Evolutionary Resampling for Multi-Target Tracking using Probability Hypothesis Density Filter

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***Abstract*—A resampling scheme is proposed for use with Sequential Monte Carlo (SMC)-based Probability Hypothesis Density (PHD) filters. It consists of two steps, first, regions of interest are identified, then an evolutionary resampling is applied for each region. Applying resampling locally corresponds to treating each target individually, while the evolutionary resampling introduces a memory to a group of particles, increasing the robustness of the estimation against noise outliers. The proposed approach is compared to the original SMC-PHD filter for tracking multiple targets in a deterministically moving targets scenario, and a noisy motion scenario. In both cases, the proposed approach provides more accurate estimates.**

I. Introduction

Driven by the wide range of applications in both military and civilian domains, Multi-Target Tracking (MTT) has received a growing interest over the last decades [1], [2]. Unlike the Single Target Tracking (STT), where a target's state is estimated, in an MTT scenario, one aims to simultaneously estimate the number of targets and their states, i.e., tracks. Similar to the STT problem, the MTT is commonly tackled through a sequential Bayesian framework. However, the MTT introduces new challenges to the Bayesian framework. First, the number of targets and measurements can be time-varying. Second, in most conventional MTT algorithms, an explicit association between measurements and targets is needed, which contributes a significant portion to the total computational complexity due to its combinatorial nature [3]–[5]. The Multiple Hypothesis Tracking (MHT) addresses the data association problem by propagating all possible associations in time [3]–[5]. Alternatively, the Joint Probability Data Association Filter (JPDAF) [1], [6] and the Probabilistic MHT (PMHT) [7] weight the observations by their association probabilities.

A different approach to tackle the MTT problem which avoids an explicit association between measurements and targets was developed using the Random Finite Set (RFS) theory [8]. An RFS approach expresses the collection of targets as a set-valued state and the collection of measurements as a set-valued observation, allowing the problem of estimating a varying number of targets using a varying number of measurements to be addressed within a full-Bayesian estimation framework [8]–[10]. This approach is compared to traditional MTT algorithms in [11]. An RFS-based fully Bayesian filter was investigated in [12] for tracking two acoustic sources. However, due to the computational complexity of the fully Bayesian MTT filter, it is not applicable for a large number of targets and one has to rely on approximations of the fully Bayesian filter by discarding the higher order moments of the RFS. This is done in the Probability Hypothesis Density (PHD) filter [8], [13], [14], where the first-order moment, called the intensity function of the RFS or PHD, is recursively propagated, in a similar fashion to the constant-gain Kalman filter. Nevertheless, the Bayesian recursion of the PHD still involves multiple integrals, precluding closed-form solutions in general. This motivated implementing PHD filters via Sequential Monte Carlo (SMC) approaches [15], [16]. SMC-based estimation methods are common tools for nonlinear system identification [17]–[19] and are often referred to as particle filtering.

In this paper, for the SMC implementation in [16], we propose to use an alternative resampling approach, that is based on evolutionary strategies [20], [21], and acts locally for an estimated number of regions. The proposed resampling step aims at increasing the robustness against outliers and to provide better estimates, especially for a lower number of particles. This is beneficial in cases where the evaluation of a single particle is already computationally expensive as in the case of complicated state transition and measurement models.

II. Overview

In this section, a brief overview of the original filter in [16] is given. However, since this overview serves as a summary of [16], numerous details related to the PHD filters and particle filters are skipped. Interested readers are referred to [8]–[10], [13], [14] for a detailed treatment of the PHD filter and to [17]–[19] for a comprehensive discussion on particle filtering.

A. The Random Finite Set Model

The varying number of targets and measurements combined with the lack of association/track identity, motivates the use of sets to represent the multi-target state at time instant k

$$X_k = \{\boldsymbol{x}_{k,1}, ..., \boldsymbol{x}_{k,N(k)}\} \in \mathcal{F}(E_s), \tag{1}$$

where $N(k)$ is the number of targets at time instant k, $\boldsymbol{x}_{k,i}$ is a single-target state vector, i.e., tracks and velocities in our simulations, defined on the single-target state space E_s and $\mathcal{F}(E_s)$ denotes all finite subsets of E_s. The $M(k)$ measurement vectors $\boldsymbol{z}_{k,1}, ..., \boldsymbol{z}_{k,M(k)}$, i.e., observed positions in our simulations, defined on the single-target measurement space E_m are represented by the RFS

$$Z_k = \{\boldsymbol{z}_{k,1}, ..., \boldsymbol{z}_{k,M(k)}\} \in \mathcal{F}(E_m), \tag{2}$$

where $\mathcal{F}(E_m)$ denotes all finite subsets of E_m.
In order to model the uncertainty in the multi-target states

