Distributed Adaptive Acoustic Contrast Control for Node-specific Sound Zoning in a Wireless Acoustic Sensor and Actuator Network

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***Abstract*—This paper presents a distributed adaptive algorithm for node-specific sound zoning in a wireless acoustic sensor and actuator network (WASAN), based on a network-wide acoustic contrast control (ACC) method. The goal of the ACC method is to simultaneously create node-specific zones with high signal power (bright zones) while minimizing power leakage in other node-specific zones (dark zones). To obtain this, a network-wide objective involving the acoustic coupling between all the loudspeakers and microphones in the WASAN is proposed where the optimal solution is based on a centralized generalized eigenvalue decomposition (GEVD). To allow for distributed processing, a gradient based GEVD algorithm is first proposed that minimizes the same objective. This algorithm can then be modified to allow for a fully distributed implementation, involving in-network summations and simple local processing. The algorithm is referred to as the distributed adaptive gradient based ACC algorithm (DAG-ACC). The proposed algorithm outperforms the non-cooperative distributed solution after only a few iterations and converges to the centralized solution, as illustrated by computer simulations.**

***Index Terms*—Acoustic Contrast Control, Sound Zoning, Wireless Sensor Network, Wireless Sensor and Actuator Network (WASAN), Generalized Eigenvalue Decomposition (GEVD)**

I. Introduction

A wireless acoustic sensor and actuator network (WASAN) [1] consists of multiple (collections of) devices that are equipped with microphones and loudspeakers and connected via wireless links. When the devices, also referred to as nodes, are able to manipulate their loudspeaker signals, new applications like active noise cancellation (ANC) [2], [3] and sound zone control [4]–[6] emerge. This paper focuses on sound zoning in a WASAN.

The goal of sound zoning is to simultaneously create zones with high signal power, i.e. bright zones, while minimizing power leakage in other zones, i.e. dark zones, and this is controlled by measuring the pressure in these zones by means of microphones. In a WASAN, each node can decide independently which of their local microphones belongs to a bright zone and which to a dark zone, hence these zones can indeed be termed node-specific. To create the desired sound zones, optimal FIR filters are designed to pre-filter all the loudspeaker signals in the WASAN. Design criteria to obtain these optimal filters, involve the resulting signal power and the signal distortion in the bright zones, and the power leakage in the dark zones. In the acoustic contrast control (ACC) method, the ratio between the power leakage in the dark zones and the signal power in the bright zones is minimized. In the pressure matching (PM) method [7], the signal distortion is explicitly taken into account, generally resulting in a much higher dark zone leakage compared to the ACC method. Hybrid methods [8] combining the trade-offs between ACC and PM are therefore also available.

This paper focuses on the ACC method where the optimal loudspeaker pre-filters are found by minimizing a network-wide objective involving the acoustic coupling between all the loudspeakers and microphones in the WASAN and where the optimal solution is based on a centralized generalized eigenvalue decomposition (GEVD). To allow for distributed processing, a gradient based GEVD algorithm is first proposed. This algorithm can then be modified to allow for a fully distributed implementation, involving in-network summations [9]–[11] and simple local processing. The algorithm is referred to as the distributed adaptive gradient based ACC algorithm (DAG-ACC). The algorithm belongs to the class of distributed algorithms that elicit cooperation between nodes in a network to simultaneously solve different but related parameter estimation problems [3], [12], [13].

The paper is organized as follows. The problem formulation and the centralized ACC method are presented in Section II and III respectively. In Section IV the gradient based GEVD algorithm is presented and Section V introduces the DAG-ACC which transforms the gradient based GEVD algorithm to allow for in-network distributed processing. Computer simulations to illustrate the convergence, are provided in Section VI. Conclusions are given in Section VII.

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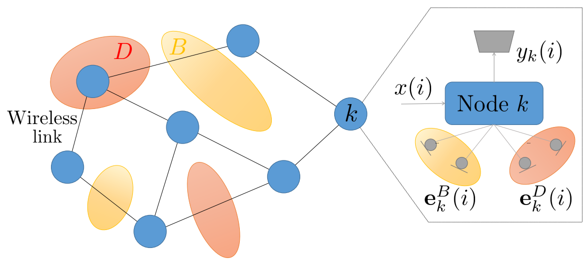


Fig. 1: WASAN with node-specific bright and dark zones.

