

On Room Impulse Response Measurement Using Perfect Sequences for Wiener Nonlinear Filters

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Abstract—In a recent paper, we have proposed a novel approach for measuring the room impulse response (RIR) robust toward the nonlinearities affecting the power amplifier or the loudspeaker. The approach is implemented by modeling the acoustic path as a Legendre nonlinear (LN) filter and by measuring the first-order kernel using perfect periodic sequences (PPSs) and the cross-correlation method. PPSs are periodic sequences that guarantee the perfect orthogonality of the basis functions of a certain nonlinear filter over a period. For LN filters, PPSs have approximately a uniform distribution. We have shown that also the Wiener Nonlinear (WN) filters, which derive from the truncation of the Wiener series, admit PPSs, whose sample distribution approximates a Gaussian distribution. Thus, WN filters and their PPSs appear more appealing for measuring the RIR. The paper discusses RIR measurement using WN filters and PPSs and explains how PPSs for WN filter suitable for RIR identification can be developed. Experimental results, using signals affected by real nonlinear devices, illustrate the effectiveness of the proposed approach and compare it with that based on LN filters.

I. INTRODUCTION

Measuring the room impulse response (RIR) is an important operation in acoustic and audio signal processing. It is used for analyzing and characterizing the room response, estimating parameters like reverberation time, early decay time, clarity, definition, interaural cross-correlation, lateral energy fraction, etc. [1]. It is also the first step of many audio applications, like room response equalization [2], spatial audio rendering [3], virtual sound [4], room geometry inference [5], and others.

Different approaches have been proposed for measuring the RIR: from the use of impulsive signals and time stretched pulses, to maximal length sequences (MLSs) [6], perfect periodic sequences (PPSs) for linear systems [7], linear sweeps, exponential sweeps (ESs) [8], [9], perfect sweeps [10], and many others. A problem affecting many of these approaches is the sensitivity to nonlinearities in the measurement systems. While the acoustic path can be considered as a linear system, the high volume of the measurement signal used to contrast noise often causes the appearance of nonlinear effects in the power amplifier or in the loudspeaker of the measurement system. These nonlinear effects are often the cause of artifacts in the measured signal, such as spikes in the measured RIR using MLSs approach [11]. ESs [8], [9] and synchronized

ESs [12] are often used to measure the RIR contrasting the effect of measurement system nonlinearities. Indeed, these techniques can be made immune to nonlinearities, provided the measurement system can be modeled as a memoryless nonlinearity followed by a linear filter, i.e., as a Hammerstein filter [13]. Unfortunately, for nonlinearities with memory, also the measurement with ES technique is affected by artifacts caused by nonlinear distortions [14], [15].

A novel approach for RIR measurement contrasting the effect of nonlinearities was proposed in [16], [17]. In this approach the entire measurement system (i.e., the power amplifier, the loudspeaker, the acoustic path and the microphone) is modeled as a Legendre nonlinear (LN) filter. LN filters are linear combinations of polynomial basis functions orthogonal for white uniform inputs [18]. They admit PPSs, i.e., periodic sequences that guarantee the perfect orthogonality of the basis functions over a sequence period. Using a PPS input, the coefficients of the LN filter can be estimated with the cross-correlation method, i.e., computing the cross-correlation between the system output and the basis functions. In [16], [17], the first-order kernel, i.e., the set of linear terms coefficients, of the LN filter representing the measurement system is first measured using a PPS and then used to estimate the RIR.

Another family of polynomial filters with orthogonal basis functions for white Gaussian inputs are the Wiener nonlinear (WN) filters, which derive from the truncation of the Wiener nonlinear series. Also WN filters admit PPSs [19], whose sample distribution approximates a Gaussian distribution. From this point of view, the PPSs for WN filters appear more appealing for measuring RIRs since for the same power they less excite the highest amplitudes and they better approximate the distribution of natural sounds. Thus, in this paper we discuss RIR measurement using WN filters and PPSs. The PPSs for WN filters developed in [19] are not suitable for RIR measurement. In fact, their period increases geometrically with the memory length of the filter and becomes prohibitively large even for small memories. Thus, here we also discuss how PPSs suitable for RIR identification, i.e., with period proportionate to the filter memory length, can be developed. Experimental results employing signals affected by real nonlinear devices illustrate the effectiveness of the proposed approach and compare it with that based on LN filters.

