A Methodology for the Estimation of Propagation Speed of Longitudinal Waves in Tone Wood

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***Abstract*—In this paper we propose a methodology for the estimation of the longitudinal wave velocity in tone wood. Differently from techniques adopted in the field of luthiery, the proposed estimation method does not require neither specific user skill nor expensive instrumentation. The introduced method exploits the impulse response of the wood block, acquired by means of accelerometers. The measured signals are processed in order to compute an estimate of the longitudinal wave velocity of the tone wood in a rake receiver fashion. We tested the technique both on synthetic data and measurements of actual tone wood blocks, showing the effectiveness of the proposed solution with respect to state-of-the-art methods.**

***Index Terms*—Material properties estimation, velocity measurement, rake receivers, tone wood, matched field processing**

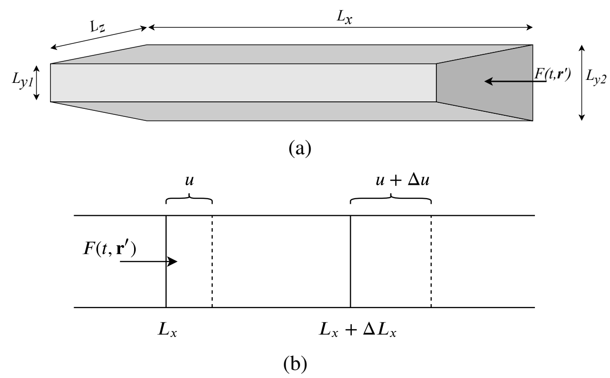


Fig. 1. The shape of a wood block used for builidng plates of string musical instruments (a). The displacement in a rectangular cross section (b).

I. Introduction

The accurate estimation of the material mechanical properties has a great impact in the field of musical acoustics because relevant mechanical parameters are used to drive the design and building process of musical instruments [1]. As an instance, the Young's modulus of wood can be directly estimated from the material by means of the tensile test [2]–[4]. This method consists in a stress test in which the Young's modulus is estimated from the slope of the linear part of the stress curve. Despite tensile test giving a direct estimate of the Young's modulus, it presents the disadvantage of being destructive and not repeatable. In order to overcome these disadvantages, we can indirectly estimate the Young's modulus from the longitudinal wave velocity. The estimation of the wave velocity in a medium can be tackled through *matched field processing*. This problem has been studied in different domains such as seismology [5], underwater navigation [6]–[8] and microphone array processing [9]–[11]. For the estimation of the velocity of longitudinal waves in tonewood, a well-known method is the *tap tone* [12]. This technique is widely adopted by luthiers because of its repeatability and non-invasive characteristic. It consists in the estimation of the resonance frequency of the wood block, from which the longitudinal velocity is derived. Unfortunately, it requires a great manual skill in order to correctly identify the resonance frequency by *tapping* the tone wood block.

Alternatively, the longitudinal wave velocity can be easily estimated by measuring the time of flight (TOF) of an impulsive wave between the extremes of the block under analysis [13], [14].

Due to the high propagation speeds, the adoption of expensive analog or digital instrumentation with sampling rate in the ultrasound bandwidth is required. Furthermore, since state-of-the-art techniques measure the TOF of the direct wave only, the estimated velocity turns out to be sensitive to measurement errors. In this paper, we propose a simple procedure for the estimation of the longitudinal wave velocity in tone wood blocks that is highly repeatable and non invasive. In addition, it does not require neither expensive instrumentation nor specific skills. The proposed solution analyzes the impulse response of the tone wood block measured with accelerometers in the audio bandwidth. The velocity is estimated in a rake receiver [15] fashion, extending the analysis of the TOF to a larger portion of the impulse response, beyond the direct wave. This allows us to work at a sampling frequency in the audible bandwidth, adopting low cost general purpose digital hardware. Results on synthetic data show that the proposed technique can provide a good estimation of the actual longitudinal wave velocity. Moreover, we show that the estimates obtained from real data provide results that match with the tap tone and the direct TOF methods.

