

Bayesian Target Enumeration and Labeling Using Radar Data of Human Gait

Freweyni K. Teklehaymanot^{*†}, Ann-Kathrin Seifert^{*}, Michael Muma^{*}, Moeness G. Amin[‡], Abdelhak M. Zoubir^{*†}

^{*} Signal Processing Group
Technische Universität Darmstadt
Merckstr. 25, 64283 Darmstadt, Germany
{seifert, muma, zoubir}@spg.tu-darmstadt.de

[†] Graduate School CE
Technische Universität Darmstadt
Dolivostr. 15, 64293 Darmstadt, Germany
teklehaymanot@gsc.tu-darmstadt.de

[‡] Center for Advanced Communications
Villanova University
Villanova, PA 19085, USA
moeness.amin@villanova.edu

Abstract—Estimating the number of clusters in an observed data set poses a major challenge in cluster analysis. In the literature, the original Bayesian Information Criterion (BIC) is used as a criterion for cluster enumeration. However, the original BIC is a generic approach that does not take the data structure of the clustering problem into account. Recently, a new BIC for cluster analysis has been derived from first principles by treating the cluster enumeration problem as a maximization of the posterior probability of candidate models given data. Based on the new BIC for cluster analysis, we propose a target enumeration and labeling algorithm. The proposed algorithm is unsupervised in the sense that it requires neither knowledge on the number of clusters nor training data. Experimental results based on real radar data of human gait show that the proposed method is able to correctly estimate the number of observed persons and, at the same time, provide labels to them with high accuracy. It is shown that, in terms of cluster enumeration performance, the proposed algorithm outperforms an existing cluster enumeration method.

I. INTRODUCTION

Model selection deals with selecting a statistical model that adequately explains the observed data from a set of candidate models. Cluster analysis is one of the prominent fields of study where model selection methods have been extensively used. Estimating the number of clusters, also called cluster enumeration, poses a major challenge in cluster analysis. Many state-of-the-art methods, e.g. [1]–[6], solve this problem using the original Bayesian Information Criterion (BIC) as derived in [7], [8]. However, the original BIC is a generic criterion that does not include information about the structure of the observed data during model selection. Recently, a new BIC has been derived from first principles for estimating the number of clusters in an unsupervised learning framework by incorporating the data structure of the clustering problem [9]. For cluster analysis, this leads to a BIC that is different from the BIC expression derived by Schwarz [7], [8]. A finite sample refinement of the BIC expression proposed in [9] has been made in [10].

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In this paper, we will demonstrate that cluster enumeration can be a valuable entity for advanced radar technologies that monitor human gait. In contrast to other remote sensing modalities, such as video cameras, radar preserves privacy, is insensitive to lighting conditions and clothing, and can penetrate common materials. By incorporating the two recently proposed Bayesian cluster enumeration criteria [9], [10], we propose a target (person) enumeration and labeling algorithm that automatically estimates the number of targets from the radar data and labels individual targets at the same time. Target enumeration and labeling is achieved by exploiting the fact that feature vectors extracted from the gait of the same target create a cluster in feature space. The proposed method is unsupervised in the sense that it does not require training data nor prior knowledge on the number of clusters (targets). Using radar data of normal human walk, we are able to estimate the correct number of targets and label them with a high accuracy despite short observation times. To the best of our knowledge, this is the first work towards utilizing unsupervised learning methods to jointly estimate the number of targets (persons) and to label them using radar-based gait data.

Previous works on radar-based sensing of humans are mostly concerned with detection or activity recognition, see e.g. [11]–[13]. On the other hand, identification of humans by the use of radar is relatively recent [14]–[17], where we note that there are similar works based on sonar data [18], [19]. However, state-of-the-art methods on human identification require knowledge of the number of targets and availability of training data. These requirements are stringent in real world applications, where the number of observed targets is mostly unknown and possibly time varying. That is why, amongst other reasons (see [20] for a survey on human sensing), automatic identification of human subjects remains a challenging task for many ambient intelligent systems with application to surveillance, security, and smart homes.

The remainder of the paper is organized as follows. Sec. II formulates the problem of estimating the number of targets and labeling them at the same time using radar human gait measurements and Sec. III discusses the proposed method. Experimental results on target enumeration and labeling using data from a 24 GHz radar system is provided in Sec. IV. Finally, conclusions are drawn and future work is briefly discussed in Sec. V.