The rest of the paper is organized as follows. In Section II, WN filters are reviewed and their identification with PPSs

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TABLE I
 BASIS FUNCTIONS OF WN FILTERS.

Order 1:	$x(n), x(n-1), \dots, x(n-N+1)$
Order 2:	$H_2[x(n)], H_2[x(n-1)], \dots, H_2[x(n-N+1)],$ $x(n)x(n-1), \dots, x(n-N+2)x(n-N+1),$ \vdots $x(n)x(n-D), \dots, x(n-N+D+1)x(n-N+1).$
Order 3:	$H_3[x(n)], H_3[x(n-1)], \dots, H_3[x(n-N+1)],$ $H_2[x(n)]x(n-1), \dots,$ $\dots, H_2[x(n-N+2)]x(n-N+1),$ \vdots $x(n)x(n-1)x(n-2), \dots,$ $\dots, x(n-N+3)x(n-N+2)x(n-N+1),$ \vdots $x(n)x(n-D+1)x(n-D), \dots,$ $\dots, x(n-N+D+1)x(n-N+2)x(n-N+1),$

is discussed. The proposed RIR measurement technique is presented in Section III. PPSs for WN filters suitable for RIR measurement are derived in Section IV. Experimental results are discussed in Section V. Conclusions are presented in Section VI.

The following notation is used: $\mathcal{N}(0, \sigma_x^2)$ is the zero mean, variance σ_x^2 , Gaussian distribution, $*$ indicates convolution, $\langle \cdot \rangle_L$ denotes average over L consecutive samples.

II. WN FILTERS AND PPSs

The WN filters are polynomial filters that derive from the double truncation, with respect to order and memory, of the Wiener series [20]. According to the Stone-Weierstass theorem, they can arbitrarily well approximate any discrete time, time-invariant, finite memory, continuous, nonlinear system,

$$y(n) = f[x(n), x(n-1), \dots, x(n-N+1)] \quad (1)$$

for any memory N , where f is a continuous function from \mathbb{R}^N to \mathbb{R} . WN filters are the linear combination of polynomial basis functions that are orthogonal for any white Gaussian input signal $x(n) \in \mathcal{N}(0, \sigma_x^2)$. For $N = 1$, the basis functions can be obtained from the Gram-Schmidt orthogonalization for $x(n) \in \mathcal{N}(0, \sigma_x^2)$ of the monomials

$$\{1, x(n), x^2(n), x^3(n), \dots\}, \quad (2)$$

obtaining the set of orthogonal polynomials.

$$\{1, x(n), x^2(n) - \sigma_x^2, x^3(n) - 3\sigma_x^2 x(n), \dots\}. \quad (3)$$

According to [21], these are Hermite polynomials of variance σ_x^2 and can be generated with the recursive relation

$$H_{j+1}[x(n)] = x(n)H_j[x(n)] - j\sigma_x^2 H_{j-1}[x(n)], \quad (4)$$

with $H_0[x(n)] = 1$ and $H_1[x(n)] = x$, and $H_j[x(n)]$ the Hermite polynomial of degree j . In what follows, for compactness the Hermite polynomials of order 0 and 1, will be indicated as 1 and $x(n)$, respectively, while the other polynomials will be indicated as $H_j[x(n)]$, with $j = 2, 3, \dots$

For $N > 1$, the basis functions can be obtained with the same procedure of [22], [18], by first writing the Hermite polynomials for $x(n), x(n-1), \dots, x(n-N+1)$ and them

multiplying the polynomials of different variables in any possible manner, taking care of avoiding repetitions. The basis functions of orders 1, 2, 3, memory length N , and diagonal number D are reported in Table I. The diagonal number D is defined as the maximum time difference between the input samples involved in each basis functions. The complete set of basis functions of memory N can be obtained by setting $D = N - 1$, but the basis functions for $D \ll N - 1$ are often sufficient to accurately model many real systems [18].

