Optimal Microphone Placement for Localizing Tonal Sound Sources

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***Abstract*—This work is concerned with determining optimal microphone placements that allow for an accurate location estimate of the sound sources, taking into account the expected signal structure of voiced speech, as well as the expected location areas and the typical range of the fundamental frequencies of the speakers. To determine preferable microphone placements, we propose a scheme that minimizes a theoretical lower bound on the variance of the location estimates over the possible sensor placements, while taking into account the expected variability in the impinging signals. Numerical examples and real measurements illustrate the performance of the proposed scheme.**

***Index Terms*—Sensor placement, worst case Cramér-Rao lower bound, Convex optimization**

I. Introduction

Reliable localization, estimation, and/or detection of a partly known signal source is a problem of notable interest in many applications, ranging from telecommunications and metrology, to radar, sonar, surveillance, and military applications. No matter the application, the topic of sensor placement is of outmost importance in the resulting estimation problems, as it sets the boundaries for the accuracy that may be achieved [1]–[5].

The problem of optimal sensor placement is a diverse and multi-faceted problem that allows for a rich plethora of different problem setups; [6] discusses the problem of optimal sensor placement for passive stationary sensors, [7] treats the optimal geometry for multi-static sensors using time of arrival (ToA) estimates, [8] explores optimal sensor configuration in a 2-D setting with mobile sensors/targets, and [9] presents an algorithm for a near optimal sensor placement, under a minimum redundancy constraint. When discussing the problem of optimal sensor placement, one has to be precise in defining the premises of the problem formulation, as the setup will differ greatly depending on factors such as if the sensors are passive or active, if the sensor environment is monostatic or multistatic, and if the sensors and/or target are stationary or moving, to mention some of the aspects.

Another matter to take into account when formulating the optimal sensor placement scheme is the desired parameter estimates. Often, the time difference of arrival (TDoA) is used to determine suitable locations, such as in [10], where optimal sensor placements were selected to minimise the corresponding Cramér-Rao lower bound (CRLB). Other sought parameters include the direction of arrival (DoA), which was employed in [11], the ToA as was used in [7], and the frequency difference of arrival (FDoA), which was used in combination with the TDoA in [8], where the latter also demonstrated that the optimal sensor setup differs depending on the desired parameters.

Yet another aspect of the sensor placement problem is the definition of optimal, as there are many different notions of optimality which give rise to an abundance of optimization methods and frameworks. Some widely used criteria of optimality are related to the Fisher information matrix (FIM) and its inverse; the uncertainty ellipse of an estimator is proportional to the determinant of the inverse FIM, the mean square error (MSE) is proportional to the trace of the inverse FIM, whereas E-optimality relates to the eigenvalues of the FIM. The choice of optimality condition will affect the resulting optimization scheme, and each of the notions offer different advantages and drawbacks. In [12], the authors formulated the sensor selection problem as a convex minimization, maximizing the power of the received signal. Later in [13]–[15], the sensor placement was formulated as a minimization of the MSE of the localization estimates. In [3], microphone placement was considered from the perspective of a minimum variance distortionless response. Typically, the chosen algorithm will affect the solution and the computational complexity of the proposed method. Often some sort of convex relaxation is required to allow for a computationally feasible solution, although other methods are employed, such as in [16], where a genetic algorithm utilizing the CRLB as the fitness function was presented. Generally, the sensor selection problem can be formulated as a combinatorial problem, but as the number of possible locations grows, such a solution quickly become unfeasible, and other methods or relaxations have to be utilized. An example of this is found in [12], wherein the authors formulate the optimization problem as one of maximizing the signal energy, and thereafter proceed to solve the non-convex optimization problem using a convex relaxation of the constraints.