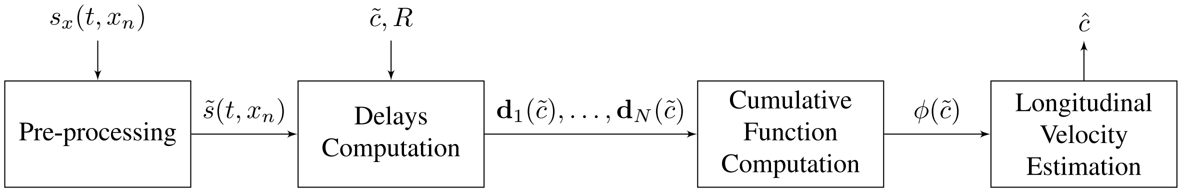


Fig. 2. The block diagram of the longitudinal velocity estimation procedure. The processing chain is divided in four stages: *Pre-processing*, *Delays Computation*, *Cumulative Function Computation* and *Longitudinal Velocity Estimation*.

The rest of the manuscript is organized as follows. In Sec. II the data model adopted in this work is introduced. In Sec. III a detailed description of the proposed solution for the estimation of longitudinal velocity in tone wood blocks is presented. The performance of the technique is analyzed in Sec. IV, evaluating the accuracy of the estimation on synthetic data and comparing the results on actual measurements with respect to the tap tone and time-of-flight methods. Finally, Sec. V draws some conclusions.

II. Data model

Let us consider a tone wood block with length L_x. The cross-section is trapezioidal, with dimensions L_{y1}, L_{y2}, L_z, so that the cross section area is $A = \frac{(L_{y1}+L_{y2})\times L_z}{2}$, as depicted in Fig. 1(a). We measure the signal of N accelerometers, placed at $\boldsymbol{r}_n = [x_n, y_n, z_n]^T$, with $n = 1, \ldots, N$. The signal acquired by the nth sensor can be modelled, in absence of noise, as

$$s(t, \boldsymbol{r}_n) = h(t, \boldsymbol{r}', \boldsymbol{r}_n) * \eta(t, \boldsymbol{r}'), \tag{1}$$

where t is the time index, $\eta(t, \boldsymbol{r}')$ is the source signal given by an axial load $F(t, \boldsymbol{r}')$ placed at $\boldsymbol{r}' = [x', y', z']^T$, $h(t, \boldsymbol{r}', \boldsymbol{r}_n)$ is the impulse response (IR) of the block and $*$ is the linear convolution operator. The IR takes into account both the direct path from $\boldsymbol{r}'$ to $\boldsymbol{r}_n$ and the reflections given by the block boundaries. Therefore, the signal $s(t, \boldsymbol{r}_n)$ in (1) contains delayed and attenuated versions of $\eta(t, \boldsymbol{r}')$, whose delays are determined by the distance travelled by the wavefronts and the wave velocity. In solids such as tone wood blocks we can identify three types of waves (*longitudinal* or *axial*, *transverse* waves, and *bending* waves) defined according to the direction of displacement in the medium with respect to the wave propagation. In case of the longitudinal wave, the displacement in the medium is observed along the direction of propagation of the wave, i.e. longitudinal waves propagating along the x consist in local displacements of particles so that wavefronts are parallel to the yz plane.

Here, we are interested in the estimation of the longitudinal wave velocity. Hence, we assume that the measurement and the excitation points are positioned at the endpoints of the wood block and aligned on the x axis, i.e. $x_n = \{0, L_x\}, y_n = y', z_n = z'$, $\forall n = 1, \ldots, N$. Additionally, we consider the longitudinal component of the acceleremerometers only. This is denoted in (1) by

$$s_x(t, x_n) = h(t, x', x_n) * \eta(t, x'). \tag{2}$$

The longitudinal wave equation [16] is

$$\frac{\partial^2 u(t,x)}{\partial t^2} = c^2 \frac{\partial^2 u(t,x)}{\partial x^2}, \tag{3}$$

where c is the longitudinal wave velocity and $u(t, x)$ is the longitudinal displacement field that corresponds to an elongation or contraction from L_x to $L_x + \Delta L_x$ caused by the axial load $F(t, \boldsymbol{r}')$ (see Fig. 1(b)).