II. Problem formulation: Sound zoning in a WASAN

Consider a WASAN consisting of K nodes deployed in a region where different bright zones and dark zones are defined. As depicted in Fig. 1, each node consists of a loudspeaker[1] and a collection of microphones defining the node-specific bright zone signal vector $\mathbf{e}_k^B(i) \in \mathbb{R}^{M_k^B}$ and dark zone signal vector $\mathbf{e}_k^D(i) \in \mathbb{R}^{M_k^D}$, with i the time index. The letters B and D refer to the bright and dark zone respectively. Node-specific means that nodes can decide independently which of their local microphones belongs to the bright zone and which to the dark zone. Moreover, each node has access to the same reference signal $x(i)$.[2]

The goal of the ACC method implemented at node k is to define the loudspeaker signal as a filtered version of the reference signal $x(i)$ such that the power in the bright zone microphones is as high as possible whereas the dark zone microphones should ideally measure a power of 0.[3] Considering that the acoustic coupling between the loudspeaker of node $\bar{k}$ and the microphones of node k are modeled as M_k^m L-th order FIR filters $\forall m \in \{B, D\}$, i.e. each time domain impulse response is stacked as one row in $\mathbf{H}_{\bar{k},k}^m = [\mathbf{H}_{\bar{k},k}^m(1) \; ... \; \mathbf{H}_{\bar{k},k}^m(L)] \in \mathbb{R}^{M_k^m \times L}$, the microphone signal vector $\mathbf{e}_k^m(i)$ of node k is described as

$$\begin{aligned} \mathbf{e}_k^m(i) &= \sum_{\bar{k}=1}^{K} \sum_{l=1}^{L} \mathbf{H}_{\bar{k},k}^m(l) y_{\bar{k}}(i-l+1) \\ &= \sum_{\bar{k}=1}^{K} \sum_{l=1}^{L} \mathbf{H}_{\bar{k},k}^m(l) \mathbf{x}(i-l+1)^T \mathbf{w}_{\bar{k}} \\ &= \sum_{\bar{k}=1}^{K} \mathbf{H}_{\bar{k},k}^m \mathbf{X}(i)^T \mathbf{w}_{\bar{k}} \qquad \forall m \in \{B, D\} \end{aligned} \tag{1}$$

[1]The derivations can easily be extended to multiple loudspeakers per node, as long as the number of loudspeakers is smaller than the number of bright microphones as well as the number of dark zone microphones of the node.

[2]An extension to multiple reference signals where the node-specific sound zones can be different for each reference signal, is possible. The resulting local loudspeaker signal is then simply the sum of the obtained loudspeaker signals when considering only one reference signal at the time.

[3]This will however introduce a high signal distortion in the bright zone microphones, which is a typical side-effect of the ACC method.

where

- $y_{\bar{k}}(i)$ is the loudspeaker signal at node $\bar{k}$ at time instant i constructed as $\mathbf{x}(i)^T \mathbf{w}_{\bar{k}}$ with $(.)^T$ the transpose operator,
- $\mathbf{w}_{\bar{k}} \in \mathbb{R}^M$ denotes the ACC filter of M coefficients applied by node $\bar{k}$ to the reference signal,
- $\mathbf{x}(i) = [x(i) \; ... \; x(i-M+1)]^T \in \mathbb{R}^M$,
- $\mathbf{X}(i) = [\mathbf{x}(i) \; ... \; \mathbf{x}(i-L+1)] \in \mathbb{R}^{M \times L}$.

In equation (1), the ACC filters $\{\mathbf{w}_k\}_{k=1}^K$ are considered to be time-invariant [14]. Note that this assumption is approximately satisfied if the coefficients of the ACC filters change slowly compared to the timescale of the system to be controlled, i.e. the impulse responses $\{H_{\bar{k},k}^m\}_{k=1}^K$.