Neglecting the constant term, a WN filter of order 3, memory N , and diagonal number D is the linear combination of the basis functions in Table I. In the form of a filter bank the filter is given by the following relation

$$\hat{y}(n) = h_1(n) * x(n) + \sum_{i=0}^D h_{2,i}(n) * b_{2,i}(n) + \sum_{i=0}^D \sum_{j=i}^D h_{3,i,j}(n) * b_{3,i,j}(n) \quad (5)$$

where $b_{2,i}(n)$ and $b_{3,i,j}(n)$ are the zero-lag basis functions of 2-nd and 3-rd order. Specifically, $b_{2,0}(n) = H_2[x(n)]$, $b_{2,i}(n) = x(n)x(n-i)$ with $i = 1, \dots, D$, $b_{3,0,0}(n) = H_3[x(n)]$, $b_{3,0,j}(n) = H_2[x(n)]x(n-j)$ with $j = 1, \dots, D$, $b_{3,i,i}(n) = x(n)H_2[x(n-i)]$ with $i = 1, \dots, D$, and $b_{3,i,j}(n) = x(n)x(n-i)x(n-j)$ with $i = 1, \dots, D-1$ and $j = i+1, \dots, D$. $h_1(n)$ is the first order kernel, i.e., a length N sequence collecting the coefficients of the linear terms $x(n-i)$. $h_{2,i}(n)$ for $i = 0, \dots, D$ are the diagonals of the second order kernel and are sequences of length $N-i$. $h_{3,i,j}(n)$ for $i = 0, \dots, D$ and $j = i, \dots, D$ are the diagonals of the third order kernel with length $N-j$. The term ‘‘diagonals’’ follows the naming conventions of Volterra filters. In practice, the nonlinear kernels are band matrices with matrix bandwidth D [23].

Note that the first order kernel does not coincide with the nonlinear filter impulse response, which is

$$\hat{h}(n) = \lim_{A \rightarrow 0} \frac{\hat{y}[A\delta(n)]}{A}$$

where $\hat{y}[A\delta(n)]$ is the filter response to a pulse of amplitude A . In fact, all basis functions $H_{2k+1}[x(n)]$ with $k \geq 1$ include a linear term that contributes to the impulse response.

The WN filters admit PPSs. Using a PPS input, the coefficients of the WN filter can be efficiently estimated with the cross-correlation method, with the first order kernel given by

$$h_1(m) = \frac{\langle \hat{y}(n)x(n-m) \rangle_L}{\langle x^2(n) \rangle_L}. \quad (6)$$

III. RIR MEASUREMENT

Consider the scheme of the RIR measurement system in Fig. 1. The system is composed of a power amplifier, a loudspeaker, a room acoustic path, and a microphone. The measurement aims at estimating the room impulse response $h_R(n)$, which is assumed to have length M . The power amplifier and the loudspeaker system at high volumes are often the source of nonlinear effects. It is assumed in the following that the system composed of the power amplifier and loudspeaker can be modeled as a WN filter of order K , memory N , and diagonal number D . For $K = 3$ the input

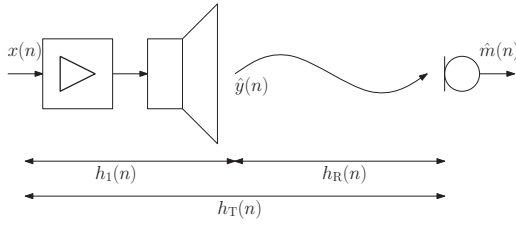


Fig. 1. The measurement system.

output relationship of this model is given in equation (5). The microphone can be considered as a linear system, due to the low level of the acquired signals. Its effect will be neglected in the following or better included in the model of the power amplifier and loudspeaker system. In these conditions, the measurement system of Fig. 1 can be modeled as a WN filter of order K , memory $N_T = N + M - 1$ and diagonal number D . For $K = 3$ the system has the following input-output relationship,

$$\begin{aligned} \hat{m}(n) = & h_R(n) * \hat{y}(n) = h_R(n) * h_1(n) * x(n) \\ & + \sum_{i=0}^D h_R(n) * h_{2,i}(n) * b_{2,i}(n) \\ & + \sum_{i=0}^D \sum_{j=i}^D h_R(n) * h_{3,i,j}(n) * b_{3,i,j}(n). \end{aligned} \quad (7)$$

Using as input of the measurement system a PPS for WN filters of order K , memory N_T , and diagonal number D , all coefficients in (7) can be estimated with the cross-correlation method. Actually, for RIR measurement, it suffices to estimate the first order kernel of the model, which according to (7) is $h_T = h_R(n) * h_1(n)$. The first order kernel of the power amplifier and loudspeaker system $h_1(n)$ can be measured and characterized in an anechoic chamber using the same PPS and the same reproduction volume.