The longitudinal velocity in (3) can be expressed in terms of the material characteristics [16]

$$c = \sqrt{\frac{EA}{m(1-\nu^2)}} = \sqrt{\frac{E}{\rho(1-\nu^2)}}, \tag{4}$$

where m is the mass of the block, while E, ν and $\rho = \frac{A}{m}$ are respectively the Young's modulus, the Poisson ratio and the density of the material. Note that the longitudinal velocity (4) is assumed to be frequency independent.

III. Proposed method

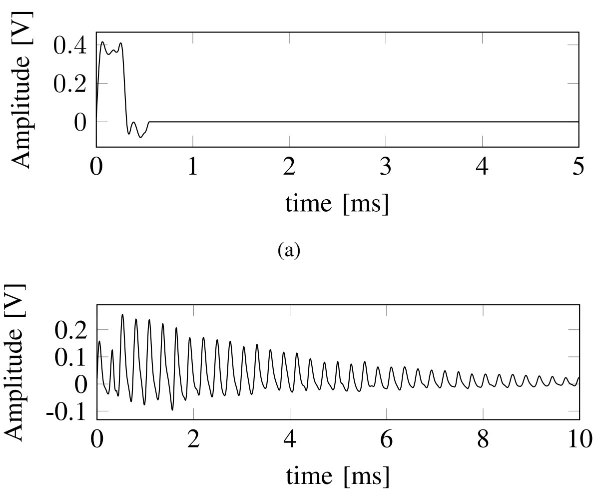
In this section we introduce the proposed technique for the estimation of the longitudinal wave velocity in tone wood blocks. The procedure adopts a model fitting approach and given the measured signals, it is able to estimate in a rake receiver fashion, the longitudinal velocity exploiting a priori information provided by the signal model (2). The procedure receives as input the signals s_n with $n = 1, \ldots, N$ and an integer R that indicates the number of reflections to be considered during the estimation. The system provides as output an estimate $\hat{c}$ of the longitudinal velocity c by testing a set of hypothetical values of the velocity. The block diagram of the longitudinal wave velocity estimation procedure is depicted in Fig. 2.

A. Pre-processing

In this step, we process the input signals (2) in order to improve the signal-to-noise ratio of the signals and avoid leakage phenomena [17]. In practice, the samples before the occurrence of the impulse are discarded and an exponential smoothing window $w(t)$ is applied to each nth signal

$$\tilde{s}(t, x_n) = w(t) * s_x(t, x_n) \tag{5}$$

with $w(t) = e^{-\frac{t}{\tau}}$ and $w(t) = 0$, $\forall t < t_i$ where t_i is the time instant at which the impulse occurs. In Fig. 3(a) an excitation signal is depicted, while Fig. 3(b) shows the response after the pre-processing stage.



(a)

(b)

Fig. 3. Example of the signals after the pre-processing stage (Sec. III-A). The hammer signal (a) and the response measured by an accelerometer (b).

B. Delays computation

Considering an hypothetical velocity $\tilde{c}$ and the required number of reflections R, a set of hypothetical delays from the impact point $\boldsymbol{r}'$ to the nth sensor location $\boldsymbol{r}_n$ are computed for all the N measurement points. Let us define the vector $\mathbf{d}_n$ of the delays associated to the nth sensor as

$$\mathbf{d}_n(\tilde{c}) = \left[\frac{l_0^n}{\tilde{c}}, \dots \frac{l_{R-1}^n}{\tilde{c}} \right] \in \mathbb{R}^{1 \times R} \tag{6}$$

where $l_k^n = 2L_x k + l_0^n$ with $k = 0 \dots, R-1$ is the length of the path related to the kth reflection seen by the nth sensor. The index $k = 0$ refers to the direct path between $\boldsymbol{r}'$ and $\boldsymbol{r}_n$. This corresponds to the distance $l_0^n = \|\boldsymbol{r}' - \boldsymbol{r}_n\| = |x' - x_n|$. Each element of the vector $\mathbf{d}_n(\tilde{c}) = l_k^n / \tilde{c}$ corresponds to the delay, expressed in seconds, given by the hypothetical velocity $\tilde{c}$ and the distance l_k^n travelled after k reflections.