One limitation of the noted works is that these do not exploit any available *a priori* knowledge of the expected source signals. Often, the structure of the impinging signals is at least partly known [17], [18], which may be exploited in determining a suitable sensor placement. Reminiscent to the

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optimal sampling scheme presented in [19], wherein the expected signal structure of spectroscopic signals was exploited to develop an optimal multi-dimensional sampling scheme allowing for the there typically partly known signals, we here propose an optimal microphone placement that exploits the expected signal structure of tonal audio sources, such as voiced speech. In order to do so, we formulate an optimal microphone placement scheme that minimizes the CRLB of the source location estimation problem over the range of potential sensor placements, taking into account the spherical propagation of the acoustic waves, expected location areas of the sources, and the expected harmonic structure of tonal sounds. In this sense, the work expands on the source localization problem examined in [18], wherein the harmonic structure of tonal sounds was exploited to localize the sound sources.

II. SIGNALMODEL

Consider a setup where M microphones measure the wavefront imping from K (near-field) tonal sound sources. Without loss of generality, we here consider each of the sound sources to have a harmonically related structure, such as may be expected in voiced speech or tonal audio, implying that the kth sound source may be deemed to be well modeled as[1] [21]

$$z_k(t) = \sum_{\ell=1}^{L_k} A_{k,\ell} e^{i\omega_{k,\ell} t + \phi_{k,\ell}} \tag{1}$$

where $\omega_{k,\ell}$, $A_{k,\ell} \in \mathbb{R}$, and $\phi_{k,\ell} \in [0, 2\pi)$ denote the frequency, amplitude, and phase for the ℓth harmonic component of the kth source, respectively, and $t = 0, \ldots, N-1$. For tonal sounds, typically the ℓth frequency may be well modelled as $\omega_{k,\ell} = \ell\omega_{k,0}$, where $\omega_{k,0}$ denotes the fundamental frequency, or pitch, of the kth sound source [21]. It should further be noted that for most sound sources, the number of overtones, L_k, is unknown, and typically varies over time. Assuming that the kth sound source is located at

$$\mathbf{s}_k = [\ x_k^s \quad y_k^s \quad z_k^s\ ]^T \tag{2}$$

the wavefront from source k impinging on the qth microphone, located at

$$\mathbf{m}_q = [\ x_q^m \quad y_q^m \quad z_q^m\ ]^T \tag{3}$$

may then be expressed as

$$y_{k,q}(t) = \sum_{\ell=1}^{L_k} \chi_{\ell,k,q} + n_k(t) \tag{4}$$

with the noise-free signal component defined as

$$\chi_{\ell,k,q} = a_{k,q}(\omega_{k,\ell}) A_{k,\ell} e^{i\omega_{k,\ell} t + \phi_{k,\ell}} \tag{5}$$

where $n_k(t)$ is an additive noise, here assumed to be well modelled as being zero-mean and circularly symmetric white Gaussian distributed, and $a_{k,q}(\omega)$ details the spherical propagation gain and phase rotation of frequency ω for the signal impinging from the kth source onto the qth sensor, i.e., [22]

$$a_{k,q}(\omega) = r_{k,q}^{-(d-1)/2} e^{-i\omega r_{k,q}/c} \tag{6}$$

where $r_{k,q}$ is the distance between the kth source and the qth microphone, i.e.,

$$r_{k,q} = \|\mathbf{s}_k - \mathbf{m}_q\|_2 \tag{7}$$

and c is the propagation speed of the wavefront, for acoustic sources generally being about $c = 343$ m/s, $\forall\omega$. Finally, d denotes the dimensionality of the considered space, i.e., $d = 2$ in a flat space and $d = 3$ in 3-D.

