On the time-frequency reassignment of interfering modes in multicomponent FM signals

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***Abstract*—The paper presents a first attempt to correct the time-frequency reassignment of a multicomponent signal having non separable individual components. In particular, the case of a 2-components signal has been investigated in depth. It has been proved that the integral (along frequencies) of the spectrogram is still a multicomponent signal with specific instantaneous frequencies. As a result, the spectrogram of this signal allows us to disentangle the frequencies and recover the missing information in the non separability region. Preliminary results show that the proposed method is able to correctly reassign the information in the interference region by separating the two individual components, with a very moderate computational effort.**

***Index Terms*—Time-frequency reassignment, multicomponent signals, instantaneous frequency, spectrogram**

I. Introduction

The representation and analysis of non stationary signals is a very interesting and challenging research topic; at the same time, it is fundamental in applications involving chirp-like signals, i.e. frequency modulated signals as radar, audio and speech signals and more recently, gravitational waves. Linear and quadratic time-frequency analysis [1], [7], [11], [12], [14], [16] offers a valid response to that problem. Its aim is to reveal the frequency content of a signal as well as the law regulating its variation with respect to time. More precisely, a multicomponent frequency modulated signal is defined as [12]

$$f(t) = \sum_{k=1}^{N} f_k(t) = \sum_{k=1}^{N} a_k(t) e^{i\phi_k(t)}, \tag{1}$$

where f_k is the $k-$th mode and a_k and ϕ_k respectively are its amplitude and phase functions of the time variable t; hence, the desired aim is to have a transform which allows for a straightforward estimation of the instantaneous frequency of each mode composing the signal, where the instantaneous frequency is defined as the positive value of the phase derivative [4], [5]. Unfortunately, time-frequency transforms are not always able to provide a compact and not ambiguous representation of signal time-frequency features. Among the methods proposed to enhance the readability of time-frequency representations, reassignment method is probably the most powerful [2]. Since time-frequency transforms are mainly windowed transforms, it consists of assigning the energy of each point in the time-frequency plane to the energy center of mass within a domain having the amplitude of the analysis window. To this aim, information from the phase spectrum is employed. In the case of Short Time Fourier Transform (STFT), this has the effect of significantly sharpening spectrogram appearance. Moreover, it is computationally efficient since the use of phase information corresponds to a combination of STFTs with suitable analysis windows. Similar properties can be found in the synchrosqueezing transform [6], [13]. The latter can be considered a special case of reassignment method but it also allows for signal reconstruction. Time-frequency reassignment can also be view as the estimation of the instantaneous frequency and group delay for each point on the time-frequency plane. That is why, most of instantaneous frequency estimation methods can be seen as particular cases or modifications of the basic reassignment method [9], [10], [15].

Unfortunately, one of the main constraints in a reassignment-like method is the separability condition on the individual signal components [3]. The time-frequency points are correctly reassigned to their corresponding mode if the following condition is satisfied [2], [3]

$$|\phi'_k(t) - \phi'_j(t)| \geq \Delta\omega, \quad j, k \in [1, N], \quad k \neq j$$

where $\Delta\omega$ is the frequency bandwidth of the analysis window. It is worth observing that such condition can be made true with a suitable choice of the analysis window whenever

$$\Delta\phi'_{k,j}(t) = \phi'_k(t) - \phi'_j(t) > 0, \quad \forall\, t, \tag{2}$$

or viceversa. If this property is not met, it is not possible to achieve the desired multicomponents decomposition. Two examples of spectrograms of non separable and separable components are respectively shown in Fig.1. a and b.

The aim of this paper is to contribute to overcome this limit in the reassignment method. With regard to a two components signal, the integral along frequency axis of its spectrogram, referred as $E(u)$, has been investigated (Fig. 1.c). As detailed in the next section, it is a function of the time variable and can be modeled as a multicomponent signal whose instantaneous frequency is $\Delta\phi'_{1,2}(t)$, as defined in Eq. (2). In addition, it provides information concerning the non separability regions (interference between two modes in the time-frequency plane) because its amplitude is concentrated in the region where the frequencies of the individual components are the closest. As a