C. Cumulative Function Computation

This step represents the core of the proposed method for the estimation of the longitudinal velocity. We exploit the delays computed in the previous step (6) in order to evaluate the supposed longitudinal velocity $\tilde{c}$. We test $\tilde{c}$ computing a cumulative function of the measurements defined as

$$\phi(\tilde{c}) = \sum_{n=1}^{N} \sum_{k=0}^{R-1} \tilde{s}(\mathbf{d}_{n,k}(\tilde{c}), x_n), \tag{7}$$

where $\mathbf{d}_{n,k}(\tilde{c})$ refers to the kth element in (6).

In practice, given a velocity, in (7) we sum the values of the N signals $\tilde{s}(t, x_n)$ in correspondence of the time instants in (6), which are determined by the combinations of the candidate velocity $\tilde{c}$ and the R reflection paths. Hence, the computation of the cumulative function (7) requires the computation of $N \times R$ summations for each velocity candidate $\tilde{c}$. It is worth noticing that in actual scenarios we work with discrete signals, consequently, the delay values in (6) may not perfectly correspond to sampled time instants. In order to compute (7) at the desired delays (6), a parabolic interpolation [18] is applied to the samples of $\tilde{s}(t, x_n)$.

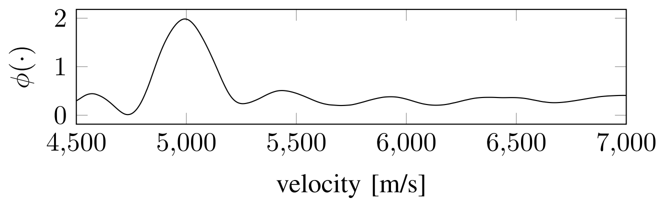


Fig. 4. An example of the cumulative function $\phi(\cdot)$ (7) evaluated at different velocity values.

D. Longitudinal Velocity Estimation

Finally, in the last step an estimate of the longitudinal velocity is computed from (7). Inspecting Fig. 4, we can notice that the graph presents a single prevailing peak and the cumulative function (7) attains its maximum where

$$\tilde{c}^\star = \arg\max_{\tilde{c}} \quad \phi(\mathbf{d}(\tilde{c})). \tag{8}$$

This can be interpreted as the fact that for a specific velocity value $\tilde{c}^\star$, the relative delay vectors match the actual reflection delays in the measurements. As a consequence, the values $\tilde{s}(\mathbf{d}_n(\tilde{c}^\star), x_n)$ in (7) will correspond to the peaks in the signals (see Fig. 4). Therefore, we assume as an estimate of the longitudinal velocity c, the value $\tilde{c}^\star$ for which the cumulative function is maximized (8). In practice, we evaluate (8) on a discrete set of J candidate velocities such that

$$\hat{c} = \tilde{c}^\star \in \{\tilde{c}_1, \dots, \tilde{c}_J\}. \tag{9}$$

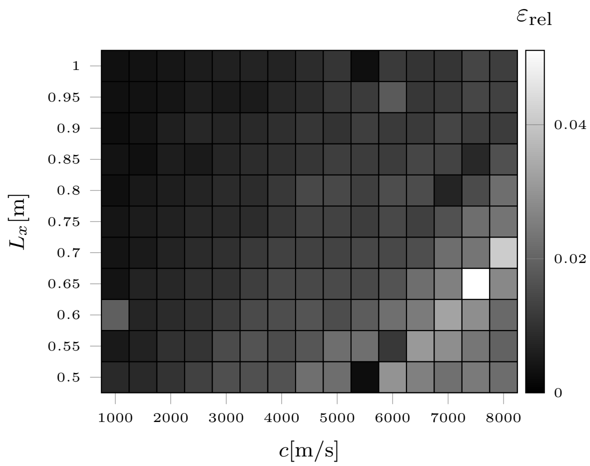
It is worth noting that from the inversion of (4), it is possible to exploit $\hat{c}$ for the estimation of the material properties e.g. the Young's modulus.