As the locations of the microphones will strongly affect how well the source locations may be estimated, we here propose a placement scheme that incorporates any available prior knowledge of the expected source locations as well as the expected sound characteristics to determine suitable sensor locations. In order to do so, we propose minimizing a weighted form of the CRLB associated with the estimate of the K source locations given the M microphone locations, the latter being selected out of a set of possible sensor placements, $\mathcal{M}$. Clearly, the CRLB for the source localization problem will depend on the locations of the M microphones, the source locations, and the source signals. Let

$$\boldsymbol{\theta} = [\ \boldsymbol{\theta}_1^T \ \ldots \ \boldsymbol{\theta}_K^T \quad \sigma_n^2\ ]^T \tag{8}$$

where σ_n^2 denotes the variance of the additive noise $n_k(t)$ (which, for notational simplicity, is here assumed to be the same for each sensor), and with

$$\boldsymbol{\theta}_k = [\ \mathbf{s}_k \quad \omega_{k,0}\ ]^T \tag{9}$$

which thus implicitly makes an assumption that the number of sources, K, and the number of overtones for each source, L_k, for $k = 1, \ldots, K$, are known. Typically, this cannot be assumed in most practical cases, and the microphone placement should thus be designed to allow for robustness to these assumptions. Due to space limitations, we will here limit the discussion of robustness to the unknown source parameters $\boldsymbol{\theta}_k$, formulating the minimization such that it allows for the expected source locations and range of pitches for human voices.

III. PLACEMENT SCHEME

Proceeding, we formulate the sensor placement scheme such that we minimise the expected minimum variance of the location estimation problem, while still allowing for uncertainty in the assumed parameters. Let $\mathbf{Y}_{\boldsymbol{m}_n;\boldsymbol{\theta}}$ denote N samples measured from M sensors, with probability density function $p(\mathbf{Y}_{\boldsymbol{m}_n;\boldsymbol{\theta}})$, parameterised by $\boldsymbol{\theta} \in \mathbb{R}^P$, and introduce the $|\mathcal{M}|$ dimensional weight vector $\mathbf{w}$ indicating if a candidate sensor placement in the set $\mathcal{M}$ is used or not. As a starting point,

[1] For notational simplicity, we here use a complex-valued notation, noting that the complex-valued form of the measured signal may be easily formed using the discrete-time analytic signal [20].

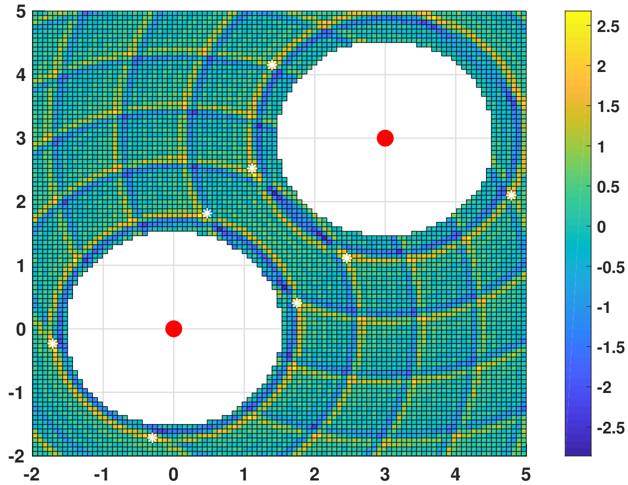

Fig. 1. The figure illustrates the found optimal sensor placement (white) for two audio sources (red). Shown in the background is the expected signal strength for the possible positions. As seen, the sensor placement scheme has selected sensor placement where the signal strength is expected to be strong, while spread around the expected source locations.

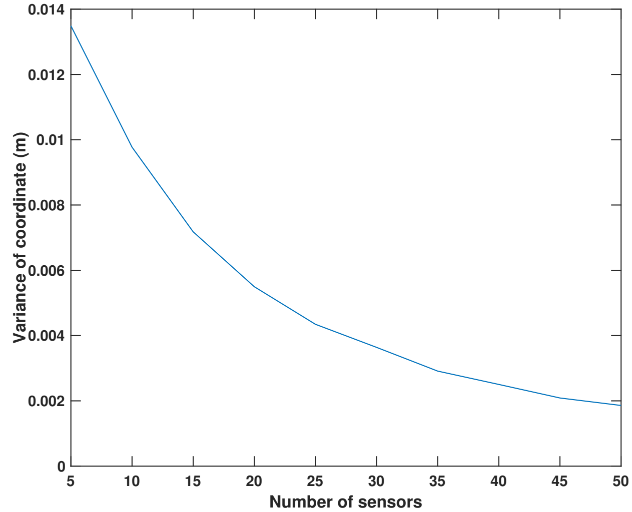

Fig. 2. The expected variation of the source position estimates as a function of number of sensors.