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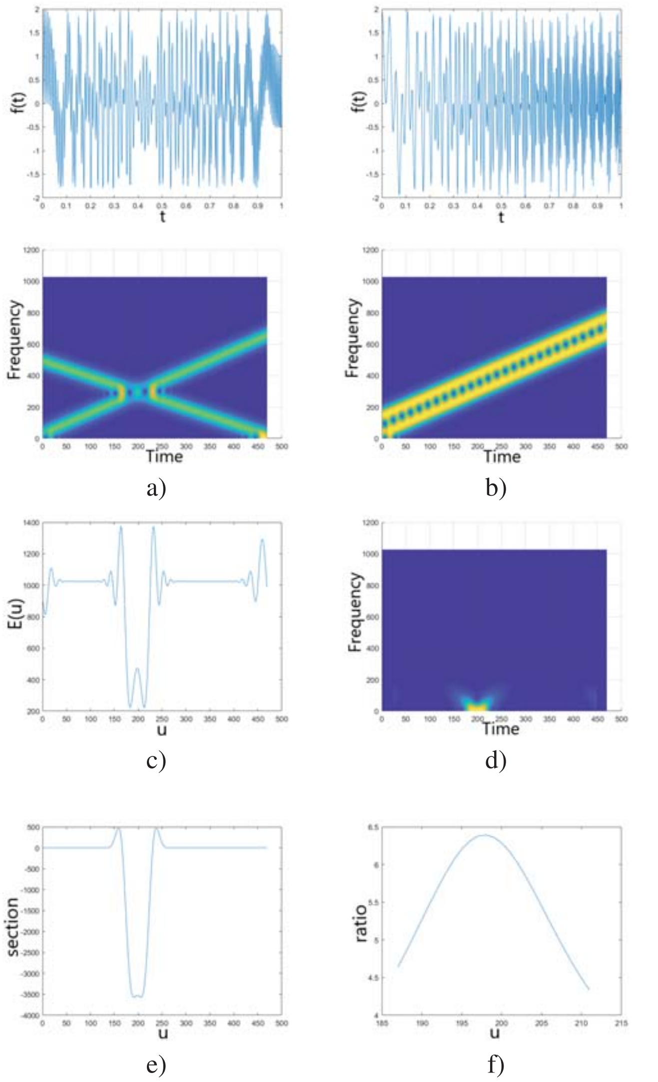

Fig. 1. Two components FM signals and their spectrogram: (*a*) non separable components; (*b*) separable components; (*c*) Plot of $E(u)$, as in Proposition 1, of the spectrogram in (a); (*d*) Spectrogram of $E(u)$; (*e*) Section at frequency equal to 0 of the real part of the STFT of $E(u)$; (*f*) Plot of the ratio as in Eq. (17) restricted to the detected interference region.

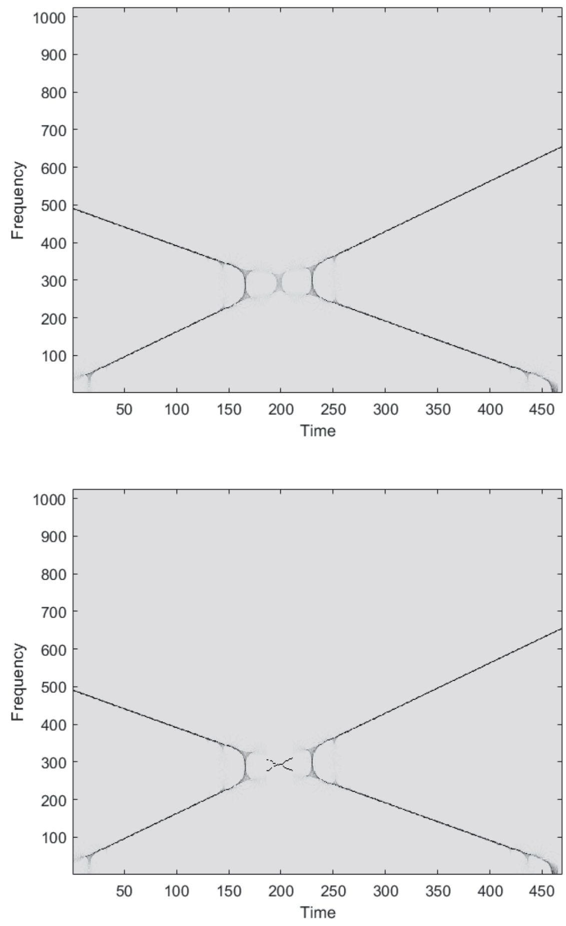

Fig. 2. (*Top*) Reassignment of the spectrogram in Fig. 1.a); (*Bottom*) Corrected spectrogram via the proposed approach.

consequence, the spectrogram of this energy signal (Fig. 1.d) can be used for estimating $\Delta\phi'_{1,2}(t)$ in the interference region. Preliminary experimental results show that the proposed method is able to resolve the individual signal components in the interference region with greater precision than classical methods, requiring a very moderate computational effort. The remainder of the paper is the following. Next section presents the proposed model and details concerning the proposed reassignment procedure in interference regions. Section III shows some experimental results along with some concluding remarks.