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and measurements, we use the RFSs Ξ_k and Σ_k defined as finite subsets of E_s and E_m, respectively. The evolution of the multi-target states is modeled by

$$\Xi_k = S_k(X_{k-1}) \cup H_k(X_{k-1}), \tag{3}$$

where X_{k-1} denotes a realization of Ξ_{k-1}, $S_k(X_{k-1})$ is the RFS of targets which survived from time instant $k-1$ to k and $H_k(X_{k-1})$ is the RFS of the new targets at time instant k, modeled as

$$H_k(X_{k-1}) = B_k(X_{k-1}) \cup \Gamma_k, \tag{4}$$

in which, $B_k(X_{k-1})$ is the RFS of targets spawned from X_{k-1}, e.g., target splitting, while Γ_k denotes the RFS of new targets appearing spontaneously. The multi-target measurements are modeled by

$$\Sigma_k = \Theta(X_k) \cup C_k(X_k), \tag{5}$$

where $\Theta_k(X_k)$ denotes the RFS of measurements generated by X_k whereas $C_k(X_k)$ denotes the false alarms and clutter measurements. After defining appropriate probability measures and densities, one arrives at the optimal multi-target Bayes filter equations [16]

$$\begin{aligned} p_{k|k-1}(X_k|Z_{1:k-1}) &= \int f_{k|k-1}(X_k|X) p_{k-1|k-1}(X|Z_{k-1}) \lambda_s(dX) \\ p_{k|k}(X_k|Z_{1:k}) &= \frac{g_k(Z_k|X_k) p_{k|k-1}(X_k|Z_{1:k-1})}{\int g_k(Z_k|X) p_{k|k-1}(X|Z_{1:k-1}) \lambda_s(dX)}, \end{aligned} \tag{6}$$

in which λ_s is a dominating measure on the Borel subsets of $\mathcal{F}(E_s)$ (see [8]), $f_{k|k-1}(\cdot|X_{k-1})$ denotes the multi-target transition density, which encapsulates all information on targets evolution, such as a target survival, a new target birth and the propagation of remaining targets. $g_k(\cdot|X_k)$ is the multi-target likelihood, that accounts for measurement noise, detection probability and clutter-generated measurements. $p_{k|k-1}(X_k|Z_{1:k-1})$ is the multi-target predictive density. Finally, $p_{k|k}(X_k|Z_{1:k})$ denotes the multi-target posterior density. However, constructing a multi-target transition density and likelihood requires advanced mathematical tools, which are usually not familiar to engineers. This encourages the use of Finite Set Statistics (FISST) which is engineering-friendly, in the sense of representing these tools in a form which is familiar and accessible for engineers [9], and allows for a systematic construction of multi-target states and measurements descriptions from (3) and (5) [8]–[10].

B. The PHD Filter

As mentioned earlier, propagating the entire posterior can be computationally taxing. Therefore, the PHD filter propagates the intensity function D_Ξ, instead. The function D_Ξ is defined on E_s and satisfies $\int_A D_\Xi(\boldsymbol{x}) d\boldsymbol{x} = \mathcal{E}\left[|\Xi \cap A|\right]$ [10], which is the expected number of targets in a given measurable region $A \subseteq E_s$ [8], [16]. This propagation is described by two operators, the prediction operator $\Phi_{k|k-1}$ and the update operator Ψ_k. Despite the apparent computational advantage of propagating $D_{k|k}(\boldsymbol{x})$, which is the intensity function associated with the multi-target posterior $p_{k|k}$ and takes on values from E_s, the propagation operators still involve a number of integrals, excluding a closed-form solution in general (a closed form solution may be obtained by forcing further assumptions as it is done in the Gaussian Mixture PHD filter [22]). This encourages the use of an SMC-based approximation to implement the PHD filter [16].

