m^1,\theta_m^2 \in \mathcal{C}_P} \big| \mathbf{h}_k^H \mathbf{t}_m (e^{j\theta_m^1} - e^{j\theta_m^2}) \big|$$

$$\geq 2\big|\mathbf{h}_k^H \mathbf{t}_{m_k}\big| \sin\left(\frac{\pi}{P}\right) - 2 \sum_{m \neq m_k} \big|\mathbf{h}_k^H \mathbf{t}_m\big| \tag{6b}$$

Note that if (6b) is greater than d_k, then (5b) is satisfied by (6a) automatically. Hence, we can use (6b) in place of (6a) for a simpler problem as follows,

$$\min_{\{\mathbf{t}_m\}_{m=1}^M} \quad \sum_{m=1}^{M} ||\mathbf{t}_m||^2 \tag{7a}$$

$$\text{s.t.} \quad \big|\mathbf{h}_k^H \mathbf{t}_{m_k}\big| \sin\left(\frac{\pi}{P}\right) - \sum_{m \neq m_k} \big|\mathbf{h}_k^H \mathbf{t}_m\big| \geq \frac{d_k}{2}, \ \forall k \in \mathcal{K} \tag{7b}$$

The condition in (7b) requires (5c) to be satisfied. Hence, (5c) is removed for a more compact problem. Note that the feasible set of the problem (7a-b) is dependent on only the amplitude of the terms $\mathbf{h}_k^H \mathbf{t}_m$ without any need for strict phase alignment unlike symbol-level beamforming problem in the previous section. As shown in the simulations, this results significantly better performance compared to the conventional and symbol-level approach. Furthermore, (7a-b) should be solved only once for a transmission frame rather than for each symbol time slot. In order to solve nonconvex problem (7a-b), we propose an ADMM based algorithm which has been an appealing optimization technique for nonconvex problems with multiple constraints like the one in (7). In [4], an effective problem formulation is adopted for ADMM in a unified framework together with convex-concave procedure to tackle conventional multi-group multicast beamforming problem. In this paper, we will use the same auxiliary variables to reformulate the problem (7a-b). However, we will deal with the nonconvex form directly without applying successive convex approximation. This will simplify the algorithm by avoiding two nested loop iterations in [4]. Finally, we will derive closed-form update equations for the steps of the proposed ADMM algorithm.

Let us define additional auxiliary variables $\Gamma_{k,m} \triangleq \mathbf{h}_k^H \mathbf{t}_m$, $\forall k \in \mathcal{K}$, $\forall m \in \mathcal{M}$ using the same approach in [4]. This method reduces the computational complexity of ADMM algorithm compared to the consensus formulation in [3]. Using the new variables, the problem in (7) can be reformulated as follows,

$$\min_{\left\{\mathbf{t}_m, \{\Gamma_{k,m}\}_{k=1}^K\right\}_{m=1}^M} \quad \sum_{m=1}^{M} ||\mathbf{t}_m||^2 \tag{8a}$$

$$\text{s.t.} \quad \Gamma_{k,m} = \mathbf{h}_k^H \mathbf{t}_m, \quad \forall k \in \mathcal{K},\ \forall m \in \mathcal{M} \tag{8b}$$

$$\big|\Gamma_{k,m_k}\big| \sin\left(\frac{\pi}{P}\right) - \sum_{m \neq m_k} \big|\Gamma_{k,m}\big| \geq \frac{d_k}{2}, \ \forall k \in \mathcal{K} \tag{8c}$$

The steps of the ADMM algorithm in scaled-form [10] are given below. Here $\{\{\lambda_{k,m}\}_{k=1}^K\}_{m=1}^M$ are the scaled dual variables corresponding to the equality constraints in (8b). $\rho > 0$ is the penalty parameter used in augmented Lagrangian [3], [10].

Algorithm 1: ADMM for the Problem (8)

Initialization: Initialize $\mathbf{t}_m^0 \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_N)$, $\lambda_{k,m}^0 \leftarrow 0$, $\forall k \in \mathcal{K}$, $\forall m \in \mathcal{M}$. Set the iteration number $j \leftarrow 0$ and the penalty parameter ρ.