II. The Proposed Model

Spectrogram reassignment is a sort of post-processing method whose aim is to improve its readability. In particular, it is useful for separating and distinguishing the individual modes of a multicomponent signal and for making the spectrogram less blurred around the single instantaneous frequencies. Let us denote with $P_S f(u,\xi) = |Sf(u,\xi)|^2$ the spectrogram of the signal $f(t) = a(t)\cos\phi(t)$, where $Sf(u,\xi)$ stands for the short-time Fourier transform (STFT) computed with a simmetric and real window $g(t)$, with $\int_{-\infty}^{+\infty} g(t)dt = 1$, i.e.

$$Sf(u,\xi) = \int_{-\infty}^{+\infty} f(t)g(t-u)e^{-i\xi t}\,dt\,. \tag{3}$$

The reassigned spectrogram [8] is defined as

$$\hat{P}_S(u,\xi) = \int_{\mathbf{R}^2} P_S f(u',\xi')\delta(u' - \hat{u}_f(u',\xi'), \xi' - \hat{\xi}_f(u',\xi'))\frac{du'd\xi'}{2\pi}\,. \tag{4}$$

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$\hat{u}_f$ and $\hat{\xi}_f$ define the local centroids of the Wigner Ville distribution of f and they can be efficiently computed as

$$\hat{u}_f(u,\xi) = u + \Re\left(\frac{Sf^{tg}(u,\xi)}{Sf^g(u,\xi)}\right) \tag{5}$$

$$\hat{\xi}_f(u,\xi) = \xi - \Im\left(\frac{Sf^{g'}(u,\xi)}{Sf^g(u,\xi)}\right) \tag{6}$$

where Sf^* denotes the STFT computed using $*$ as analysis function [8] and $\Re(z)$, $\Im(z)$ respectively stand for the real and imaginay part of the complex number z. The computation of the reassigned spectrogram consists of assigning the total mass to the center of the distribution within the domain. An example is shown in Fig. 2.top. Unfortunately, if the individual components of the analysed signal are such that their instantaneous frequency curves intersect, reassignment fails in correspondence to the interference. In the next section we will prove that it is possible to recover the lost information in the interference region. The following model for the STFT of a signal $f(t) = a(t)\cos\phi(t)$ has been adopted [12].

$$Sf(u,\xi) = \frac{1}{2}a(u)e^{i(\phi(u)-\xi u)}(\hat{g}(\xi-\phi'(u)) + \epsilon(u,\xi)) \tag{7}$$

with $\xi \geqslant 0$. $\epsilon(u,\xi)$ is a corrective term which is negligible if $a(t)$ and $\phi'(t)$ have small relative variations over the support of the window g and if $\phi'(u) \geq \Delta\omega$, where $\Delta\omega$ is the bandwidth of $\hat{g}$. In the sequel, $\epsilon(u,\xi)$ will be considered negligible.

A. The model

Let $f(t) = \sum_k a_k cos\phi_k(t)$ be a multicomponent signal with constant amplitudes; then, the following Proposition holds.

Proposition 1 The integral $E(u) = \int_{-\infty}^{\infty} P_S f(u,\xi)d\xi$ of the spectrogram $P_S f(u,\xi)$ computed w.r.t. frequency direction is such that

$$E(u) = c + \sum_{k\neq j} A_{k,j}(u)\cos\theta_{k,j}(u) \tag{8}$$

with $A_{k,j}(u)=\frac{a_k a_j}{2}\int\limits_{-\infty}^{+\infty}\hat{g}(\xi-\phi'_k)\hat{g}(\xi-\phi'_j)d\xi$ and $\theta_{k,j}(u) = \phi_k(u)-\phi_j(u)$, $c = \frac{\pi}{2}\sum_k a_k^2 \int\limits_{-\infty}^{+\infty} g(t)^2 dt$.

The Proof is in Appendix.

Prop. 1 proves that the energy of the spectrogram computed over the frequency domain is a multicomponent signal with respect to the time variable, having non constant amplitude — see Fig. 1.c. The instantaneous frequencies of the individual modes of this new signal are the difference between couples of instantaneous frequency of the modes in the original signal. In addition, the amplitudes depend on these differences and they vanish as these differences increase. In other words, the energy of these modes is mostly concentrated in the interference regions we are interested in. In order to estimate $\Delta\phi'_{k,j}(u) = \phi'_k(u) - \phi'_j(u)$, we will exploit the spectrogram of $E(u)$, shown in Fig. 1.d.