Assume that the impulse responses $\{H_{\bar{k},k}^m\}_{k=1}^K$ are known by node k or can be estimated by node k in a calibration phase [15] and tracked during the operation [16]. Since the loudspeaker signal played by a node is also observed by the other nodes, the objective of the nodes is to cooperate in order to produce the optimal loudspeaker outputs $\{y_k(i)\}_{k=1}^K$ by finding the ACC filters $\{\mathbf{w}_k\}_{k=1}^K$ that minimize

$$J_{ACC}(\{\mathbf{w}_k\}_{k=1}^K) = \frac{\sum_{k=1}^K E\{\mathbf{e}_k^D(i)^T \mathbf{e}_k^D(i)\}}{\sum_{k=1}^K E\{\mathbf{e}_k^B(i)^T \mathbf{e}_k^B(i)\}} \tag{2}$$

where $E\{.\}$ is the expected value operator.

Note that $E\{\mathbf{e}_k^D(i)^T \mathbf{e}_k^D(i)\}$ and $E\{\mathbf{e}_k^B(i)^T \mathbf{e}_k^B(i)\}$ denote the sum of the microphone signal powers in the dark zone and bright zone of node k respectively. The global objective is therefore the ratio between the sum of the powers of all the dark zone microphone signals and the sum of the powers of all the bright zone microphone signals in the WASAN.

III. Centralized ACC method

Stacking all the ACC filters $\{\mathbf{w}_k\}_{k=1}^K$ into one augmented $\tilde{M}$-dimensional vector

$$\mathbf{w} = [\mathbf{w}_1^T \; ... \; \mathbf{w}_K^T]^T \tag{3}$$

where $\tilde{M} = KM$, equation (1) can be rewritten as

$$\mathbf{e}_k^m(i) = \mathbf{U}_k^m(i)\mathbf{w} \tag{4}$$

where

$$\mathbf{U}_k^m(i) = [\mathbf{H}_{1,k}^m \mathbf{X}(i)^T \; ... \; \mathbf{H}_{K,k}^m \mathbf{X}(i)^T] \quad \forall m \in \{B, D\}. \tag{5}$$

By substituting (4) in (2), the optimal time-invariant centralized ACC filters correspond to the $\mathbf{w}^*$ that minimizes

$$\begin{aligned} J_{ACC}(\mathbf{w}) &= \frac{\sum_{k=1}^K E\{\mathbf{w}^T \mathbf{U}_k^D(i)^T \mathbf{U}_k^D(i)\mathbf{w}\}}{\sum_{k=1}^K E\{\mathbf{w}^T \mathbf{U}_k^B(i)^T \mathbf{U}_k^B(i)\mathbf{w}\}} \\ &= \frac{\mathbf{w}^T \sum_{k=1}^K \mathbf{R}_k^D \mathbf{w}}{\mathbf{w}^T \sum_{k=1}^K \mathbf{R}_k^B \mathbf{w}} \\ &= \frac{\mathbf{w}^T \mathbf{R}^D \mathbf{w}}{\mathbf{w}^T \mathbf{R}^B \mathbf{w}} \end{aligned} \tag{6}$$

where $\mathbf{R}_k^m \triangleq E\{\mathbf{U}_k^m(i)^T \mathbf{U}_k^m(i)\}$ and $\mathbf{R}^m \triangleq \sum_{k=1}^K \mathbf{R}_k^m$. Node k can estimate the correlation matrices $\{\mathbf{R}_k^m\}_{m \in \{B,D\}}$ locally by time-averaging over a certain time-period, if the signal $x(i)$ is considered to be (quasi-)stationary and ergodic.

The objective in (6) is a Rayleigh quotient of symmetric positive definite matrices $\{\mathbf{R}^D, \mathbf{R}^B\}$ and so the optimal solution $\mathbf{w}^*$ is (up to a scalar multiplication) given by the generalized eigenvector corresponding to smallest generalized eigenvalue of the matrix pencil $\{\mathbf{R}^D, \mathbf{R}^B\}$ [17]. However, to compute a centralized solution, each node has to transmit the $(\tilde{M} \times \tilde{M})$-dimensional matrices $\{\mathbf{R}_k^m\}_{m\in\{B,D\}}$ to a fusion center where the ACC filters $\{\mathbf{w}_k\}_{k=1}^K$ are computed via a generalized eigenvalue decomposition (GEVD) and then transmitted back to the nodes. This is not a robust method, as it introduces a single point of failure, and furthermore, requires a high computational capacity of the fusion center to compute the GEVD of a matrix pencil whose dimension grows linearly with the number of nodes K in the WASAN. To alleviate these high computational and communication requirements, a gradient based GEVD algorithm is first proposed in Section IV and then it will be shown in Section V that this algorithm allows for an efficient distributed implementation.