Notation: lower- and upper-case boldface letters denote column vectors and matrices, respectively; calligraphic letters represent sets; \mathbb{R} denotes the set of real numbers; \mathbb{Z}^+ represents the set of positive integers; \ln denotes the natural logarithm; $^\top$ stands for vector or matrix transpose; $|\mathbf{Y}|$ denotes the determinant of the matrix \mathbf{Y} ; $\text{vec}(\mathbf{Y})$ represents the stacking of the columns of the matrix \mathbf{Y} into a long column vector; \otimes denotes the Kronecker product.

II. PROBLEM FORMULATION

Radar data of human motions is typically analyzed in the time-frequency domain, since it contains multiple signal components and is highly non-stationary. The spectrogram is frequently used to represent the characteristic micro-Doppler signatures of humans (targets) in the joint time-frequency domain [11], [12], [21]. For further processing, the spectrogram of a target's motion is often considered as an image, which is denoted by $\mathcal{S} \in \mathbb{R}^{f \times t}$, where f is the number of pixels along the Doppler frequency axis and t is the number of pixels along the time axis. Assuming that there are K targets, we collect N_k images per target. The total number of images is given by N , such that $N = \sum_{k=1}^K N_k$. Each image $\mathcal{S}_n, n = 1, \dots, N$, is vectorized to create a long column vector $\mathbf{s}_n \in \mathbb{R}^{d \times 1}$, where $d = ft$, which is referred to as feature vector. In the set of feature vectors $\mathcal{S} \triangleq \{\mathbf{s}_1, \dots, \mathbf{s}_N\} \subset \mathbb{R}^{d \times N}$, often, $d > N$, which creates a sample scarce scenario.

In order to solve this problem, we use a probabilistic Principal Component Analysis (PCA) that automatically reduces the dimension of \mathcal{S} from d to r , where $r < d$ [22]. The new set of feature vectors with reduced dimensions is given by

$$\mathcal{X} = \mathbf{V}^\top \mathcal{S}, \quad (1)$$

where $\mathcal{X} \subset \mathbb{R}^{r \times N}$ and the column vectors of $\mathbf{V} \in \mathbb{R}^{d \times r}$ are the eigenvectors of \mathcal{S} corresponding to the first r eigenvalues such that $\lambda_1 > \lambda_2 > \dots > \lambda_r > 0$. The set of feature vectors \mathcal{X} can be partitioned into K clusters, where a cluster corresponds to the feature vectors collected from a single target, such that $\mathcal{X} \triangleq \{\mathcal{X}_1, \dots, \mathcal{X}_K\}$. The clusters $\mathcal{X}_k, k \in \mathcal{K} \triangleq \{1, \dots, K\}$, are independent, mutually exclusive, and non-empty. A family of candidate models $\mathcal{M} \triangleq \{M_{L_{\min}}, \dots, M_{L_{\max}}\}$ is given, where L_{\min} and L_{\max} are the specified minimum and maximum number of clusters, respectively. Each candidate model $M_l \in \mathcal{M}, l = L_{\min}, \dots, L_{\max}$ and $l \in \mathbb{Z}^+$, represents a partition of \mathcal{X} into l clusters with associated parameters $\Theta_l = [\theta_1, \dots, \theta_l] \in \mathbb{R}^{q \times l}$. Our research goal is to estimate the number of clusters (targets), \hat{K} , in \mathcal{X} given that the true number of clusters satisfies the constraint $L_{\min} \leq K \leq L_{\max}$. Once the number of clusters is estimated, we provide unique labels to the clusters (targets). Hence, the proposed method combines automatic cluster enumeration and labeling in an unsupervised learning framework.

III. PROPOSED METHOD

We propose a target enumeration and labeling algorithm that automatically estimates the number of targets and provides them with unique labels using radar data of human gait.

A. Choice of the Number of Principal Components

We automatically reduce the dimension of our set of feature vectors \mathcal{S} from d to r , where $r < d$, using a probabilistic PCA [23] that approximates the likelihood function of \mathcal{S} given the number of principal components $c = C_{\min}, \dots, C_{\max}$ [22]. The likelihood function is given by

$$p(\mathcal{S}|c) \approx N^{-\frac{1}{2}(z+c)} \left(\frac{\sum_{a=c+1}^d \lambda_a}{d-c} \right)^{-\frac{1}{2}N(d-c)} \left(\prod_{a=1}^c \lambda_a \right)^{-\frac{1}{2}N}, \quad (2)$$

where $\lambda_a, a = 1, \dots, d$, are the eigenvalues and $z = d(d-1)/2 - (d-c)(d-c-1)/2$.