B. $\Delta\phi'$ estimate

The following procedure can be used to estimate $|\Delta\phi'_{k,j}(u)| = |\phi'_k(u) - \phi'_j(u)|$, under the hypothesis of separated interference regions w.r.t. ξ. Without loss of generality, let us consider an FM signal having two components, f_1 and f_2, with constant amplitude $a = 1$, i.e.

$$f(t) = \cos\phi_1(t) + \cos\phi_2(t). \tag{9}$$

Proposition 2 Let $Sf(u,\xi)$ be the STFT of $f(t)$ as defined in Eq. (9), computed by a Gaussian window g with variance equal to σ^2 and $\int_{-\infty}^{+\infty} g(t)dt = 1$. Set $f_{int}(u) = E(u) - c$ according to Eq. (8) and let $Sf_{int}(u,\xi)$ be the corresponding STFT computed using the same window g. If $\Delta\phi'(u) = \phi'_2(u) - \phi'_1(u)$, then

$$|\Delta\phi'(u)| = \frac{2}{\sigma^2}\sqrt{-\ln\left(\frac{2}{\sqrt{2\pi\sigma^2}}\frac{\Re(Sf(u,\xi))}{f_{int}(u)}\right)}. \tag{10}$$

Proof is in Appendix.

C. Reassignment in the interference region

In case of multiple frequencies which are not sufficiently apart, we can't correctly estimate the local centroids in Eqs.(5) and (6) at interference regions. The estimate in Eq. (10) can be used in order to partially reconstruct the center of mass, in interference regions, and then to improve the resolution of the reassigned spectrogram as follows. Let us denote by G the center of mass of the reassigned distribution, along the frequency direction, i.e.

$$G(u) = \frac{1}{E(u)}\int\limits_{-\infty}^{+\infty} \xi \cdot \hat{P}_S f(u,\xi)d\xi. \tag{11}$$

Since the ridge curves are the center of mass of the reassigned spectrogram, it can be also written as $G(u) = \frac{\phi'_1(u)+\phi'_2(u)}{2}$. Then, it is possible to determine the points of ridge curves in the interference region simply computing the quantities

$$\bar{\xi}(u) = G(u) \pm \frac{|\Delta\phi'(u)|}{2}. \tag{12}$$

D. Algorithm

Summing up, the proposed method consists of the following steps. Let f be a two-component signal as in Eq. (9).

1) Compute the spectrogram $P_S f(u,\xi)$ of f using a Gaussian function g as analysis window (Fig. 1.a).
2) Estimate the integral of $P_S f(u,\xi)$ w.r.t. the frequency axis as the sum along frequencies and denote it by $E(u)$ (Fig. 1.c).
3) Set $f_{int}(u) = E(u) - c$, according to Proposition 1.
4) Compute the STFT of $f_{int}(u)$ (Figs. 1.d, 1.e).
5) Compute the ratio in Eq. (17) (see Fig. 1.f) and estimate the frequencies $\Delta\phi'(u)$ as in Eq. (10).
6) Estimate the compact interference region $\bar{\Omega}$ by retaining those points such that the energy $E(u)$ over-exceeds the 10% of its maximum value.
7) Compute the quantity in Eq. (11) and reassign the estimated frequencies as in Eq. (12) $\forall\, u \in \bar{\Omega}$.

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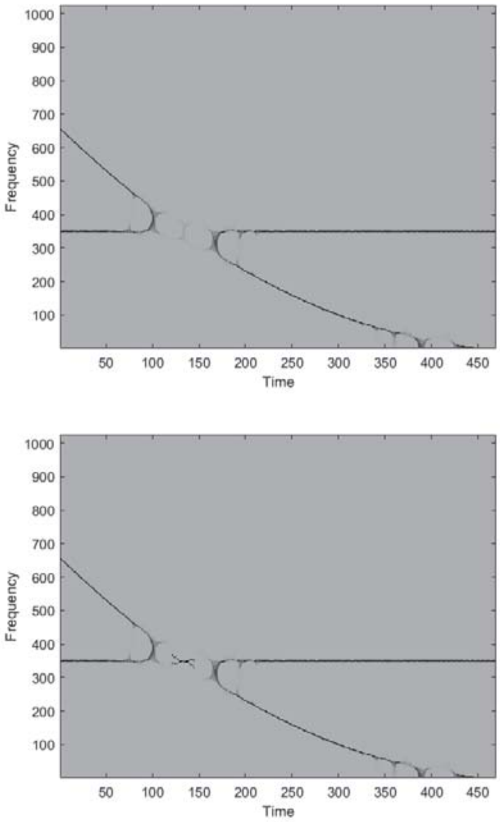

Fig. 3. (*Top*) Reassignment of the spectrogram of a signal whose modes respectively are a constant FM signal and a quadratic FM signal; (*Bottom*) Corrected spectrogram via the proposed approach.