IV. Gradient based GEVD algorithm

The gradient of (6) is given by

$$\begin{aligned}\nabla_{\mathbf{w}} J_{ACC} &= \frac{\mathbf{R}^D\mathbf{w}}{\mathbf{w}^T\mathbf{R}^B\mathbf{w}} - \frac{\left(\mathbf{w}^T\mathbf{R}^D\mathbf{w}\right)\mathbf{R}^B\mathbf{w}}{\left(\mathbf{w}^T\mathbf{R}^B\mathbf{w}\right)^2} \\ &= \alpha\left(\mathbf{R}^D\mathbf{w} - \frac{\left(\mathbf{w}^T\mathbf{R}^D\mathbf{w}\right)}{\left(\mathbf{w}^T\mathbf{R}^B\mathbf{w}\right)}\mathbf{R}^B\mathbf{w}\right)\end{aligned} \tag{7}$$

where α is a scalar. Note that the gradient is equal to zero in every generalized eigenvector of the matrix pencil $\{\mathbf{R}^D, \mathbf{R}^B\}$. The gradient provides a direction of ascent, so if the optimization variable $\mathbf{w}$ takes small steps in the direction of the negative gradient, a decrease in the objective J_{ACC} is guaranteed. Therefore Algorithm 1 is proposed to solve (6) iteratively. Here n denotes the iteration index.

Algorithm 1: Gradient based GEVD algorithm

1 - Initialize $\mathbf{w}(0)$ randomly.
- $n \leftarrow 0$.

2 Perform a gradient update with stepsize μ:

$$\sigma(n) = \frac{\mathbf{w}(n)^T\mathbf{R}^D\mathbf{w}(n)}{\mathbf{w}(n)^T\mathbf{R}^B\mathbf{w}(n)} \tag{8}$$

$$\mathbf{p}(n) = \mathbf{R}^D\mathbf{w}(n) - \sigma(n)\mathbf{R}^B\mathbf{w}(n) \tag{9}$$

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \mu\mathbf{p}(n). \tag{10}$$

3 - $n \leftarrow n+1$ and return to step 2 until some stopping criterion is met.

A procedure similar to [18] can be followed to show that Algorithm 1 converges to the unique (up to a scalar multiplication) generalized eigenvector $\mathbf{w}^*$ corresponding to the smallest generalized eigenvalue of the matrix pencil $\{\mathbf{R}^D, \mathbf{R}^B\}$. Necessary requirements are:

- the stepsize μ should be smaller than $2/(\lambda_{max}^{\mathbf{R}^B} S)$, with $\lambda_{max}^{\mathbf{R}^B}$ the largest eigenvalue of $\mathbf{R}^B$ and S the difference between the largest and smallest generalized eigenvalue of the matrix pencil $\{\mathbf{R}^D, \mathbf{R}^B\}$,
- $\mathbf{w}(0)^T\mathbf{w}^*$ is not equal to zero,
- $\mathbf{R}^B$ and $\mathbf{R}^D$ are symmetric positive definite matrices.

Note that $\sigma(n)$ tracks the evolution in the objective J_{ACC} for the current estimate $\mathbf{w}(n)$. Determining an optimal stepsize

$$\mu^*(n) = \arg\min_{\mu} J_{ACC}(\mathbf{w}(n) - \mu\mathbf{p}(n)) \tag{11}$$

for faster convergence as in [17], [19] requires computing the smallest Ritz value and corresponding Ritz vector of the matrix pencil $\{\mathbf{R}^D, \mathbf{R}^B\}$ with respect to $\{\mathbf{w}(n), \mathbf{p}(n)\}$, which complicates the distributed implementation of the next section and will only be briefly touched upon further in the paper.