we consider an optimization based on A-optimality (see, e.g., [23], [24])

$$\begin{aligned} \underset{\mathbf{w}}{\text{minimize}} \quad & \text{trace}\Big(\big(\sum_{n=1}^{N} w_n \mathbf{F}(\mathbf{Y}_{\boldsymbol{m}_n;\boldsymbol{\theta}})\big)^{-1}\Big) \\ \text{subject to} \quad & \|\mathbf{w}\|_1 \leq \gamma, \\ & w_n \in \{0,1\}, \quad n = 1,2,\cdots N \end{aligned} \tag{10}$$

Where $\mathbf{F}(\mathbf{Y}_{\boldsymbol{m}_n;\boldsymbol{\theta}})$ denotes the FIM for the localization problem at hand, defined as

$$\mathbf{F}(\mathbf{Y}_{\boldsymbol{m}_n;\boldsymbol{\theta}}) = \mathbb{E}\left\{\nabla_{\boldsymbol{\theta}} \log\left(p(\mathbf{Y}_{\boldsymbol{m}_n;\boldsymbol{\theta}})\right) \nabla_{\boldsymbol{\theta}} \log\left(p(\mathbf{Y}_{\boldsymbol{m}_n;\boldsymbol{\theta}})\right)^H\right\} \tag{11}$$

where $\mathbb{E}\{\cdot\}, \nabla_{\boldsymbol{\theta}}$, and $(\cdot)^H$ denote the statistical expectation, the gradient with respect to $\boldsymbol{\theta}$, and the conjugate transpose, respectively, and $\gamma > 0$ constitutes an upper bound on the sum of the weight vector. Further details on the FIM for the problem at hand are provided in the appendix.

The formulation in (10) is not convex, but a convex approximation can be obtained by relaxing the binary constraint on the weights. The relaxed formulation is still limited in the sense that it does not allow for parameters with significantly different variances, or for the case when one is primarily interested in a subset of the parameters. To extend the formulation in (10) to also account for these cases, we proceed to instead perform the optimization using weighted FIMs, in analogy with [19], by introducing the linearly transformed parameters $\tilde{\boldsymbol{\theta}} = \mathbf{A}(\boldsymbol{\theta})\boldsymbol{\theta}$. Then, the weighted FIM becomes $\tilde{\mathbf{F}}(\mathbf{Y}_{\boldsymbol{m}_n;\boldsymbol{\theta}}) = \mathbf{A}(\boldsymbol{\theta})\mathbf{F}(\mathbf{Y}_{\boldsymbol{m}_n;\boldsymbol{\theta}})\mathbf{A}(\boldsymbol{\theta})$. Reminiscent to the formulation in [19], we are able to reformulate (10) as[2]

$$\begin{aligned} \underset{\boldsymbol{\mu},\mathbf{w}}{\text{minimize}} \quad & \sum_{p=1}^{P} \psi_p \mu_p \\ \text{subject to} \quad & \begin{bmatrix} \sum_{n=1}^{|\mathcal{M}|} w_n \mathbf{F}(\mathbf{Y}_{\boldsymbol{m}_n;\boldsymbol{\theta}}) & \mathbf{e}_p \\ \mathbf{e}_p^T & \mu_p \end{bmatrix} \succeq \mathbf{0}, \\ & \sum_{n=1}^{|\mathcal{M}|} w_n \mathbf{F}(\mathbf{Y}_{\boldsymbol{m}_n;\boldsymbol{\theta}}) \succ \mathbf{0} \\ & \mathbf{1}^T\mathbf{w} \leq \gamma, \quad \mu_p \leq \lambda_p \end{aligned} \tag{12}$$