C. The SMC Implementation of the PHD Filter

Let α_{k-1} denote an approximation of the intensity function $D_{k-1|k-1}$ using L_{k-1} particles and their weights $\{\boldsymbol{x}_{k-1}^{(i)}, w_{k-1}^{(i)}\}_{i=1}^{L_{k-1}}$ such that [16]

$$\alpha_{k-1}(\boldsymbol{x}_{k-1}) = \sum_{i=1}^{L_{k-1}} w_{k-1}^{(i)} \delta_{\boldsymbol{x}_{k-1}^{(i)}}(\boldsymbol{x}_{k-1}), \tag{7}$$

where $\delta_{\boldsymbol{x}}(\cdot)$ denotes the delta-Dirac mass located at $\boldsymbol{x}$.
Let $\phi_{k|k-1}(\boldsymbol{x}_k, \boldsymbol{x}_{k-1})$ denote the intensity function which accounts for target survival and propagation and for targets spawning given their previous state $\boldsymbol{x}_{k-1}$, while $\gamma_k(\boldsymbol{x}_k)$ describes the spontaneous birth RFS intensity function. By following the importance sampling principle, which allows samples to be drawn from the proposal densities $q_k(\cdot|\boldsymbol{x}_{k-1}^{(i)}, Z_k)$ and $p_k(\cdot|Z_k)$, which satisfy certain conditions [17]–[19], one arrives at the following approximation of $D_{k|k-1}$

$$(\Phi_{k|k-1}\alpha_{k-1})(\boldsymbol{x}_k) = \sum_{i=1}^{L_{k-1}+J_k} w_{k|k-1}^{(i)} \delta_{\boldsymbol{x}_k^{(i)}}(\boldsymbol{x}_k), \tag{8}$$

$$\boldsymbol{x}_k^{(i)} \sim \begin{cases} q_k(\cdot|\boldsymbol{x}_{k-1}^{(i)}, Z_k), & i = 1, ..., L_{k-1} \\ p_k(\cdot|Z_k), & i = L_{k-1}+1, ..., L_{k-1}+J_k \end{cases}$$

$$w_{k|k-1}^{(i)} = \begin{cases} \frac{\phi_{k|k-1}(\boldsymbol{x}_k^{(i)}, \boldsymbol{x}_{k-1}^{(i)})}{q_k(\boldsymbol{x}_k^{(i)}|\boldsymbol{x}_{k-1}^{(i)}, Z_k)} w_{k-1}^{(i)}, & i = 1, ..., L_{k-1}. \\ \frac{\gamma_k(\boldsymbol{x}_k^{(i)})}{J_k p_k(\boldsymbol{x}_k^{(i)}|Z_k)}, & i = L_{k-1}+1, ..., L_{k-1}+J_k. \end{cases}$$

Note that the number of particles grew from L_{k-1} to $L_{k-1}+J_k$ by adding J_k new particles from the birth process. Moreover, let $\kappa_k(\cdot)$ denote the clutter RFS intensity function, $v(\boldsymbol{x}_k)$ is the probability of not detecting the target $\boldsymbol{x}_k$ whereas $\psi_{k,\boldsymbol{z}}(\boldsymbol{x}_k)$ describes the likelihood of a target $\boldsymbol{x}_k$, we arrive at

$$(\Psi_k \alpha_{k|k-1})(\boldsymbol{x}_k) = \sum_{i=1}^{L_{k-1}+J_k} w_k^{(i)} \delta_{\boldsymbol{x}_k^{(i)}}(\boldsymbol{x}_k), \tag{9}$$

$$w_k^{(i)} = \left[v(\boldsymbol{x}_k^{(i)}) + \sum_{\boldsymbol{z} \in Z_k} \frac{\psi_{k,\boldsymbol{z}}(\boldsymbol{x}_k^{(i)})}{\kappa_k(\boldsymbol{z}) + C_k(\boldsymbol{z})} \right] w_{k|k-1}^{(i)}, \tag{10}$$

$$C_k(\boldsymbol{z}) = \sum_{i=1}^{L_{k-1}+J_k} \psi_{k,\boldsymbol{z}}(\boldsymbol{x}_k^{(i)}) w_{k|k-1}^{(i)}. \tag{11}$$

However, a continuous increment of the number of particles will render the scheme computationally inefficient. This is addressed in [16] by varying the number of particles according to the estimated mass for the number of targets $\hat{N}_{k|k} = \sum_{i=1}^{L_{k-1}+J_k} w_{k|k}^{(i)}$, assigning ρ particles for each target according to

$$L_k = \lfloor \rho \hat{N}_{k|k} \rfloor, \tag{12}$$

where $\lfloor\cdot\rfloor$ denotes the floor function. The new number of particles is used in the resampling step, by redistributing the total weights mass $\hat{N}_{k|k}$ over L_k resampled particles equally, i.e., all particles will be equally weighted by $\hat{N}_{k|k}/L_k$. This is done by discarding low-weighted particles while replicating particles with sufficiently large weights [17].