V. Distributed Adaptive Gradient based ACC algorithm

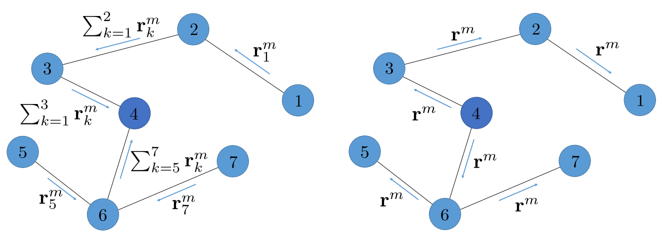
Algorithm 1 allows for a distributed implementation. To see this, note that the matrix vector products of (8) and (9) in Algorithm 1 can be written as:

$$\begin{aligned}\mathbf{R}^m\mathbf{w}(n) &= \sum_{k=1}^K E\{\mathbf{U}_k^{mT}(i)\mathbf{U}_k^m(i)\mathbf{w}(n)\} \\ &= \sum_{k=1}^K E\{\mathbf{U}_k^{mT}(i)\mathbf{e}_k^m(i)\} = \sum_{k=1}^K \mathbf{r}_k^m = \mathbf{r}^m\end{aligned} \tag{12}$$

where $\mathbf{r}_k^m = E\{\mathbf{U}_k^{mT}(i)\mathbf{e}_k^m(i)\}$ can be estimated locally by node k when $\mathbf{w}(n)$ is used as ACC filters in the WASAN, since node k has access to both $\mathbf{U}_k^m(i)$ and $\mathbf{e}_k^m(i)$.[4] If the nodes are able to construct the in-network sum $\sum_{k=1}^K \mathbf{r}_k^m$ to obtain $\{\mathbf{r}^m\}_{m\in\{B,D\}}$ and if each node has a local copy of the current ACC filters $\mathbf{w}^k(n)$, where superscript k refers to the local copy at node k, then Algorithm 1 can be executed in a fully distributed fashion.

Algoritms to obtain in-network summations with minimal communication requirement are abundantly available in literature, e.g., relying on gossip or consensus methods [9], [10]. However, these methods typically need many iterations to converge, as well as multiple broadcasts of intermediary summed variables. Therefore, based on [11], a method is proposed to calculate the in-network summation which relies on a tree topology that is formed from the set of available links in the network, and that has a minimal communication requirement (only 2 transmissions for each link). Fig. 2 shows the network flow. In a first phase, a tree is formed using the available links in the network [20], consisting of one root node and $K-1$ leaf nodes. A leaf node k with only one neighbor starts by transmitting $\{\mathbf{r}_k^m\}_{m\in\{B,D\}}$ to its neighbor. A node k with more than one neighbor waits until it receives signals from all its neighbors except one denoted by k' and transmits $\{\mathbf{r}_k^m + \sum_{\bar{k}\in\{\mathcal{N}_k\setminus k'\}} \mathbf{r}_{\bar{k}}^m\}_{m\in\{B,D\}}$ to node k', where $\mathcal{N}_k$ denotes the set of neighbors of node k. This continues until the root

[4]Note that $\mathbf{r}_k^m$ can also be estimated as $E\{\mathbf{U}_k^{mT}(i)\mathbf{U}_k^m(i)\mathbf{w}(n)\}$, however using $E\{\mathbf{U}_k^{mT}(i)\mathbf{e}_k^m(i)\}$ should be more robust to possible modeling errors in $\mathbf{U}_k^m(i)$. Extra simulations are needed to confirm this statement.



(a) Phase 1 (b) Phase 2

Fig. 2: Tree based in-network summation

node receives signals from all its neighbors. The root node then computes $\{\mathbf{r}^m = \mathbf{r}_k^m + \sum_{\bar{k}\in\mathcal{N}_k} \mathbf{r}_{\bar{k}}^m\}_{m\in\{B,D\}}$ in the next phase and broadcasts this back to all the nodes using the available communication tree.

The resulting distributed adaptive gradient based ACC algortihm (DAG-ACC) is presented in Algorithm 2. It is adaptive in the sense that in between two consecutive gradient steps, the nodes re-estimate their correlation vectors $\{\mathbf{r}_k^m\}_{m\in\{B,M\}}$ using a frame of I samples, so that any change in the signal statistics of $x(i)$ or $\{H_{k,k}^m\}_{m\in\{B,M\}}$ can indeed be tracked. In this way, n now corresponds both to the iteration index as well as the frame index.

Algorithm 2: Distributed Adaptive Gradient based ACC algorithm (DAG-ACC)

1 - Initialize $\mathbf{w}^k(0)$ with the same random vector $\forall k$.
- $n \leftarrow 0$.