As was proposed for linear systems [24], the RIR could be obtained from h_T using the Kirkeby algorithm,

$$h_R(n) = \text{IFFT} \left[\frac{\text{FFT}[h_T(n)] \cdot \text{FFT}[h_1(n)]^*}{\text{FFT}[h_1(n)] \cdot \text{FFT}[h_1(n)]^* + \epsilon(\omega)} \right], \quad (8)$$

where $\text{FFT}[\cdot]$ and $\text{IFFT}[\cdot]$ are direct and inverse FFT operators, respectively, $\epsilon(\omega)$ is a frequency-dependent regularization parameter. Actually, the Kirkeby algorithm provides the RIR $h_R(n)$ only when the loudspeaker has a uniform response at all directions in the frequency range of interest. However, since the amplifier and loudspeaker affect the measurement in a known, mild manner, it is common to approximate $h_R(n)$ with $h_T(n)$. Furthermore, since the first order kernel $h_1(n)$ of the amplifier and loudspeaker system does not differ much from the impulse response of the loudspeaker, the same approximation is acceptable also in the RIR measurement using PPSs for WN filters. For the orthogonality properties of the PPSs, the measurements of $h_T(n)$ and $h_R(n)$ are not affected by the nonlinear kernels of the amplifier and loudspeaker systems, i.e., by $h_{2,i}(n)$ and $h_{3,i,j}(n)$ for all i and j , provided a PPS of sufficient order and memory is used. Thus, the proposed RIR measurement system is immune to

the nonlinearities of the amplifier and loudspeaker, even when they have memory.

IV. PPSs SUITABLE FOR RIR MEASUREMENT

In this section, we discuss how a PPS $x_p(n)$ of period L for a WN filter of order K , memory N_T , diagonal number D , and Gaussian input variance σ_x^2 , suitable for RIR identification, can be developed. The PPS should be bounded by 1, i.e., $|x_p(n)| < 1 \forall n$, for faithfully reproducing it with digital to analog converters. The PPSs for WN filters developed in [19] are not suitable for RIR measurement. Indeed, their period increases geometrically with the memory N_T of the filter, and already for small N_T is unreasonably high. In [19], L depends geometrically on N_T because the PPSs allow the estimation of all kernels of WN filter. In RIR measurement, as shown in Section III, only the first order kernel needs to be estimated, relaxing the constraints imposed on the PPS.

In what follows the approach of [19] is adapted to estimate only the first order kernel. It should be noted that the approach for developing PPSs for WN filter is different from that used for LN filters in [16], [17]. In particular, to develop PPSs for RIR measurement we impose that all joint moments estimated over a period involved in the measurement of the first order kernel assume the ideal values of a Gaussian noise $\mathcal{N}(0, \sigma_x^2)$. Thus, the following system of nonlinear equations is imposed

$$\langle x_p^{r_0}(n) \dots x_p^{r_{N_T-1}}(n - N_T + 1) \rangle_L = \mu_{r_0} \dots \mu_{r_{N_T-1}}, \quad (9)$$

for all $r_0, \dots, r_{N_T-1} \in \mathbb{N}$ with $r_0 > 0$ (for the periodicity of the sequence), $r_0 + \dots + r_{N_T-1} \leq K + 1$, and with $x_p^{r_0}(n) \dots x_p^{r_{N_T-1}}(n - N_T + 1) = x_p^{v_0}(n - i) x_p^{v_1}(n - j) \dots x_p^{v_D}(n - D - j)$, for some $i, j \in [0, N_T - 1]$ and $v_0 + \dots + v_D \leq K$; μ_r is the r -th moment of the Gaussian process $\mathcal{N}(0, \sigma_x^2)$,