Repeat

$$\left\{\{\Gamma_{k,m}^{j+1}\}_{k=1}^K\right\}_{m=1}^M \leftarrow \arg \min_{\left\{\{\Gamma_{k,m}\}_{k=1}^K\right\}_{m=1}^M} \sum_{k=1}^{K} \sum_{m=1}^{M} |\Gamma_{k,m} - \mathbf{h}_k^H \mathbf{t}_m^j + \lambda_{k,m}^j|^2$$

$$\text{s.t.} \quad \big|\Gamma_{k,m_k}\big| \sin\left(\frac{\pi}{P}\right) - \sum_{m \neq m_k} \big|\Gamma_{k,m}\big| \geq \frac{d_k}{2}, \ \forall k \in \mathcal{K} \tag{9a}$$

$\{\mathbf{t}_m^{j+1}\}_{m=1}^M \leftarrow$

$$\arg\min_{\{\mathbf{t}_m\}_{m=1}^M} \sum_{m=1}^M ||\mathbf{t}_m||^2 + \rho \sum_{k=1}^K \sum_{m=1}^M |\Gamma_{k,m}^{j+1} - \mathbf{h}_k^H \mathbf{t}_m + \lambda_{k,m}^j|^2 \tag{9b}$$

$\lambda_{k,m}^{j+1} \leftarrow \lambda_{k,m}^j + \Gamma_{k,m}^{j+1} - \mathbf{h}_k^H \mathbf{t}_m^{j+1}, \quad \forall k \in \mathcal{K},\ \forall m \in \mathcal{M}$

Set $j \leftarrow j+1$.

Until convergence criterion is met.

Note that the optimization problem in (9a) can be decomposed into K independent subproblems where the k^{th} subproblem is given by,

$$\min_{\{\Gamma_{k,m}\}_{m=1}^M} \sum_{m=1}^M |\Gamma_{k,m} - \mathbf{h}_k^H \mathbf{t}_m^j + \lambda_{k,m}^j|^2 \tag{10a}$$

$$\text{s.t.} \ \ \left|\Gamma_{k,m_k}\right| \sin\left(\frac{\pi}{P}\right) - \sum_{m \neq m_k} \left|\Gamma_{k,m}\right| \geq \frac{d_k}{2} \tag{10b}$$

Now, let us express the optimization variables $\{\Gamma_{k,m}\}_{m=1}^M$ in terms of their amplitude and phase, i.e., $\Gamma_{k,m} = \beta_{k,m} e^{j\varphi_{k,m}}$ where $\beta_{k,m} \geq 0,\ \forall k \in \mathcal{K},\ \forall m \in \mathcal{M}$. If we further define $\zeta_{k,m}^j \triangleq \mathbf{h}_k^H \mathbf{t}_m^j - \lambda_{k,m}^j$ for ease of notation, we can reformulate (10) as follows,

$$\min_{\{\beta_{k,m},\varphi_{k,m}\}_{m=1}^M} \sum_{m=1}^M \left(\beta_{k,m}^2 - 2\beta_{k,m} \mathrm{Re}(\zeta_{k,m}^j e^{-j\varphi_{k,m}})\right) \tag{11a}$$

$$\text{s.t.} \ \ \beta_{k,m_k} \sin\left(\frac{\pi}{P}\right) - \sum_{m \neq m_k} \beta_{k,m} \geq \frac{d_k}{2} \tag{11b}$$

$$\beta_{k,m} \geq 0, \quad \forall m \in \mathcal{M} \tag{11c}$$

Note that the phase variables $\{\varphi_{k,m}\}_{m=1}^M$ do not affect any constraint in the problem (11a-c). Hence, the optimum $\{\varphi_{k,m}^\star\}_{m=1}^M$ can be easily obtained as $\varphi_{k,m}^\star = \angle\zeta_{k,m}^j,\ \forall m \in \mathcal{M}$ which minimizes the objective function in (11a). After inserting the optimum $\{\varphi_{k,m}^\star\}_{m=1}^M$ into (11a), the problem (11) can be reformulated in terms of $\{\beta_{k,m}\}_{m=1}^M$ as follows,

$$\min_{\{\beta_{k,m}\}_{m=1}^M} \sum_{m=1}^M \left(\beta_{k,m}^2 - 2\beta_{k,m}|\zeta_{k,m}^j|\right) \tag{12a}$$

$$\text{s.t.} \ \ (11\text{b}), (11\text{c}) \tag{12b}$$

The optimum solution of (12) can be obtained by Kuhn-Tucker necessary optimality conditions which are given as

$$\beta_{k,m_k} - |\zeta_{k,m_k}^j| = \frac{\mu_k}{2} \sin\left(\frac{\pi}{P}\right) + \frac{\tilde{\mu}_{k,m_k}}{2} \tag{13a}$$

$$\beta_{k,m} - |\zeta_{k,m}^j| = -\frac{\mu_k}{2} + \frac{\tilde{\mu}_{k,m}}{2}, \quad m \neq m_k,\ \forall m \in \mathcal{M} \tag{13b}$$

$$\mu_k \geq 0, \quad \tilde{\mu}_{k,m} \geq 0, \quad \forall m \in \mathcal{M} \tag{13c}$$