2 - Each node k selects the part $\mathbf{w}_k^k(n)$ corresponding to its local ACC filter of $\mathbf{w}^k(n)$ to produce the loudspeaker signals:

$$y_k(nI+i) = \mathbf{x}^T(nI+i)\mathbf{w}_k^k(n), \quad i = 1..I. \tag{13}$$

3 - Each node k collects I observations of their bright and dark zone microphones and estimates $\mathbf{r}_k^B(n)$ and $\mathbf{r}_k^D(n)$ via time averaging in the frame:

$$\mathbf{r}_k^m(n) = \frac{1}{I}\sum_{i=1}^{I} \mathbf{U}_k^{mT}(nI+i)\mathbf{e}_k^m(nI+i) \quad \forall m \in \{B,D\}. \tag{14}$$

4 - The nodes interchange $\mathbf{r}_k^B(n)$ and $\mathbf{r}_k^D(n)$ such that each node obtains locally the in-network sum vectors:

$$\mathbf{r}^m(n) = \sum_{k=1}^{K} \mathbf{r}_k^m(n) \quad \forall m \in \{B,D\}. \tag{15}$$

5 Each node k performs a gradient update of its local estimation of the centralized ACC filter with the same stepsize μ:

$$\sigma^k(n) = \frac{\mathbf{w}^k(n)^T\mathbf{r}^D(n)}{\mathbf{w}^k(n)^T\mathbf{r}^B(n)} \tag{16}$$

$$\mathbf{p}^k(n) = \mathbf{r}^D(n) - \sigma^k(n)\mathbf{r}^B(n) \tag{17}$$

$$\mathbf{w}^k(n+1) = \mathbf{w}^k(n) - \mu\mathbf{p}^k(n). \tag{18}$$

6 - $n \leftarrow n+1$ and return to step 2.

As each iteration of the algorithm needs only two in-network summations to compute vectors $\{\mathbf{r}^m\}_{m\in\{B,M\}}$, the communication requirement is much lower than when the full network correlation matrices $\{\mathbf{R}^m\}_{m\in\{B,M\}}$ need to be formed. Also the local memory requirement are reduced since the matrices $\{\mathbf{R}_k^m \in \mathbb{R}^{\tilde{M}\times\tilde{M}}\}_{m\in\{B,M\}}$ do not need to be stored. The local processing requirement is also very low, compared to performing a GEVD of the matrix pencil $\{\mathbf{R}^D, \mathbf{R}^B\}$. The important drawback is that the algorithm needs some iterations to converge to the optimal filter $\mathbf{w}^*$.

It is also possible to perform T gradient iterations with the same batch of data in frame n. In this case, however, every node needs to keep track of the full local correlation matrix $\mathbf{R}_k^m(n)$ defined as

$$\mathbf{R}_k^m(n) = \frac{1}{I}\sum_{i=1}^{I} \mathbf{U}_k^m(nI+i)^T\mathbf{U}_k^m(nI+i) \quad \forall m \in \{B,D\} \tag{19}$$

which significantly increases the memory requirement in the node. For each frame n, the nodes then compute an in-network summation of $\mathbf{r}_k^m(n,t) = \mathbf{R}_k^m(n)\mathbf{w}^k(n,t)$ denoted by $\mathbf{r}^m(n,t)$ for $m \in \{B,D\}$, update the gradient as

$$\begin{aligned}\mathbf{w}^k(n,t+1) =& \mathbf{w}^k(n,t) - \mu\Bigg(\mathbf{r}^D(n,t) \\ &- \frac{\mathbf{w}^k(n,t)^T\mathbf{r}^D(n,t)}{\mathbf{w}^k(n,t)^T\mathbf{r}^B(n,t)}\mathbf{r}^B(n,t)\Bigg)\end{aligned} \tag{20}$$

and do this for all $t = \{0,...,T-1\}$. Using $\mathbf{w}^k(n+1,0) = \mathbf{w}^k(n,T)$, one can then proceed to the next frame. This procedure will increase the convergence speed, but also increases the communication requirement due to the extra in-network summations and increases the memory requirement for each node (cfr supra).

If one is able to compute $\mathbf{r}^m(n,t) = \sum_{k=1}^{K} \mathbf{R}_k^m(n)\mathbf{w}^k(n,t)$ in a distributed fashion, then one can also compute $\mathbf{R}^m(n)\mathbf{p}^k(n) = \sum_{k=1}^{K} \mathbf{R}_k^m(n)\mathbf{p}^k(n)$ for $m \in \{B,D\}$, which is needed to compute the Ritz vector for an optimal stepsize selection (11). This again increases the convergence speed but again at the cost of an increased memory and communication requirement.