$$\mu_r = E[x^r(n)] = \begin{cases} 0 & \text{for } r \text{ odd,} \\ \sigma_x^r (r-1)!! & \text{for } r \text{ even,} \end{cases} \quad (10)$$

with $q!! = q \cdot (q-2) \cdot (q-4) \dots 1$. It can be proved that the number Q of nonlinear equations in (9) increases exponentially with the order K , geometrically with the diagonal number D , but linearly with the memory length N_T . For sufficiently large L , the system (9) is underdetermined and may have infinite solutions in the variables $x_p(n)$. The Newton-Raphson method has proved particularly effective for solving (9). The method was implemented as in [25, ch. 9.7], starting from a random distribution of $x_p(n)$ in $\mathcal{N}(0, \sigma_x^2)$, and reflecting $x_p(n)$ in $[-1, +1]$ every time they exceed the range to obtain a sequence in $[-1, +1]$, as desired. Convergence was obtained in all simulations, provided σ_x^2 was sufficiently small. Indeed, the PPS sample distribution is similar to a Gaussian and intuitively convergence is possible only when the probability of having samples outside $[-1, +1]$ is sufficiently small. Solutions to (9) were found for $L = 3Q \div 4Q$ and $\sigma_x^2 \leq 1/10$. Since the Newton-Raphson method has a computational complexity that increases with Q^3 , as discussed in [26] and [18], it is very useful to impose some structural conditions to the PPS. For example, the following conditions allow to almost halve the number of equations and variables:

TABLE II
PPSS FOR RIR ESTIMATION (N_T RIR LENGTH, K WN FILTER ORDER,
 D DIAGONAL NUMBER, L PPS PERIOD, S MEANS SYMMETRY,
O ODDNESS, O-P ODDNESS FROM 1 TO P)

Seq.	N_T	K	D	L	$\log_2(L)$	exploits
1	8192	3	0	131060	17	S,O,O-1
2	8192	3	1	262132	18	S,O,O-1
3	8192	3	2	524276	19	S,O,O-1
4	8192	3	3	1048560	20	S,O,O-4
5	8192	3	4	2228208	21	O,O-4
6	8192	5	0	262132	18	S,O,O-1
7	8192	5	1	1114104	20	O,O-1,O-2
8	16384	3	0	262140	18	O,O-1
9	16384	3	1	524276	19	S,O,O-1
10	16384	3	2	1048564	20	S,O,O-1
11	16384	3	3	2097136	21	S,O,O-4
12	16384	5	0	524276	19	S,O,O-1
13	32768	3	2	2097136	21	S,O,O-4
14	65536	3	2	8388576	23	S,O,O-8

– Symmetry: for any N -tuple of samples a_1, a_2, \dots, a_N , there is also the reversed one a_N, a_{N-1}, \dots, a_1 . Thus, for every couple of symmetric joint moments (e.g., $\langle x(n)x^2(n-1) \rangle_L$ and $\langle x^2(n)x(n-1) \rangle_L$) only one is needed.

– Oddness: for any N -tuple a_1, a_2, \dots, a_N , there is also the opposite one $-a_1, -a_2, \dots, -a_N$. Consequently, all odd joint moments are *a priori* zero.

– Oddness-1: For any N -tuple a_1, a_2, \dots, a_N , there is also the one obtained by alternatively negating the samples $a_1, -a_2, a_3, \dots, -a_N$. Consequently, all odd-1 joint moments are *a priori* zero.

Odd-1 are all those joint moments that change sign by alternatively negating the samples, as $\langle x(n)x(n-1) \rangle_L$ [19]. Oddness-2, oddness-4, ..., conditions could be similarly defined and used to halve the number of equations.

PPSSs suitable for RIR measurement can be downloaded from the website [27]. Table II provides the characteristics of the PPSs currently available.