IV. VALIDATION

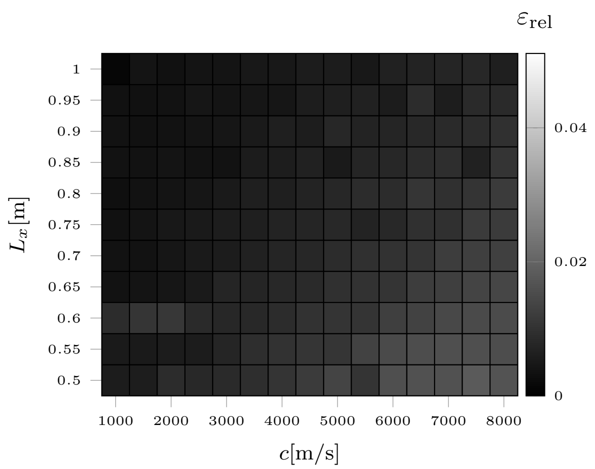
In order to validate the proposed technique, we tested the longitudinal velocity estimation both on simulated synthetic data and signals measured from actual tone wood blocks. The whole estimation procedure described in Sec. III is implemented in *MATLAB* [19].

A. Simulation setup

In order to evaluate the performance of the proposed technique we simulated rectangular blocks of homogeneous isotropic material with length $L_x \in \{0.5, 1\}\,\mathrm{m}$, width $L_y = 0.15\,\mathrm{m}$ and height $L_z = 0.03\,\mathrm{m}$. The $N = 2$ impulse responses (2) were computed using the image source method [20]–[23] at two different sampling frequencies $Fs_1 = 22.05\,\mathrm{kHz}$ and $Fs_2 = 44.1\,\mathrm{kHz}$. An additive sensor noise is simulated using a random white Gaussian noise, whose variance is set so that the desired signal to noise ratio at each sensor is $60\,\mathrm{dB}$. For each block we varied the wave speed in the range $c \in \{1000, 8000\}\,\mathrm{m/s}$ with a step of $500\,\mathrm{m/s}$. The estimation



(a)



(b)

Fig. 5. The relative error ε_{rel} (10) for the simulations with $Fs_1 = 22.05\,\text{kHz}$ (a) and $Fs_2 = 44.1\,\text{kHz}$ (b).

of the longitudinal wave velocity is evaluated in terms of the relative error

$$\varepsilon_{\text{rel}}(c) = \left| \frac{c - \hat{c}}{c} \right|, \tag{10}$$

where c is the actual longitudinal velocity and $\hat{c}$ is the estimate given by (9) considering $J = 4096$ uniformly sampled candidates $\tilde{c} \in \{c/1.2, 1.2c\}$. As regards the algorithm parameters, in the pre-processing stage we adopted $\tau = 10\,\text{ms}$ (5) while for the delay estimation (6) $R = 15$ reflections are considered.

B. Measurement setup

The impulsive excitation and the response of the wood block (2) are recorded using $N = 2$ accelerometers *ADXL326* by *Analog Devices* [24]. The sensors are connected to the *Bela Mini* [25], an acquisition board that performs AD conversion with sampling rate $Fs = 22.05\,\text{kHz}$. It is worth noticing that a calibration step in order to guaranteed the synchronization of the sensors. The algorithm parameters were set as in the simulation setup (see Sec. IV-A). We measured the longitudinal wave velocity in $K = 7$ tone wood blocks made of red spruce coming from woods of Trentino South Tyrol, in Italy. The length of the K blocks are reported in the last column of Table I. A total number of 5 measurements have been performed for each tone wood block and the average of the obtained values have been considered as the longitudinal wave velocity estimate. In order to asses the performance of the proposed procedure, we compare the obtained results with the tap tone and when available with the TOF methods.

Block	$\hat{c}$[m/s]							[m]
	Rep. 1	Rep. 2	Rep. 3	Rep. 4	Rep. 5	Average	Std. Deviation	L_x
1	6236	6187	6220	6123	5990	6151	100	0.64
2	5806	5947	5831	5983	5891	5892	75	0.54
3	4937	4985	5083	5174	5217	5079	120	0.30
4	4932	5192	4879	5152	5283	5088	173	0.41
5	6212	6153	6359	5912	5960	6119	184	0.45
6	5025	5299	5245	5211	5274	5211	109	0.45
7	5888	5862	6026	5601	5511	5778	214	0.45

TABLE I

LONGITUDINAL VELOCITY ESTIMATES OBTAINED FROM THE MEASUREMENTS OF THE TONE WOOD BLOCKS.