VI. Simulations

To demonstrate the effectiveness of the DAG-ACC, a WASAN scenario with $K = 5$ nodes is considered. Each node has one bright zone microphone, one dark zone microphone and one loudspeaker. The reference signal $x(i)$ is a Gaussian noise source with zero mean and unit variance of size 3000. The ACC filters $\{\mathbf{w}_k\}_{k=1}^{K}$ have length $M = 20$. The acoustic coupling between the loudspeakers and the microphones is modeled by means of random impulse responses with zero mean and unit variance of length $L = 10$. Moreover, at frame $n = 150$ and $n = 300$, a random Gaussian component with zero mean and variance 0.1 is added to the impulse responses to model a possible re-estimation of the local impulse responses. To demonstrate convergence, algorithm 2 is run in batch mode, meaning that the reference signal $x(i)$

is periodic with period 3000 and $I = 3000$. The stepsize μ is set to 0.01.

In the simulations, the proposed DAG-ACC is compared with the centralized ACC method presented in section III. Also the result of a non-cooperative ACC method is provided, obtained when each node solves a local ACC problem based on:

$$J_{ACC,k}(\mathbf{w}_k) = \frac{E\{\mathbf{w}_k^T \mathbf{X}(i)\mathbf{H}^D_{k,k}\mathbf{H}^D_{k,k}{}^T\mathbf{X}(i)^T\mathbf{w}_k\}}{E\{\mathbf{w}_k^T \mathbf{X}(i)\mathbf{H}^B_{k,k}\mathbf{H}^B_{k,k}{}^T\mathbf{X}(i)^T\mathbf{w}_k\}}. \quad (21)$$

Note that this $J_{ACC,k}$ does not take into account the coupling between nodes, and will automatically generate suboptimal ACC filters. In Fig. 3, the objective J_{ACC} is plotted for each iteration of Algorithm 2 and J_{ACC} is also plotted for the centralized and non-cooperative ACC method. Here the median over 100 independent experiments is shown. The bumps in the 3 graphs are due to the abrupt changes of the impulse responses as described earlier. As expected, note that the DAG-ACC converges to the same optimal value $J_{ACC}(\mathbf{w}^*)$ as the centralized ACC method. The median MSE between the normalized $\mathbf{w}^*$ and normalized $\mathbf{w}(n)$ reaches the machine precision after 50000 iterations, which confirms convergence (not shown in Fig 3). Moreover, although the J_{ACC} for the non-cooperative ACC method is lower than the the J_{ACC} for the random initialization in Algorithm 2 initially, the DAG-ACC outperforms the non-cooperative ACC method already after a few iterations. As discussed, this degradation occurs due to the absence of cooperation among the nodes when they are solving ACC problems that are indeed coupled.

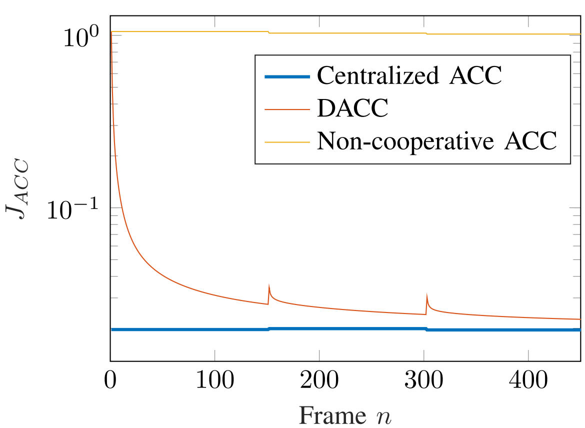


Fig. 3: Evolution of the objective J_{ACC} for the DAG-ACC and the centralized and non-cooperative ACC method.

VII. Conclusion

In this paper a node-specific ACC sound zoning has been considered in a WASAN. A distributed adaptive gradient-based ACC algorithm has been proposed, avoiding the high communication and computational requirements of a centralized ACC method. The algorithm has been shown to converge to the solution of the centralized method, by means of computer simulations.

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