V. EXPERIMENTAL RESULTS

In order to test the robustness towards nonlinearities of the novel sequences, we have considered an emulated scenario. PPSs for WN and LN filters, MLSs and ESs having different periods (or lengths) L have been applied to a real device, a Behringer MIC 100 vacuum tube preamplifier, at a sampling frequency of 44.1 kHz. The preamplifier emulates the nonlinearities introduced by a power amplifier or a loudspeaker. It has a potentiometer that allows to control the amount of nonlinear distortion introduced. Ten different settings have been considered and Fig. 2 shows the second, third, and total harmonic distortion in percent on a 1 kHz tone at the maximum amplitude of the sequences. The same peak amplitude has been considered for all sequences. Clearly, many of the harmonic distortions of Fig. 2 are larger than those expected in a measurement system, but they have been selected so large to stress the robustness of the proposed approach. The recorded output of the preamplifier has been convolved with a known RIR and a white Gaussian noise has been added to the output to have a signal to noise ratio of 40 dB. The known RIR allows us to measure the log-spectral distance (LSD) [28], [29]

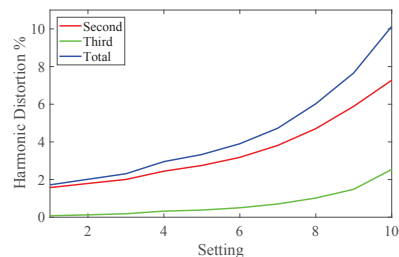


Fig. 2. Seconds, third, and total harmonic distortion.

between the measured RIR and its actual value. The LSD is defined in the band $B = [k_1 \frac{F_S}{T}, k_2 \frac{F_S}{T}]$, with F_S the sampling frequency and T the number of the samples of the discrete Fourier Transform (DFT), as follows:

$$\text{LSD} = \sqrt{\frac{1}{k_2 - k_1 + 1} \sum_{k=k_1}^{k_2} \left[10 \log_{10} \frac{|H_R(k)|^2}{|\hat{H}_R(k)|^2} \right]^2}, \quad (11)$$

where $|H_R(k)|$ is the actual room magnitude response and $|\hat{H}_R(k)|$ is the measured room magnitude response.

For the measurement with WN PPSs, the first 5 sequences of Tab. II have been used. Sequences with identical characteristics have been considered also for LN filters. For MLS and ES, sequences with order $\log_2(L)$ ranging from 15 to 21 have been applied. None of the PPSs have order 15 or 16, but these orders have still been used for the MLSs and the ESs because they allow the identification of a RIR of 8192 samples.

Fig. 3 shows the LSD in dB of the measured RIR without any compensation of the pre-amplifier, computed in the band [100, 18000] Hz. The MLS is clearly affected by the nonlinearity, while the ES provides robust results for order greater than 16. The PPSs for LN and WN filters provide similar robust results, with the WN filter showing slightly better results at the highest setting, due to its lower signal power. The same experiment was repeated for an equal power of the LN and WN PPSs and also for larger nonlinear distortions. Similar results were obtained: for the largest distortions, the LSD of the WN PPSs slightly increased compared with Fig. 3.(a), but remained always lower than that of the LN PPSs. The result is consistent with fact that the PPS for a WN filter has an almost Gaussian distribution, while that for an LN filter has an almost uniform distribution. The main advantage of WN PPSs over LN PPSs is that for equal power they excite less the highest amplitudes and are less affected by nonlinear distortions.

Fig. 4 shows the LSD in dB of the measured RIR when the pre-amplifier is compensated with the Kirkeby algorithm in (8). The LSD improves in all the methods, particularly in PPSs for LN and WN filters and in MLSs for order larger than 17. On the contrary, the improvement of LSD, while still relevant, is significantly worse for the ES.

VI. CONCLUSIONS

The paper discusses a novel technique for RIR measurement based on PPSs for WN filters. The technique is robust towards the nonlinearities affecting the power amplifier or