for $p = 1, \ldots, P$ and $w_n \in [0,1]$, where $\mathbf{X} \succ \mathbf{0}$ indicates that the matrix $\mathbf{X}$ is positive definite, with w_n denoting the nth element in $\mathbf{w}$, $\mathbf{1}$ being an $|\mathcal{M}| \times 1$ vector of ones, and $\mathbf{e}_p$ the pth canonical basis vector, i.e., a vector with all elements equal to zero except the pth element being equal to one.

The minimization is here formed over $\boldsymbol{\theta} \in \Theta$, where Θ denotes the set of possible parameter vectors, in this case the expected source locations and range of pitch parameters. The weight parameters $\psi_p \geq 0$ allow for putting emphasis on different parameters in $\boldsymbol{\theta}$, whereas λ_p introduce an upper tolerance bound on the pth parameter $\boldsymbol{\theta}$. The vector $\boldsymbol{\mu}$, with elements μ_p, for $p = 1, \ldots, P$, forms an upper bound of the inverse FIM, coinciding at the optimum with the CRLB of the selected sensor placement.

As a result, the solution to (12) will minimize the worst case CRLB for the parameters of interest, when $\boldsymbol{\theta}$ is known to be in Θ. The minimization in (12) constitutes a semi-definite program that may be solved efficiently using standard convex solvers, such as SeDuMi [25] or SDPT3 [26].

IV. NUMERICAL

To illustrate the proposed sensor placement scheme, we examine the results using both simulated and experimental data. Initially, consider a hypothetical setup where two

[2]Given the page limitation, we refer the reader to [19] for a more detailed motivation of the formulation.

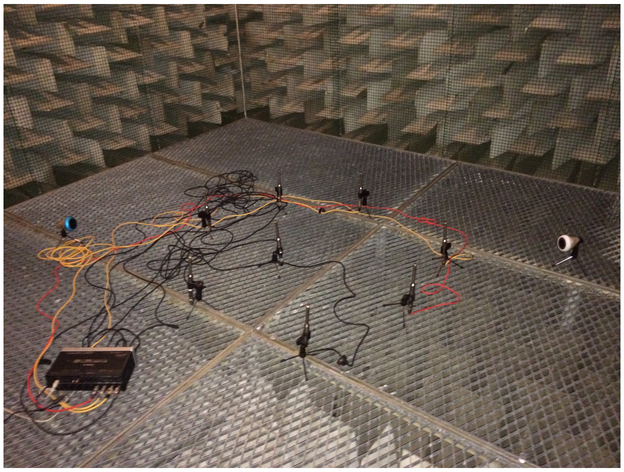

Fig. 3. The setup of sources and sensors in the anechoic chamber.

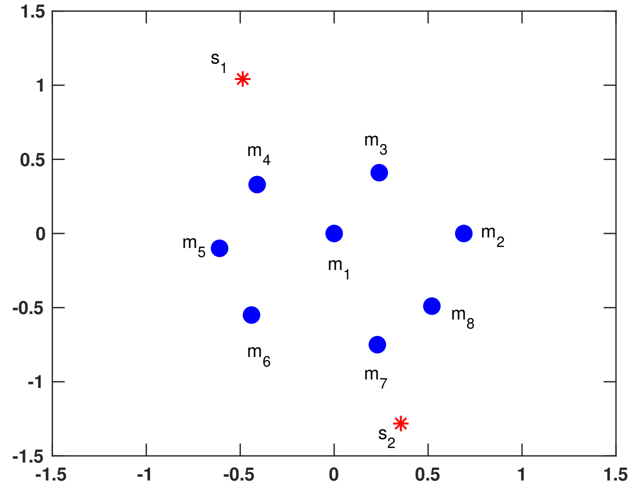

Fig. 4. The setup of sources and sensors used in the real experiment.