$$\mu_k \left(\beta_{k,m_k} \sin\left(\frac{\pi}{P}\right) - \sum_{m \neq m_k} \beta_{k,m} - \frac{d_k}{2}\right) = 0 \tag{13d}$$

$$\tilde{\mu}_{k,m} \beta_{k,m} = 0, \quad \forall m \in \mathcal{M} \tag{13e}$$

$$(11\text{b}), (11\text{c}) \tag{13f}$$

where μ_k and $\{\tilde{\mu}_{k,m}\}_{m=1}^M$ are the Lagrange multipliers corresponding to the inequalities in (11b) and (11c), respectively. Note that the optimum β_{k,m_k} should be strictly greater than zero by (11b). Hence, $\tilde{\mu}_{k,m_k} = 0$ by (13e) and $\beta_{k,m_k} = |\zeta_{k,m_k}^j| + \frac{\mu_k}{2} \sin\left(\frac{\pi}{P}\right)$. Now, let us consider two cases for μ_k in sequel.

Case 1: $\mu_k = 0$.

In this case, $\tilde{\mu}_{k,m} = 0\ \forall m \in \mathcal{M}$ in order to satisfy (13b) and (13e). Hence, $\beta_{k,m} = |\zeta_{k,m}^j|\ \forall m \in \mathcal{M}$. Since this solution is optimum for (12a) without any constraints, if it satisfies the inequality in (11b), it is the optimum solution of (12a-b). If it does not, consider the following case.

Case 2: $\mu_k > 0$.

In this case, the inequality in (11b) is satisfied with equality by (13d). If $|\zeta_{k,m}^j| < \frac{\mu_k}{2}$, the corresponding $\tilde{\mu}_{k,m}$ should be strictly greater than zero by (13b) and (11c). In this case, $\beta_{k,m} = 0$ by (13e). Otherwise if $|\zeta_{k,m}^j| \geq \frac{\mu_k}{2}$, $\tilde{\mu}_{k,m} = 0$ in order to satisfy (13b) and (13e). This can be expressed as $\beta_{k,m} = \max\{0, |\zeta_{k,m}^j| - \frac{\mu_k}{2}\}$ for $m \neq m_k,\ \forall m \in \mathcal{M}$. If we insert $\{\beta_{k,m}\}_{m=1}^M$ into the equality in (13d), we obtain

$$\sin\left(\frac{\pi}{P}\right)|\zeta_{k,m_k}^j| + \sin^2\left(\frac{\pi}{P}\right)\frac{\mu_k}{2} - \frac{d_k}{2} = \sum_{m \neq m_k} \max\left\{0, |\zeta_{k,m}^j| - \frac{\mu_k}{2}\right\}. \tag{14}$$

μ_k that satisfies (14) can be found by a simple binary search as in the water filing algorithm. Let $\mu_k^\star$ denote this value. If we show the function inside the parentheses in (13d) by $f_k(\{\beta_{k,m}\}_{m=1}^M) = \beta_{k,m_k} \sin\left(\frac{\pi}{P}\right) - \sum_{m \neq m_k} \beta_{k,m} - \frac{d_k}{2}$, the optimum solution for the update in (9a) is given by

$$\Gamma_{k,m_k}^{j+1} \leftarrow \begin{cases} \zeta_{k,m_k}^j & \text{if } f_k(\{|\zeta_{k,m}|\}_{m=1}^M) \geq 0 \\ \zeta_{k,m_k}^j + \frac{\mu_k^\star}{2} \sin\left(\frac{\pi}{P}\right) e^{j\angle\zeta_{k,m_k}^j} & \text{otherwise} \end{cases}$$

$$\Gamma_{k,m}^{j+1} \leftarrow \begin{cases} \zeta_{k,m}^j & \text{if } f_k(\{|\zeta_{k,m}|\}_{m=1}^M) \geq 0 \\ \max\{0, |\zeta_{k,m}^j| - \frac{\mu_k^\star}{2}\} e^{j\angle\zeta_{k,m}^j} & \text{otherwise} \end{cases}$$

$$m \neq m_k,\ \forall m \in \mathcal{M}$$

$$\forall k \in \mathcal{K} \tag{15}$$

The optimum update for the optimization problem in (9b) is given as follows,

$$\mathbf{t}_m^{j+1} = \left(\mathbf{I}_N + \rho \sum_{k=1}^K \mathbf{h}_k \mathbf{h}_k^H\right)^{-1} \left(\rho \sum_{k=1}^K (\Gamma_{k,m}^{j+1} + \lambda_{k,m}^j)\mathbf{h}_k\right)$$

$$\forall m \in \mathcal{M} \tag{16}$$

Now, all the steps of ADMM Algorithm are expressed in closed-form. In the following section, we will compare our proposed method with the above-listed benchmarks.