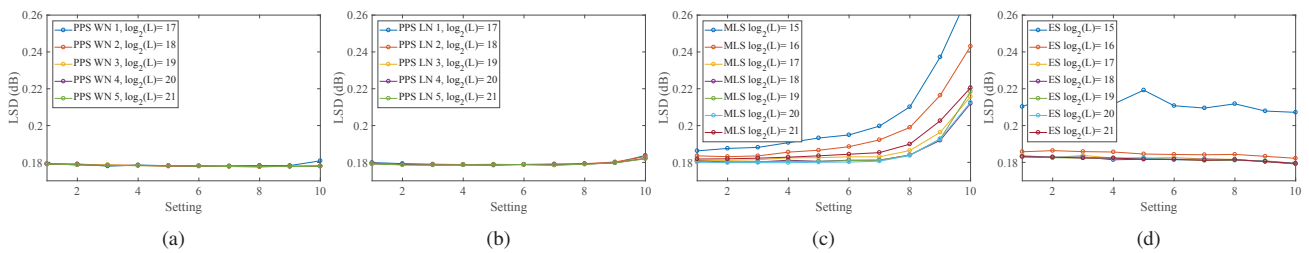


Fig. 3. LSD between measured and real RIRs without pre-amplifier compensation: (a) PPSs for WN filter, (b) PPSs for LN filter, (c) MLSs, (d) ESs.

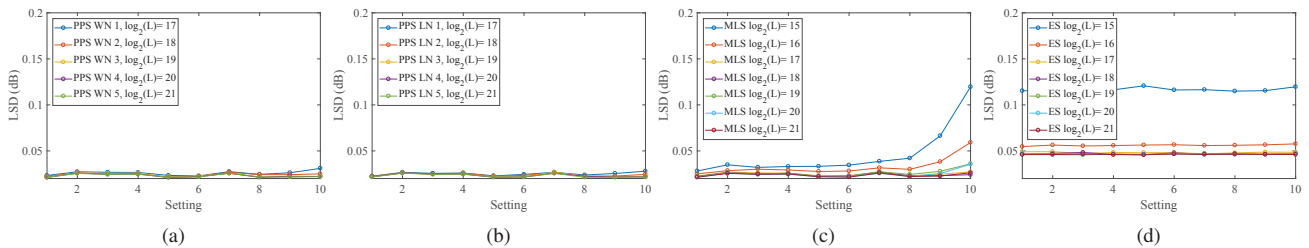


Fig. 4. LSD between measured and real RIRs with pre-amplifier compensation: (a) PPSs for WN filter, (b) PPSs for LN filter, (c) MLSs, (d) ESs.

the loudspeaker of the measurement systems. It provides an improvement in comparison with the techniques based on PPSs for LN filters, since the almost Gaussian distribution of the novel PPSs less excites the nonlinearities of the measurement system. PPSs for WN filters suitable for RIR measurement have also been developed within the paper. The experimental results, employing signals affected by a real nonlinear device, highlight the robustness towards nonlinearities of the proposed RIR measurement technique.