speakers are expected to be located at coordinates (0,0) and (3,3), respectively, such as could be expected for instance in an auditorium. The fundamental frequencies of the two speakers are assumed to be 200 ± 5 Hz and 180 ± 5 Hz, respectively, although the exact pitches are unknown. The sources have $L_1 = L_2 = 5$ overtones. For simplicity, we limit our attention to the planar scenario, so that $d = 2$. We consider an equidistant grid of potential microphone locations consisting of $|\mathcal{M}| = 9820$ candidate locations in the region $x, y \in [-2, 5]$, except, for presentational clarity, within a ≤ 1.5 m distance from the sources. Figure 1 illustrates the setup, where each pixel represents a candidate microphone location (except in the vicinity of ≤ 1.5 form each source), and the actual source locations are marked in red. The white marks indicate the optimal placements if placing $M = 10$ microphones, together with the expected strength of the sound field at each location. As can be seen from the figure, the optimization has selected microphone placements where the expected signal is strong, while spreading them out on both sides of the sources to be able to better locate these. Here, we have used $\gamma = 10$, $\lambda = 0.10$, $\psi_p = 1$, $\forall p$.

Figure 2 illustrates how the variance of the estimated source location decreases with the number of placed microphones, when placing the sensor optimally for each number of sensors using (12). The figure shows the minimum expected variance over the x- and y-directions for a setup with one source, with 5 harmonics, placed at the origin as the number of microphones grow, thereby also allowing one to determine the minimum required number of microphones to use to allow for a desired localization accuracy. As can be expected, the marginal gain of adding a sensor decreases as the number of sensors increases, with the amount of added useful information available in the localization problem decreasing for each additional sensor.

Next, we consider a real data experimental setup, using the data set reported on in [18]. This data was collected in an anechoic chamber of approximate dimensions $4 \times 4 \times 3$ meters. The sensor/source setup can be seen in Figures 3 and 4, where the sources are located at $\mathbf{s_1} = [-0.4866, 1.0420]$ and $\mathbf{s_2} = [0.3548, -1.2818]$. The sources each play the same signal, a SQAM violin signal. As a first step, we used the HALO method presented in [18] to estimate the signal parameters and locations. The estimates indicated that the violin signal may be well described as having fundamental frequency $\hat{\omega}/2\pi = 198.0$ Hz with $\hat{L} = 14$ overtones. Next, we proceeded to use these estimates in the sensor placement scheme to find an optimal sensor placement. The result for placing 6 sensors is shown in Figure 5, where the uncertainties in pitch have been allowed to be $\pm 5 Hz$. We proceed to examine which of the available sensors that yield the most information, comparing to the combinatorial solution, which is here feasible due to the limited number of candidate locations. Table I indicates the resulting placements for both approaches. As can be seen the sensor selections are the same for both approaches.

V. Conclusion

In this paper we have proposed a method for determining a sensor placement that minimizes the worst case CRLB for the corresponding localization problem. The scheme minimizes a relaxation of the resulting A-optimality formulation of the localization problem, and can be efficiently implemented as a semi definite program. Numerical experiments illustrate the proposed sensor placements for both simulated and experimental data.

Appendix

Introduce

$$y_{:,q}(t) \triangleq \sum_{k=1}^{K} y_{k,q}(t) \quad \text{and} \quad y_{k,:}(t) \triangleq \sum_{q=1}^{M} y_{k,q}(t) \tag{13}$$

and let $\tilde{\boldsymbol{y}}_t$ denote the measured data vector at time t, i.e.,

$$\tilde{\boldsymbol{y}}_t = \begin{bmatrix} y_{:,1}(t) & \dots & y_{:,M}(t) \end{bmatrix}^T = \tilde{\boldsymbol{u}}_t + \tilde{\boldsymbol{n}}_t \tag{14}$$

where $\tilde{\boldsymbol{n}}$ is defined similar to $\tilde{\boldsymbol{y}}$, and

$$\tilde{\boldsymbol{u}}_t \triangleq \begin{bmatrix} \Psi_1(t) & \dots & \Psi_M(t) \end{bmatrix}^T \tag{15}$$