VI. Simulation Results

The number of antennas and the multicast groups are set as $N = 32$ and $M = 3$, respectively. The channels for

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all the users are assumed to be independent and zero-mean unit variance complex Gaussian vectors. The noise variance is $\sigma_k^2 = 1, \forall k$. The target SNR is the same for all the users, i.e., $\gamma_k = \gamma, \forall k$. In the figures, each point presents the average of randomly generated 100 channels. We compare our method with conventional and symbol-level beamforming optimization in (3a-b) and (4a-b), respectively. The current state-of-the-art ADMM algorithm in [4] is used for solving (3a-b). Convex programming solver CVX is used for solving the convex problem (4a-b). In the figures, PM, CB and SLB stand for the proposed method, conventional beamforming and symbol-level beamforming, respectively. For PM and SLB, two PSK modulation schemes which are BPSK and QPSK are considered. Multicast message symbols are chosen randomly for each realization of SLB.

In the first experiment, the number of users in each multicast group is 12, hence the total number of users is $K = 3 \times 12 = 36$. Fig. 1 illustrates the transmit power for different target SNR (γ) values. There is a significant power saving compared to SLB especially for QPSK showing the effectiveness of rotated constellation approach. CB performs only a little better than PM for $\gamma = 10$ dB. However, as γ increases, PM outperforms CB with almost 2 dB power gain. This shows that PM is more advantageous when the SNR need of the users are high.

In Fig. 2, γ is kept constant at 20 dB and the number of users per multicast group, K/M is varied. As the number of users increases, the power gap between CB and PM approaches approximately 5.7 dB and 4.7 dB for BPSK and QPSK, respectively. Similarly power gap between PM and SLB increases with K/M. For $K/M = 16$, SLB for BPSK results lower transmit power than CB. Still, PM provides 2.5 dB power saving compared to SLB.

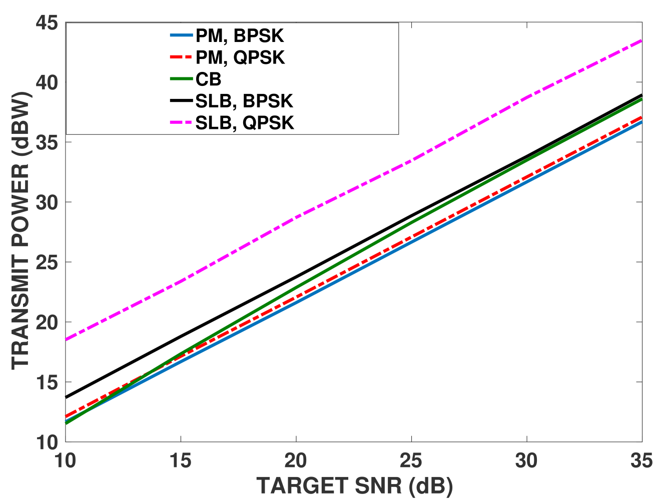

Fig. 1. Transmit power versus target SNR, γ for $K = 3 \times 12$ users.

VII. Conclusion

In this paper, a new beamforming design method which exploits constructive interference is proposed for multi-group multicasting systems. The proposed method keeps the received PSK symbols inside the safe region of constellation for any set of symbols contrary to the symbol-level beamforming. The considered non-convex optimization problem is reformulated such that ADMM can be applied in a computationally efficient manner. The closed-form optimum solutions are derived for ADMM steps. The proposed algorithm performs significantly better in terms of transmit power compared to the existing benchmarks especially when the constraints are relatively demanding and the feasible region is narrow.

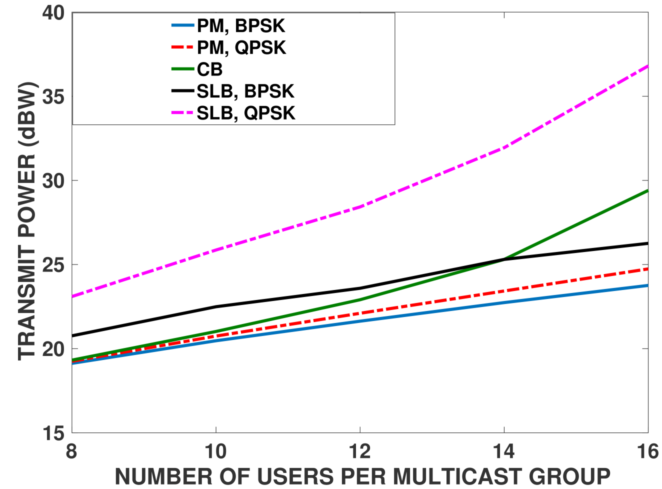

Fig. 2. Transmit power versus number of users per multicast group, K/M for $\gamma = 20$ dB.

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