III. The Evolutionary Resampling

The resampling step in the above SMC implementation is similar to what is done in the conventional Sequential Importance Sampling/Resampling Particle Filter (SIR-PF) [17]–[19], [23]. However, the conventional resampling scheme suffers from the diversity decay effect due to the replicating/discarding process. Moreover, since all particles are resampled, particle memory is reset at each iteration. We propose a modified resampling scheme, where the intensity function is approximated using a Gaussian Mixture Model (GMM), each component of which is resampled individually using evolutionary strategies [20], [21]. The number of components in the GMM corresponds to the number of what we call Regions of Interest (ROI) in the following.

A. The Regions of Interest Identification

ROIs are regions that probably enclose a single target. This loose definition implies the potential of having more ROIs than actual targets, which is allowed, since some regions will enclose clutter measurements and thus, the identification of ROIs is not unique. To identify ROIs, we first initialize the number of ROIs with $R = L_{k-1} + J_k$, so that each ROI is represented by a single particle. Afterwards, starting randomly at region j, characterized by the set Λ_j of all particles in the region, the nearest particle to the region center

$$\boldsymbol{\mu}_j = \frac{\sum_{i:\boldsymbol{x}_k^{(i)}\in\Lambda_j} w_k^{(i)}\boldsymbol{x}_k^{(i)}}{\sum_{i:\boldsymbol{x}_k^{(i)}\in\Lambda_j} w_k^{(i)}}, \tag{13}$$

w.r.t. the Euclidean metric, is included into the set Λ_j, and is consequently removed from other regions. Afterwards, the region center is updated and the process is repeated until one of the following conditions is satisfied:

- The sum of the particles weights for the j-th ROI is $\hat{N}_{k|k}^{(j)} = \sum_{i:\boldsymbol{x}_k^{(i)}\in\Lambda_j} w_k^{(i)} > 1$, this implies that a single region can not contain more than a single target.
- The Euclidean distance $||(\boldsymbol{x}_k^{(i)} - \boldsymbol{\mu}_j)||_2$ between the nearest particle and the region center is larger than a predefined limit d_{max}.

This procedure is then iterated for non-empty regions. A simple outlier rejection is performed to remove insignificant regions, i.e., a region with one particle and a weight below a threshold (chosen to be 10^{-3} in this paper). The weights of the remaining L_{rem} particles have to be renormalized to keep the weights mass $\sum_{i=1}^{L_{\text{rem}}} w_k^{(i)} = \hat{N}_{k|k}$ fixed.

B. The Local Resampling

After identifying the ROIs, an evolutionary resampling step for each ROI is applied. Assuming that ROI j is under consideration, the number of particles needed in this region is calculated $L_k^{(j)} = \lfloor\rho\sum_{i:\boldsymbol{x}_k^{(i)}\in\Lambda_j} w_k^{(i)}\rfloor = \lfloor\rho\hat{N}_{k|k}^{(j)}\rfloor$. Afterwards, the set Λ_j is divided into an elitist subset $\mathcal{W}_{j,k}$ and a non-elitist subset $\mathcal{M}_{j,k}$

$$\boldsymbol{x}_k^{(i)} \in \begin{cases} \mathcal{W}_{j,k}, & \text{if} \quad w_k^{(i)} \geq w_{\text{th}}, \\ \mathcal{M}_{j,k}, & \text{if} \quad w_k^{(i)} < w_{\text{th}}, \end{cases} \tag{14}$$

where $w_{\text{th}} = 1/|\Lambda_j|\sum_{i:\boldsymbol{x}_k^{(i)}\in\Lambda_j} w_k^{(i)}$ [21] is a weight threshold, in which $|\Lambda_j|$ denotes the number of particles in Λ_j. While the non-elitist subset $\mathcal{M}_{j,k}$ is always discarded, three cases are distinguished for the elitist subset $\mathcal{W}_{j,k}$:

1) The number of elitist particles $|\mathcal{W}_{j,k}|$ is larger than the needed number of particles in the ROI $|\mathcal{W}_{j,k}| > L_k^{(j)}$. In this case, only the $L_k^{(j)}$ elitist particles with the highest weights are kept, all other particles are discarded.
2) The number of elitist particles is smaller than the needed number of particles in the ROI $|\mathcal{W}_{j,k}| < L_k^{(j)}$. In this case, $L_k^{(j),\text{new}} = L_k^{(j)} - |\mathcal{W}_{j,k}|$ new particles are needed. In order to generate these, elitist particles are used to construct a Gaussian density $\mathcal{N}(\overline{\boldsymbol{x}}_{k,j}, \boldsymbol{Q}_{k,j})$ with a mean vector and a covariance matrix obtained according to

$$\overline{\boldsymbol{x}}_{k,j} = \frac{\sum_{i:\boldsymbol{x}_k^{(i)}\in\mathcal{W}_{j,k}} w_k^{(i)}\boldsymbol{x}_k^{(i)}}{\sum_{i:\boldsymbol{x}_k^{(i)}\in\mathcal{W}_{j,k}} w_k^{(i)}}, \tag{15}$$

$$\boldsymbol{Q}_{k,j} = \frac{\sum_{i:\boldsymbol{x}_k^{(i)}\in\mathcal{W}_{j,k}} w_k^{(i)}(\boldsymbol{x}_k^{(i)} - \overline{\boldsymbol{x}}_{k,j})(\boldsymbol{x}_k^{(i)} - \overline{\boldsymbol{x}}_{k,j})^{\text{T}}}{\sum_{i:\boldsymbol{x}_k^{(i)}\in\mathcal{W}_{j,k}} w_k^{(i)}}. \tag{16}$$

The $L_k^{(j),\text{new}}$ new particles are then drawn from $\mathcal{N}(\overline{\boldsymbol{x}}_{k,j}, \boldsymbol{Q}_{k,j})$ and, since $\mathcal{N}(\overline{\boldsymbol{x}}_{k,j}, \boldsymbol{Q}_{k,j})$ is an approximation of the target intensity function, the new particles are assigned equal weights $\hat{N}_{k|k}^{(j)}/L_k^{(j),\text{new}}$.
3) The number of elitist particles $|\mathcal{W}_{j,k}|$ is equal to the needed number of particles in the cluster $|\mathcal{W}_{j,k}| = L_k^{(j)}$. Then, the elitist particles are kept unchanged.

In order to keep the estimated number of targets within the region fixed, weights are renormalized such that $\sum_{i:\boldsymbol{x}_k^{(i)}\in\Lambda_j} w_k^{(i)} = \hat{N}_{k|k}^{(j)}$. The GMM approximation of the intensity function accounts for the multi-modality of the multi-target intensity function, whereas resampling regions individually corresponds to an individual treatment of targets. The evolutionary resampling scheme [21] will introduce a memory to the elitist particles, due to the fact that it does not resample all particles, thereby increasing robustness against outliers. In addition, drawing new particles from a continuous density, such as a Gaussian, increases the particle diversity compared to the original implementation in [16], counteracting the diversity decay problem. Finally, we will refer to the SMC-PHD implementation using the proposed resampling scheme as the Evolutionary Resampling PHD (ER-PHD) in the following.

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Filter	Deterministic motion			Noisy motion		
	$\overline{e}_p$	$\overline{e}_{\text{loc},p}$	$\overline{e}_{\text{card},p}$	$\overline{e}_p$	$\overline{e}_{\text{loc},p}$	$\overline{e}_{\text{card},p}$
SMC-PHD	61	42	19	76	36	40
ER-PHD	54	38	16	66	41	25

TABLE I: OSPA values of SMC-PHD and ER-PHD estimates.

IV. Experimental Results

In this section, the proposed resampling scheme is evaluated and compared to the original approach in [16] using a non-linear multi-target tracking scenario [16] in the 2D physical space. In the following simulations, the state vector of the target t, $\boldsymbol{x}_k^{(t)} = [x_k^{(t)}, \dot{x}_k^{(t)}, y_k^{(t)}, \dot{y}_k^{(t)}, \Delta\theta_k^{(t)}]^{\mathrm{T}}$ follows the model

$$\boldsymbol{x}_k^{(t)} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & \cos(\Delta\theta_k^{(t)}) & 0 & -\sin(\Delta\theta_k^{(t)}) & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & \sin(\Delta\theta_k^{(t)}) & 0 & \cos(\Delta\theta_k^{(t)}) & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \boldsymbol{x}_{k-1}^{(t)} + \boldsymbol{u}_k, \tag{17}$$