Block	$\hat{c}$[m/s]			% Error w.r.t. tap tone	
	Tap Tone	Lucchi meter	Proposed	Lucchi meter	Proposed
1	6059	-	6151	-	1.5
2	5886	-	5892	-	0.1
3	5010	-	5079	-	1.4
4	4605	4946	5088	7.4	10.5
5	6420	5885	6119	8.3	4.7
6	5372	5127	5211	4.6	3.0
7	5763	5575	5778	3.3	0.3

TABLE II

ESTIMATED VELOCITIES GIVEN BY THE CONSIDERED TECHNIQUES. THE PERCENTAGE ERROR WITH RESPECT TO THE TAP TONE IS REPORTED.

The tap tone technique [12] estimates the longitudinal velocity $\hat{c}_{\text{T}}$ as follows

$$\hat{c}_{\text{T}} = \frac{0.973 \cdot f \cdot L_x}{h}, \tag{11}$$

where L_x and $h = \frac{L_{y1} + L_{y2}}{2}$ are the wood block length and the average thickness, respectively. The resonance frequency f is manually determined by the violin maker by tapping the block close to an antinode of the resonance mode, while holding lightly the wood on a nodal line. It is worth noting that (11), regarded by violin makers as a ground truth, is valid under the assumption that the tonewood block can be approximated by a bar. The TOF was measured using the Lucchi meter [26], an ultrasonic tester designed to measure the TOF in a piece of wood. Through the knowledge of L_x and the TOF it is possible to obtain the average longitudinal wave velocity.

C. Results

In Fig. 5 the relative error ε_{rel} (10) is reported considering the two different sampling frequencies. In general, the proposed technique provides a good estimation for both the sampling frequencies with $\varepsilon_{\text{rel}} \leq 0.051\,(5.1\%)$. Inspecting both Fig. 5(a) and Fig. 5(b) we can observe that ε_{rel} tends to increase with the wave velocity c, while it decreases with the block length L_x. As expected, the estimation greatly improves at Fs_2, with $\varepsilon_{\text{rel}} \leq 0.017\,(1.7\%)$ as depicted in Fig. 5(b).

In Table I, the estimated velocities for each measurement are reported along with the average value over the repetitions and the standard deviation. It is worth noting that the consistency and repeatability of the measurements is confirmed by the standard deviations in Table I which present relatively small values with respect to the velocity magnitudes.

In Table II, the estimated velocities are reported along with the results obtained using the tap tone and the Lucchi meter. We choose to evaluate the estimation in terms of the percentage error with respect to (11) since the tap tone method is considered to be the standard by violin makers. Inspecting the third column of Table II, we can notice that the estimates obtained with the proposed method are close to the reference ones.

Moreover the proposed estimation outperformed the estimates given by the TOF technique for all the blocks except for $k = 4$. A more detailed analysis on this specific case shows us that the block number 4 presented inhomogeneous wood grains with respect to the other samples. Hence, the results given by the tap tone (11) are less reliable due to the bar approximation. This hypothesis is confirmed by the fact that the estimates given both by the Lucchi meter and the proposed estimator are close to each other.

V. Conclusions

In this manuscript we proposed a procedure for the estimation of the longitudinal wave velocity in tone wood block. The methodology concerns the measurement of the impulse responses by means of accelerometers, from which an estimate of the longitudinal wave velocity is obtained considering not only the time of flight but also the reflections present in the signals. The technique has been evaluted on synthetic data, for which the actual longitudinal wave velocity is known, showing a good accuracy in the estimation. Furthermore, we compared the proposed velocity estimator with respect to two widely adopted techniques, namely, the *tap tone* and the TOF-based, measuring the longitudinal velocity of 7 tone wood blocks. The results were in line with the ones obtained by the other techniques, showing the effectiveness of the proposed velocity estimator.

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