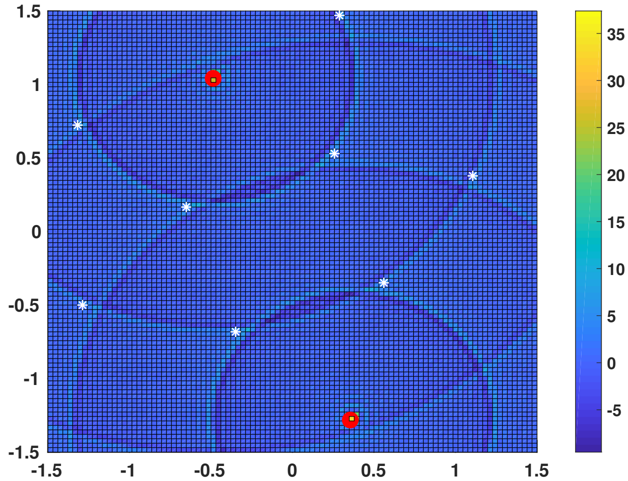

Fig. 5. The figure illustrates the found optimal sensor placement (white) for two audio sources (red) for the real data case. Shown in the background is the expected signal strength for the possible positions.

with the noise-free measurements at sensor q defined as

$$\Psi_q(t) = \sum_{k=1}^{K} \sum_{\ell=1}^{L_k} \chi_{\ell,k,q} \tag{16}$$

Then, according to Slepian-Bangs formula (see, e.g., [27]), the v, wth element in $\mathbf{F}(\mathbf{Y}_{\boldsymbol{m}_n;\boldsymbol{\theta}})$ may then be expressed as

$$[\mathbf{F}(\mathbf{Y}_{\boldsymbol{m}_n;\boldsymbol{\theta}})]_{v,w} = \frac{2}{\sigma_n^2} \Re \left\{ \sum_{t=0}^{N-1} \frac{\partial \tilde{\boldsymbol{u}}_t^H}{\partial \boldsymbol{\theta}_v} \frac{\partial \tilde{\boldsymbol{u}}_t}{\partial \boldsymbol{\theta}_w} \right\} \tag{17}$$

where $\Re(\cdot)$ denotes the real part, with

$$\frac{\partial \tilde{\boldsymbol{u}}(t)}{\partial x_k^s} = \sum_{q=1}^{M} \sum_{\ell=1}^{L_k} \left(\frac{-(d-1)\kappa_{q,k}}{2\xi_{q,k}^{(d-5)/2}} + \frac{i\omega_{k,0}\kappa_{q,k}}{c\,\|\kappa_{q,k}\|} \right) \chi_{\ell,k,q}$$

$$\frac{\partial \tilde{\boldsymbol{u}}(t)}{\partial \omega_{k,0}} = \sum_{q=1}^{M} \sum_{\ell=1}^{L_k} \ell i \left(t - \frac{r_{k,q}}{c} \right) \chi_{\ell,k,q}$$

where $\xi_{q,k} = (\boldsymbol{m}_q - \boldsymbol{s}_k)$ and $\kappa_{q,k} = (x_q^m - x_k^s)$ for the qth microphone and kth source, respectively. The derivatives with respect to the y- and z-coordinates are defined analogously.

TABLE I

SENSOR SELECTION FROM EXPERIMENTAL SETUP

Number of sensors	Proposed	Combinatorics
4	$m_1, m_2, m3, m_6$	m_1, m_2, m_3, m_6
5	m_1, m_2, m_3, m_4, m_6	m_1, m_2, m_3, m_4, m_6
6	$m_1, m_2, m_3, m_4,$ m_5, m_6	$m_1, m_2, m_3, m_4,$ m_5, m_6

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