Once the likelihood function is evaluated for each candidate number of principal components $c = C_{\min}, \dots, C_{\max}$, the correct number of principal components is selected as [22]

$$r = \arg \max_{c=C_{\min}, \dots, C_{\max}} \ln p(\mathcal{S}|c). \quad (3)$$

B. Bayesian Cluster Enumeration and Labeling Algorithm

The Bayesian cluster enumeration and labeling algorithm simultaneously estimates the number of clusters and provides them with unique labels. We use the two-step cluster enumeration algorithm proposed in [9], [10] to estimate the number of clusters in \mathcal{X} . In this algorithm, the first step is to cluster the data set \mathcal{X} via the Expectation-Maximization (EM) algorithm using the number of clusters specified by each candidate model $M_l \in \mathcal{M}$, where $l = L_{\min}, \dots, L_{\max}$. This results in cluster parameter estimates, such as cluster centroids $\hat{\boldsymbol{\mu}}_m$, covariance matrices $\hat{\boldsymbol{\Sigma}}_m$, and number of feature vectors per cluster N_m , for $m = 1, \dots, l$. In the second step, given the estimated cluster parameters, one of the two recently proposed Bayesian cluster enumeration criterion [9], [10] is calculated for each candidate model $M_l \in \mathcal{M}$ via

$$\begin{aligned} \text{BIC}_N(M_l) &= \sum_{m=1}^l N_m \ln N_m - \sum_{m=1}^l \frac{N_m}{2} \ln \left| \hat{\boldsymbol{\Sigma}}_m \right| \\ &\quad - \frac{q}{2} \sum_{m=1}^l \ln N_m, \quad \text{or} \end{aligned} \quad (4)$$

$$\begin{aligned} \text{BIC}_{\text{NF}}(M_l) &= \text{BIC}_N(M_l) + \frac{1}{4} r(r+1) l \ln 2 \\ &\quad + \frac{1}{2} \sum_{m=1}^l \ln \left| \hat{\boldsymbol{\Sigma}}_m \right| - \frac{1}{2} \sum_{m=1}^l \ln \left| \mathbf{D}^\top \hat{\mathbf{F}}_m \mathbf{D} \right|, \end{aligned} \quad (5)$$

where $q = \frac{1}{2}r(r+3)$ is the estimated number of parameters per cluster, $\mathbf{D} \in \mathbb{R}^{r^2 \times \frac{1}{2}r(r+1)}$ denotes the duplication matrix of the covariance matrix $\hat{\boldsymbol{\Sigma}}_m$ [24], and $\hat{\mathbf{F}}_m \triangleq \hat{\boldsymbol{\Sigma}}_m^{-1} \otimes \hat{\boldsymbol{\Sigma}}_m^{-1} \in \mathbb{R}^{r^2 \times r^2}$. BIC_N is an asymptotic criterion, while the penalty term of BIC_{NF} is derived for the finite sample regime.

The duplication matrix \mathbf{D} is calculated using [24]

$$\mathbf{D}^\top = \sum_{i \geq j} \mathbf{u}_{ij} \text{vec}(\mathbf{Y}_{ij})^\top, \quad (6)$$

where $1 \leq j \leq i \leq r$ and $\mathbf{u}_{ij} \in \mathbb{R}^{\frac{1}{2}r(r+1) \times 1}$ is a unit vector with one at its $(j-1)r + i - \frac{1}{2}j(j-1)$ th entry and zero elsewhere. \mathbf{Y}_{ij} is given by

$$\mathbf{Y}_{ij} = \begin{cases} \mathbf{E}_{ii}, & i = j \\ \mathbf{E}_{ij} + \mathbf{E}_{ji}, & i \neq j \end{cases}, \quad (7)$$

where \mathbf{E}_{ij} contains one at its (i, j) th entry and zero elsewhere.