where $x_k^{(t)}, y_k^{(t)}$ denote the coordinates of target t, $\dot{x}_k^{(t)}, \dot{y}_k^{(t)}$ denote the velocities on the corresponding axis and $\Delta\theta_k^{(t)}$ is the change in direction between two successive velocity vectors. The process noise is denotes by $\boldsymbol{u}_k \sim \mathcal{N}(\boldsymbol{0}_{5\times 1}, \boldsymbol{Q}_u)$, where $\boldsymbol{Q}_u$ is a zero matrix or a diagonal matrix depending on the scenario. Tracks were allowed to be generated within the range $(x_k, y_k) \in [-2000, 2000] \times [-2000, 2000]$. The single-target state vector $\boldsymbol{x}_k^{(t)}$ is observed via bearing $b_k^{(t)}$ and range $r_k^{(t)}$ measurements, captured in the vector $\boldsymbol{z}_k^{(t)} = [b_k^{(t)}, r_k^{(t)}]^{\mathrm{T}}$. Assuming that the sensor is centered at the origin of the coordinate system, the measurements are modeled by

$$b_k^{(t)} = \arctan\left(\frac{x_k^{(t)}}{y_k^{(t)}}\right) + v_{b,k}, \text{ and } r_k^{(t)} = \left\| [x_k^{(t)}, y_k^{(t)}]^{\mathrm{T}} \right\|_2 + v_{r,k}, \tag{18}$$

where $\boldsymbol{v}_k = [v_{\mathrm{b},k}, v_{\mathrm{r},k}]^{\mathrm{T}} \sim \mathcal{N}(\boldsymbol{0}_{2\times 1}, \text{diag}\left(\left[\frac{2\pi}{180}, 10\right]\right))$.

The clutter noise is uniformly distributed with an average rate of 10 points per scan, i.e., a Poisson point process with a uniform intensity function.

The PHD filters are designed to have a constant survival probability $e_{k|k-1}(\cdot) = 0.99$, [16]. For simplicity, no spawning is considered in the simulations. The spontaneous birth process is modeled by a Poisson process, with a rate of birth per scan 0.1 and a weighted Gaussian mixture intensity function consisting of 4 components. The mean vectors of the Gaussian components match the true birth, i.e., initialization, positions of the generated targets and the covariance matrices of the mixture components are set to $\boldsymbol{Q}_B = \text{diag}\left(\left[50, 50, 50, 50, \frac{6\pi}{180}\right]\right)$. Filters are initiated by drawing samples from the density $\mathcal{N}(\overline{\boldsymbol{x}}_0, \boldsymbol{Q}_0)$, where $\overline{\boldsymbol{x}}_0 = [0.1, 0, 0.10, 0.01]^{\mathrm{T}}$ and $\boldsymbol{Q}_0 = \text{diag}\left([100, 10, 100, 100, 1]\right)$. The importance densities q_k and p_k match the state transition and birth densities described above, respectively. Furthermore, details on the detection density and likelihood calculations can be found in [16]. Evaluating the performance of an MTT scheme is not intuitive and many measures can be used [24]–[26]. In this paper, the Optimal Subpattern Assignment (OSPA) metric [27] is employed, which accounts for over- or underestimating the number of targets and the quality of estimation per target. The metric includes three components $\overline{e}_{\text{loc},p}$, $\overline{e}_{\text{card},p}$ and $\overline{e}_p$, which describe the localization error of the detected targets, the cardinality error and the total error combining the previous two measures, respectively [27]. In [27], the metric has two parameters to be defined, i.e., c and p. Here, we use $c = 100$ and $p = 1$. In the ER-PHD, $d_{\text{max}} = 75$ is used. Finally for both filters, the number of particles per target is $\rho = 500$.