III. EXPERIMENTAL RESULTS AND CONCLUSIONS

The proposed method has been tested on several multicomponent FM-signals. In the following, for the sake of brevity, we will present two representative results. In the presented test, one multicomponent signal has been considered: it consists of a sum of two linear FM components, i.e. $f(t) = cos(\pi/3nt^2) + sin(\pi/4n(1-t)^2)$, where n is the signal length and $t \in [0,1]$. Starting from the result of the reassignment method, the proposed model gives the result in Fig. 2.bottom. As it can be observed, we are able to deal with the interference region independently of the external regions. We further stress that the proposed model is just a preliminary attempt to resolve modes in the interference region; that is why reassignment has been corrected just in the central part of the interference region. In fact, the cross region between non interference/interference causes some numerical instability in the proposed solution — this is currently under study. Despite this drawback, the strength of the proposed approach is proved by the ability in correctly drawing the two individual modes even in the most critical region: complete interference. To further highlight this point, Fig. 3 depicts the result obtained for a two components signal whose modes are a constant FM signal and a quadratic one. As it can be observed, time-frequency points have been correctly reassigned in the interference region of the two signal modes.

Future research will focus on dealing with more than two components making the whole framework more robust to more complicated, and then closer to real study cases. It is straightforward that such a generalization will involve both theoretical and practical aspects that have not yet been considered so far, making existing tools more and more effective.

APPENDIX

Proof of Proposition 1 By linearity of the STFT and some algebra, it follows that

$$|Sf(u,\xi)|^2 = \sum_k |Sf_k(u,\xi)|^2$$
$$+2\sum_{k\neq j} \cos\theta_{k,j}(u,\xi)|Sf_k(u,\xi)|\cdot|Sf_j(u,\xi)| \quad (13)$$

where $\theta_{i,j}$ is the angle between $Sf_k(u,\xi)$ and $Sf_j(u,\xi)$. By Eq. (7):

$$|Sf_k(u,\xi)| = \frac{a_k}{2}\hat{g}(\xi - \phi'_k(u)) \,. \quad (14)$$

Moreover, it follows that $\theta_{k,j}(u) = \phi_k(u) - \phi_j(u)$, i.e. it is independent of ξ. By substituting in Eq. (13) and by summing over ξ we obtain

$$\int_{-\infty}^{+\infty} |Sf(u,\xi)|^2 d\xi = \sum_k \frac{a_k^2}{4} \int_{-\infty}^{+\infty} \hat{g}(\xi - \phi'_k(u))^2 d\xi$$
$$+\frac{a_k a_j}{2}\cos\theta_{i,j}(u)\sum_{k\neq j}\int_{-\infty}^{+\infty} \hat{g}(\xi-\phi'_k(u))\hat{g}(\xi-\phi'_j(u))d\xi \,. \quad (15)$$

By Plancharel formula, $\int_{-\infty}^{+\infty} \hat{g}(\xi-\phi'_k(u))^2 d\xi = 2\pi \int_{-\infty}^{+\infty} g(t)^2 dt$. Hence, Eq. (8) follows by defining $c = \frac{\pi}{2}\sum_k a_k^2 \int_{-\infty}^{+\infty} g(t)^2 dt$ and $A_{k,j}(u) = \frac{a_k a_j}{2} \int_{-\infty}^{+\infty} \hat{g}(\xi-\phi'_k)\hat{g}(\xi-\phi'_j)d\xi$.

Proof of Proposition 2 From Eq. (7) we get

$$\Re(Sf_{int}(u,\xi)) = \frac{1}{2}A(u)\hat{g}(\xi - |\Delta\phi'(u)|)\cos(\Delta\phi(u)) \,. \quad (16)$$

For all u belonging to the support of f_{int}, we consider the ratio

$$\frac{\Re(Sf(u,\xi))}{f_{int}(u)} = \frac{1}{2}\hat{g}(\xi - |\Delta\phi'(u)|) \,. \quad (17)$$

Since $\hat{g}(\xi) = \sqrt{2\pi\sigma^2}\exp(-\frac{1}{2}\sigma^2\xi^2)$, by evaluating Eq. (17) at $\xi = 0$, it follows

$$\exp(-\frac{1}{2}\sigma^2\Delta\phi'^2) = \frac{2}{\sqrt{2\pi\sigma^2}}\frac{\Re(Sf(u,\xi))}{f_{int}(u)} \quad (18)$$

then, for all u such that the quantity on the right is less than 1, we get the proof.

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