Once either $\text{BIC}_N(M_l)$ or $\text{BIC}_{\text{NF}}(M_l)$ is calculated for all candidate models $M_l \in \mathcal{M}$, the number of clusters is estimated using either

$$\hat{K}_{\text{BIC}_N} = \arg \max_{l=L_{\min}, \dots, L_{\max}} \text{BIC}_N(M_l), \quad \text{or} \quad (8)$$

$$\hat{K}_{\text{BIC}_{\text{NF}}} = \arg \max_{l=L_{\min}, \dots, L_{\max}} \text{BIC}_{\text{NF}}(M_l). \quad (9)$$

Since the two-step cluster enumeration algorithm produces a cluster number estimate as well as an estimate of cluster parameters, we can provide labels to cluster centroid estimates $\hat{\boldsymbol{\mu}}_m$, for $m = 1, \dots, \hat{K}$, where \hat{K} corresponds to the cluster enumeration result of either BIC_N or BIC_{NF} . Hence, the feature vectors that are associated with a specific centroid will receive the label given to that centroid. This way, we are able to estimate the observed number of targets and, at the same time, provide unique labels to them.

The proposed framework is outlined in Algorithm 1.

Algorithm 1 Target enumeration and labeling algorithm

Inputs: set of feature vectors \mathcal{S} ; set of candidate models $\mathcal{M} \triangleq \{M_{L_{\min}}, \dots, M_{L_{\max}}\}$; minimum and maximum number of principal components C_{\min} and C_{\max}

Dimension reduction

for $c = C_{\min}, \dots, C_{\max}$ **do**

 Compute $p(\mathcal{S}|c)$ using Eq. (2)

end for

Estimate the correct number of principal components in \mathcal{S} via Eq. (3)

Create a new set of feature vectors \mathcal{X} using Eq. (1)

Target enumeration

Calculate the duplication matrix \mathbf{D} via Eq. (6)

for $l = L_{\min}, \dots, L_{\max}$ **do**

for $m = 1, \dots, l$ **do**

 Estimate $\boldsymbol{\mu}_m$ and $\boldsymbol{\Sigma}_m$ using the EM algorithm

 Estimate N_m via hard clustering [9]

end for

 Calculate either $\text{BIC}_N(M_l)$ or $\text{BIC}_{\text{NF}}(M_l)$ via Eq. (4) or Eq. (5), respectively

end for

Estimate the number of clusters, \hat{K} , in \mathcal{X} using either Eq. (8) or Eq. (9)

Target labeling

for $m = 1, \dots, \hat{K}$ **do**

 Assign unique labels to the feature vectors that belong to $\hat{\boldsymbol{\mu}}_m$

end for

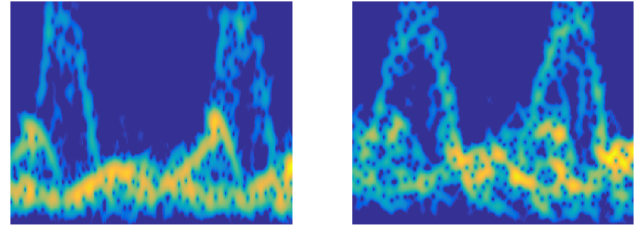


Fig. 1. Examples of micro-Doppler stride signatures of two individuals.

IV. EXPERIMENTAL RESULTS

A. Experimental Radar Data

Using a 24 GHz radar system [25], the experimental data were collected in an office environment at Technische Universität Darmstadt. The antenna feed point was positioned at approximately 1.15 m above the floor. Five test subjects were asked to walk toward the radar system starting at approximately 4.5 m in front of the radar, where only one person was present in front of the radar at a time. Data were collected at a 0° angle relative to the radar line-of-sight and with a non-oblique view on the targets. The volunteers were asked to walk slowly and without swinging their arms. In total, 65 radar measurements of 6 seconds duration are considered. The number of measurements are equal among the test subjects, i.e., the data set contains 13 gait samples per person.

B. Feature Extraction

The recorded radar return signals are processed to obtain the spectrogram (see [21] for more details). In order to detect single strides, the maxima of the envelope signal of the micro-Doppler signatures are utilized. The part of the spectrogram that shows a pair of strides, i.e., a full gait cycle, is extracted and converted to a gray scale image. All images are resized to have the same dimension, i.e., each image $\mathcal{S} \in \mathbb{R}^{f \times t}$, with $f = 100$ and $t = 128$. Examples of extracted stride pairs for two individuals are given in Fig. 1. We reduce the dimension of the spectrogram images using Eq. (3). Thus, a small number of descriptive features is automatically extracted for each radar signature. As an example, Fig. 2 shows a scatter plot of principal component scores using three principal components.