A. The Deterministic Motion Scenario

In this simulation, deterministic tracks of 10 targets are generated using (17) with $\boldsymbol{u}_k = \boldsymbol{0}$. The PHD filters' hypothesized state transition model matches (17) with

$$\boldsymbol{Q}_u = \text{diag}\left(\left[6.25, 25, 6.25, 25, 3 \cdot 10^{-4}\right]\right). \tag{19}$$

The two filters, the original SMC-PHD [16] and the ER-PHD, are compared w.r.t. their performance in tracking the different targets. The simulation is realized 20 times. The average, over time steps and realizations, OSPA performance metric $\overline{e}_p$ and its two components $\overline{e}_{\text{loc},p}$ and $\overline{e}_{\text{card},p}$, are given in Table I.
As Table I shows, the ER-PHD yields a slightly lower $\overline{e}_p$, and this advantage is maintained for both $\overline{e}_{\text{loc},p}$ and $\overline{e}_{\text{card},p}$. However, to provide an intuition of what these values mean, the target estimates produced by the two filters, the SMC-PHD and the ER-PHD, are depicted in Figure 1 and 2, respectively. As the estimates show, the ER-PHD provides more accurate estimates. This is most pronounced in the case where a target is almost not detected at all with the SMC-PHD, e.g., the tracks highlighted in red, while the ER-PHD produces a relatively accurate estimate of the respective target. For ρ below 500, the ER-PHD also starts to miss targets, while the performance difference relative to the SMC-PHD, evaluated by the OSPA metric, even increases. Finally, as the number of particles increases, the performance difference decreases and at $\rho = 1000$ becomes insignificant

B. The Noisy Motion Scenario

In this experiment, the target-tracks are generated according to the model (17) with the covariance matrix $\boldsymbol{Q}_u$ in (19). This will introduce the probability of a target motion outlier, making the task harder since the targets move abruptly and can change their turning rate. The simulations are repeated a 100 times, meaning that 100 different tracks are generated and tracked. Similar to the previous experiment, the different OSPA values are averaged over the 100 realizations and time steps. The average values are provided in Table I, where the ER-PHD has a clear advantage in estimating the number of targets, which is due to the introduction of memory to the elitist group of the particles, increasing the robustness against outliers in the motion model. Nevertheless, the memory introduced by this resampling scheme is short enough for the filter to adapt to abrupt changes in the number of targets, as seen in the simulations. Finally, the SMC-PHD has a lower localization error $\overline{e}_{\text{loc},p}$ which can be explained as follows: the SMC-PHD underestimates the number of targets severely compared to the ER-PHD, indicating that the SMC-PHD

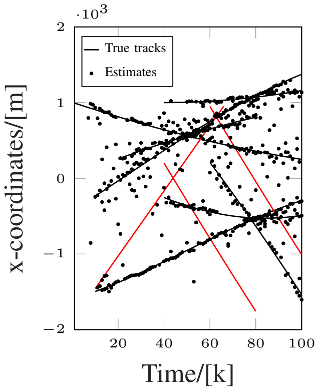

Fig. 1: SMC-PHD tracks estimates.

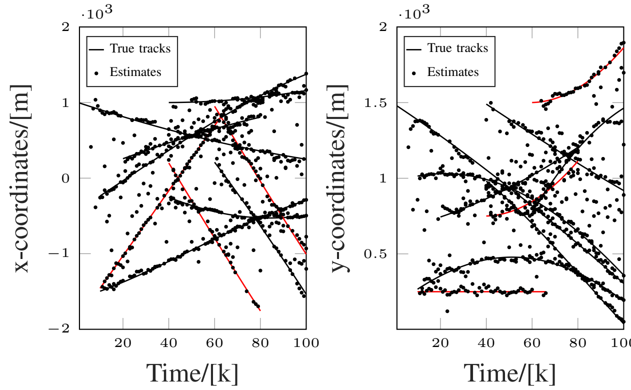

Fig. 2: ER-PHD tracks estimates.

detects the most dominant or 'clear' targets in the set. This then results in a lower localization error on average than localizing the full set including 'noisy' targets. This effect did not show up in the deterministic motion model scenario, where the difference of cardinality error between the two targets was not as large as it is in the noisy motion scenario.

V. Conclusion

In this paper, a resampling scheme is proposed to be used in an SMC-based implementation of PHD filters for MTT. In this scheme, ROIs are first identified, each ROI is then resampled using an evolutionary scheme. The individual treatment of the ROIs corresponds to resampling targets individually, while the evolutionary resampling increases robustness against outliers by introducing an extra memory to the elitist particles. This scheme was compared to the original SMC-PHD filter proposed in [16] in two scenarios, where the ER-PHD outperformed the original filter, especially for a low number of particles, which can be beneficial in cases where the evaluation of a single particle is expensive, e.g., for complicated state transition models using non-elementary functions.

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