REFERENCES

- [1] M. Kleiner, *Acoustics and Audio Technology*. Fort Lauderdale: J. Ross Publishing, 2011.
- [2] S. Cecchi, A. Carini, and S. Spors, "Room response equalization—a review," *Applied Sciences*, vol. 8, no. 1, article no. 16, 2018.
- [3] H. Hacıhabıoglu, E. De Sena, Z. Cvetkovic, J. Johnston, and J. O. Smith III, "Perceptual spatial audio recording, simulation, and rendering: An overview of spatial-audio techniques based on psychoacoustics," *IEEE Signal Processing Magazine*, vol. 34, no. 3, pp. 36–54, 2017.
- [4] J. Garas, *Adaptive 3D Sound Systems*. New York: Kluwer Academic Publishers, 2000.
- [5] F. Antonacci, J. Filos, M. Thomas, E. Habets, A. Sarti, P. Naylor, and S. Tubaro, "Inference of room geometry from acoustic impulse responses," *IEEE Transactions on Audio, Speech, and Language Processing*, vol. 20, no. 10, pp. 2683–2695, Dec. 2012.
- [6] M. R. Schroeder, "Synthesis of low-peak-factor signals and binary sequences with low autocorrelation," *IEEE Trans. on Information Theory*, pp. 85–89, Jan. 1970.
- [7] C. Antweiler, "Multi-channel system identification with perfect sequences," in *Advances in Digital Speech Transmission*, R. Martin, U. Heute, and C. Antweiler, Eds., SPIE. John Wiley & Sons, 2008, pp. 171–198.
- [8] A. Farina, "Simultaneous measurement of impulse response and distortion with a swept-sine technique," in *Proc. 108 AES Convention*, Paris, France, Feb. 2000.
- [9] S. Müller and P. Massarani, "Transfer-function measurement with sweeps," *J. Audio Eng. Soc.*, vol. 49, no. 6, pp. 443–471, Jun. 2001.
- [10] C. Antweiler and G. Enzner, "Perfect sequence lms for rapid acquisition of continuous-azimuth head related impulse responses," in *Proc. of WASPAA '09, IEEE Workshop on Applications of Signal Processing to Audio and Acoustics*, 2009, pp. 281–284.
- [11] M. Wright and J. Vanderkooy, "Comments on -Aspects of MLS measuring systems- and author's reply," *J. Audio Eng. Soc.*, vol. 43, no. 1/2, pp. 48, 49, Feb. 1995.
- [12] A. Novak, P. Lotton, and L. Simon, "Synchronized swept-sine: Theory, application, and implementation," *J. Audio Eng. Soc.*, vol. 63, no. 10, pp. 786–798, 2015.
- [13] T. Schmitz and J.-J. Embrechts, "Hammerstein kernels identification by means of a sine sweep technique applied to nonlinear audio devices emulation," *Journal of the Audio Engineering Society*, vol. 65, no. 9, pp. 696–710, 2017.
- [14] A. Torras-Rosell and F. Jacobsen, "A new interpretation of distortion artifacts in sweep measurements," *J. Audio Eng. Soc.*, vol. 59, no. 5, pp. 283–289, May 2011.
- [15] D. G. Ćirić, M. Marković, M. Mijić, and D. Šumarac Pavlović, "On the effects of nonlinearities in room impulse response measurements with exponential sweeps," *Applied Acoustics*, vol. 74, no. 3, pp. 375 – 382, Mar. 2013.
- [16] A. Carini, S. Cecchi, and L. Romoli, "Room impulse response estimation using perfect sequences for Legendre nonlinear filters," in *Proc. of EUSIPCO 2015, European Signal Processing Conference*, 2015, pp. 2541–2545.
- [17] —, "Robust room impulse response measurement using perfect sequences for legendre nonlinear filters," *IEEE/ACM Transactions on Audio, Speech, and Language Processing*, vol. 24, no. 11, pp. 1969–1982, 2016.
- [18] A. Carini, S. Cecchi, L. Romoli, and G. L. Sicuranza, "Legendre nonlinear filters," *Signal Processing*, vol. 109, pp. 84 – 94, Apr. 2015.
- [19] A. Carini, L. Romoli, S. Cecchi, and S. Orcioni, "Perfect periodic sequences for nonlinear Wiener filters," in *Proc. of EUSIPCO 2016, European Signal Processing Conference*, 2016, pp. 1788–1792.
- [20] V. J. Mathews and G. L. Sicuranza, *Polynomial Signal Processing*. New York: Wiley, 2000.
- [21] S. Roman, *The Umbral Calculus*. Academic Press, New York, 1984.
- [22] A. Carini and G. L. Sicuranza, "Fourier nonlinear filters," *Signal Processing*, vol. 94, pp. 183–194, Jan. 2014.
- [23] G. H. Golub and C. F. Van Loan, *Matrix computations*, Johns Hopkins University Press, Baltimore, MD, USA, Ed., 1996.
- [24] A. Farina, "Advancements in impulse response measurements by sine sweeps," in *Proc. 122 AES Convention*, Vienna, Austria, May 2007.
- [25] W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, *Numerical recipes in C : the art of scientific computing*. New York, NY, USA: Cambridge University Press, 1995.
- [26] A. Carini and G. L. Sicuranza, "Perfect periodic sequences for even mirror Fourier nonlinear filters," *Signal Processing*, vol. 104, pp. 80–93, Nov. 2014.
- [27] A. Carini. (2018) Perfect periodic sequences. [Online]. Available: http://www2.units.it/ipl/res_PSeqs.htm
- [28] Y. Huang, J. Benesty, and J. Chen, *Acoustic MIMO Signal Processing*. Springer, 2006.
- [29] S. Bharitkar and C. Kyriakakis, *Immersive Audio Signal Processing*. New York: Springer, 2006.