C. Person Enumeration and Labeling

1) *Scenario-1:* Considering the first four persons, $N = 187$ stride pairs are obtained from 52 radar measurements, where person A, B, C, and D are represented by $N_1 = 40$, $N_2 = 38$, $N_3 = 62$, and $N_4 = 47$ samples, respectively. For this radar data, the log-likelihood function as a function of the number of principal components is shown in Fig. 3. Here, 5 principal components are selected, such that the original set of vectorized spectrogram images, $\mathcal{S} \subset \mathbb{R}^{12800 \times 187}$, is reduced to $\mathcal{X} \subset \mathbb{R}^{5 \times 187}$.

For Scenario-1, we set the minimum and maximum number of clusters in the candidate models to $L_{\min} = 1$ and $L_{\max} = 2K$, respectively, where $K = 4$ is the true number of targets

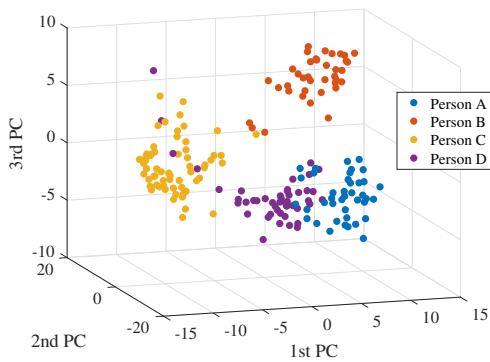


Fig. 2. Principal component scores for radar-based human gait data of four different persons using the first three principal components (PC).

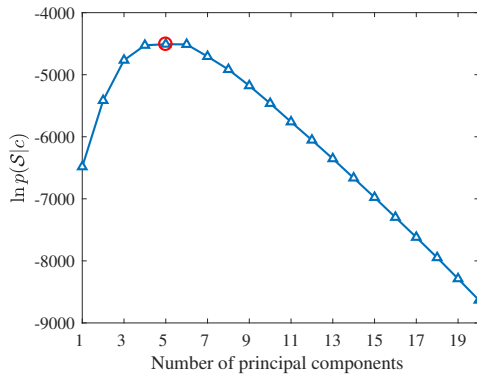


Fig. 3. Log-likelihood function as a function of the number of principal components, where the red circle indicates its maximum value.

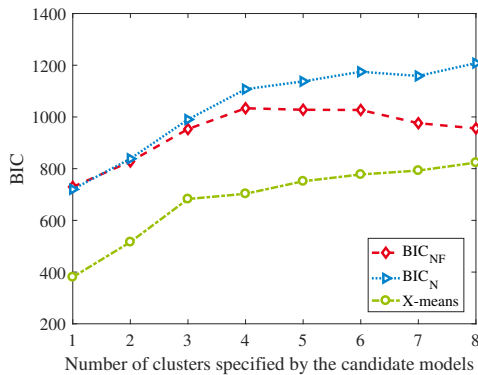


Fig. 4. The BIC of different Bayesian cluster enumeration criteria as a function of the number of clusters specified by the candidate models.

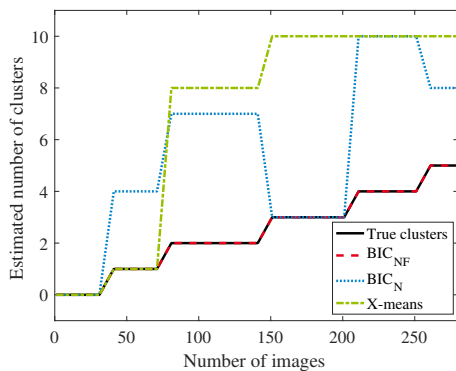


Fig. 5. Estimated number of clusters as a function of the number of observed images.

(persons) observed by the radar. Cluster enumeration and labeling is performed on the set of feature vectors $\mathcal{X} \subset \mathbb{R}^{5 \times 187}$. We compare the proposed Bayesian cluster enumeration methods with the X-means algorithm [1]. BIC_N and BIC_{NF} have the same data fidelity terms but different penalty terms. The X-means algorithm implements the original BIC [7], [8] as a wrapper around the K-means algorithm which results in a different data fidelity and penalty terms compared to BIC_N and BIC_{NF} . Note that estimating the number of clusters in \mathcal{X} is very challenging because \mathcal{X} has few feature vectors which results in even fewer feature vectors per cluster.

Fig. 4 shows the BIC computed by the different Bayesian cluster enumeration criteria as a function of the number of clusters specified by the candidate models. Only BIC_{NF} is able to estimate the correct number of clusters (persons), which corresponds to $\hat{K}_{\text{BIC}_{\text{NF}}} = 4$, while the other methods overestimate the number of clusters to $\hat{K}_{\text{BIC}_N} = 8$, and $\hat{K}_{\text{X-means}} = 8$. The asymptotic methods, BIC_N and X-means, stand at a disadvantage when the number of feature vectors is small because they are derived assuming that the number of feature vectors $N \rightarrow \infty$. In such cases, BIC_{NF} is more appropriate because its penalty term is refined for the finite sample regime.

Since BIC_{NF} is the only criterion that results in the correct estimate of the number of clusters in \mathcal{X} , we show the labeling performance of the proposed method using the cluster enumeration result of BIC_{NF} . Table I shows the confusion matrix generated by the proposed cluster enumeration and labeling algorithm. The first two persons are correctly labeled 100% of the time, while person D is often confused with the remaining targets, but is still recognized in approximately 89% of the cases. Overall, we achieve a high labeling rate using the proposed Bayesian cluster enumeration and labeling algorithm. Note that we get an average labeling rate of 95.73% without a prior knowledge on the number of clusters (targets) and no training data.

In order to underscore the performance of the proposed method, we also present results obtained using the same set of feature vectors \mathcal{X} , but a trained classifier for discriminating the four different persons. Using a simple nearest neighbor (NN) classifier, we obtain the confusion matrix as shown in Table II, where 80% of the data was used to train the classifier and the remainder was used for testing. The presented numbers are the average rates over 100 classifications, where training and test data were randomly chosen. The overall accuracy is 97.19%, where person A is correctly classified in all cases and person D shows the lowest rate with 93.5%.

We note that, the results obtained using a trained classifier and the proposed unsupervised cluster enumeration and labeling algorithm are comparable, despite the fact that 80% of the data was available to the classifier for training. In some real world applications, however, training data is unavailable. In such cases, cluster enumeration and labeling algorithms, such as the one proposed in this paper, can provide a high target labeling rate without training data and prior knowledge on the number of clusters.

TABLE I

CONFUSION MATRIX (%) FOR PERSON LABELING USING THE PROPOSED CLUSTER ENUMERATION AND LABELING ALGORITHM.

		Estimated labels			
		A	B	C	D
True labels	A	100	0	0	0
	B	0	100	0	0
	C	0	6.45	93.55	0
	D	2.13	6.38	2.13	89.36

TABLE II

CONFUSION MATRIX (%) FOR PERSON RECOGNITION USING AN NN CLASSIFIER.

		Estimated labels			
		A	B	C	D
True labels	A	100	0	0	0
	B	0	97.25	2.75	0
	C	0	1.33	98	0.62
	D	0.3	0.1	6.1	93.50

2) *Scenario-2*: In this scenario, we consider a different measurement setup. First, we observe person A and collect $N_1 = 40$ images. Next, we do the same for person B, where $N_2 = 38$. Every time a new person is observed, in total, $N = \sum_{k=1}^K N_k$, where K is the number of persons already observed. We do this for five persons in a sequential manner, resulting in $N = 252$ images. When ever a set of images is available, we re-estimate the total number of targets observed by the radar so far. For this, the number of principal components is also re-estimated based on the current set of feature vectors. We set $L_{\min} = 1$ and $L_{\max} = 10$.

Fig. 5 shows the number of estimated clusters \hat{K} as a function of the number of observed images n , where $n = 1, \dots, N$. Due to the setup described above, the true number of clusters forms a staircase. Among the compared Bayesian cluster enumeration criteria BIC_{NF} is the only criterion that is able to correctly estimate the number of persons and track the change in the number of persons as we observe more images. Due to the small number of available features, BIC_N rarely correctly estimates the number of persons even though it responds to the change in the number of observed persons. The X-means algorithm is able to correctly estimate the number of persons in the beginning but quickly overestimates K and chooses the specified maximum number of clusters as we observe more images.

V. CONCLUSION

We propose a target enumeration and labeling algorithm to estimate the number of persons and label them by monitoring their gait using radar. It outperforms an existing cluster enumeration method in terms of the correct estimation of the number of clusters (persons). Despite short observation times, the persons are labeled with a high accuracy in the absence of training data and knowledge on the number of persons. In the future, we will test the performance of our algorithm by increasing the number of targets, considering more gait classes, and extending the measurement duration.

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