Gravitational Search Algorithm for IIR Filter-Based Audio Equalization

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***Abstract*—In this paper we present an evolutionary algorithm for the design of stable IIR filters for binaural audio equalization. The filters are arranged as a cascade of second-order sections (SOS's) and the gravitational search algorithm (GSA) is used. This process seeks for optimal coefficients based on a fitness function, possibly leading to unstable filters. To avoid this, we propose two alternative methods. Experiments have been performed taking an in-car listening environment as the use case, characterized by multiple loudspeakers, thus, multiple impulse responses (IR). This technique has been compared with a previous heuristic method, achieving superior results.**

***Index Terms*—Gravitational search algorithm, Audio equalization, IIR filters, Automotive**

Audio equalization is a process aimed at increasing the sound quality in a listening environment [1]. Typical listening scenarios include indoor facilities, such as halls, venues or rooms, or small environments such as vehicle cockpits. These two categories present different issues. The in-car scenario features small volumes, very short reverberation with no separation between direct wave and reflections and peculiar materials, such as car window glass, plastic, fabric, etc. [2].

A large number of techniques for audio equalization have been proposed, based on linear filtering [1], [3]. Room response equalization, generally asks for the inversion of an impulse response. While this can be easily done for a single location, it is more difficult for multiple listening points, or even for the binaural case, since the impulse responses perceived at the typical ear spacing of 18 cm can be different [4], [5].

In the years, some nonlinear techniques have been used, including machine learning-based ones. An early work from Chang et al. [6] proposed a Time Delay Neural Network (TDNN) to process the signal sample-wise to inverse-model the room acoustics. In [7], the authors used an end-to-end Convolutional Neural Network (CNN). These approaches may have a very high computational cost, since the neural network is required to directly process the signal.

Other approaches may be envisioned, based on efficient techniques such as linear filtering, but taking advantage of machine learning techniques for the design of the filters coefficients. The design of filters for room equalization is a well-known topic and a plethora of approaches have been proposed (see, e.g., [1]). While FIR filters offer the advantage of providing linear-phase, IIR can be less expensive. Some researchers have opted for IIR graphical equalizers which allow to partition the problem in sub-bands [3]. These are composed of several peaking filter, one per octave, or one per third-octave, implemented as second-order sections (SOS's). One approach for automatic equalization was proposed in [8], where two optimization methods, the Direct Search Method (DSM) and the Rosenbrock Method were applied to the research of optimal gains for a graphic equalizer in order to obtain a given target curve. One issue with a graphic equalizer is the limited degrees of freedom it has. Other strategies can be envisioned for the design of the coefficients, where the filters are not designed by a closed-form equation, but have coefficients derived from iterative heuristic processes looking for solutions that are optimal according to some defined constraints.

Several algorithms have been proposed for filter design based on novel paradigms. Neural networks have been proposed for the filter design task. In [9] a simple neural network is trained to correct target gains for a graphic equalizer in order to reduce the error due to interference of adjacent filters. In [10] a neural network is devised to design an IIR filter. Wang et al. [11] proposes a two step optimization Frequency-Response Masking (FRM) technique based on the design of a FRM filter optimizing the subfilters, further optimized by decomposing it into several linear neural networks. Agrawal et al. [12] used Particle Swarm Optimization (PSO) with fractional derivative to design a stable IIR filter, while Foresi et al. [13] implemented PSO with fractional derivative constraints to design a quasi-linear phase IIR filter for Audio Crossover Systems, achieving the desired filter with a flat magnitude response. In [14] the authors achieved an IIR filter using the Artificial Immune Algorithm and compared the results with Genetic Algorithm (GA), the Touring Ant Colony Optimization (TACO) and Tabu Search (TS). In [15] the authors use Gravitational Search Algorithm (GSA) to model an IIR filter and a nonlinear rational filter, then they compare the technique with PSO and GA. In this case, the algorithms provide filter coefficients as outputs.

A. *Scope of the work*

Motivated by the works above, in [16], we employed the GSA algorithm for the design of FIR filters for audio equalization in a binaural scenario, with results superior to the state of the art techniques. In this work we extend the application of GSA to the design of stable IIR filters for equalization.

Some considerations must be underlined: unlike some of the works mentioned above, here we want to remove any constraint on the SOS design and let the evolutionary algorithm directly find optimal coefficients for several cascaded SOS's. As a consequence, with this approach, the coefficients are free to take any value, possibly leading to unstable impulse responses. To overcome this issue, we introduce two alternative constraints to avoid instabilities, one based on pole reflection and one on triangular stability.

The equalization scenario we consider is that of a room, specifically a car cabin. Generally, the environment is composed of $\mathcal{S}$ loudspeakers, each with an IIR filter associated. $\mathcal{M}$ listening points are placed in the environment that is, thus, characterized by $\mathcal{S}\times\mathcal{M}$ impulse responses (IR). The design of such filters is conducted by GSA. This process is performed off-line, due to the high computational cost. For this reason, the method is meant for pre-tuning. Time varying effects, including the movement of the listener head and position, are not taken into account.

For a broader comparison, we also take the DSM in consideration applying to our use case. Differently from [8] we use the DSM to directly design the filters coefficients, thus, the two stability-preserving methods proposed for GSA are also applied to DSM.

The paper outline follows: in Section I and Section II the GSA and DSM, respectively, are described. In Section III the proposed IIR filter design is exposed. In Section IV experiments details are given, while in Section V the results are reported. Finally, Section VI concludes the paper.

I. Gravitational Search Algorithm

The gravitational search algorithm (GSA) is based on *agents* subject to the Newton gravitational law [17]. The agent A attracts every other agent with a force. The i-th agent is defined by the position $X_i = [x_i^1, \cdots, x_i^d, \cdots, x_i^D]$, where D is the dimension of the search space. The mass M_i of i-th agent at n-th iteration is defined as:

$$M_i[n] = \frac{q_i[n]}{\sum\limits_{j=1}^{A} q_j[n]} \tag{1}$$

where $q_i[n]$ is defined as:

$$q_i[n] = \frac{fit_i[n] - worst[n]}{best[n] - worst[n]} \tag{2}$$

where $fit_i[n]$ is the fitness value of the i-th agent, $best[n]$ and $worst[n]$ are the best and worst fitness value of all agents, respectively.

Like the law of gravity, the force of the i-th agent F_i is calculated considering a set A_{best} of heavier masses:

$$F_i[n] = \sum_{\substack{j\in A_{best},\\ j\neq i}} rand_j \cdot G[n] \cdot \frac{M_j[n]\cdot M_i[n]}{R_{ij}[n]+\epsilon} \cdot (X_j[n] - X_i[n]) \tag{3}$$

Finally, acceleration, velocity and position of i-th agents ($\mathrm{a}_i[n]$, $v_i[n]$, $X_i[n]$, respectively) are calculated.

$$\mathrm{a}_i[n] = \frac{F_i[n]}{M_i[n]} = \sum_{\substack{j\in A_{best},\\ j\neq i}} rand_j \cdot G[n] \frac{M_j}{R_{ij}[n]+\epsilon} \cdot (X_j[n] - X_i[n]) \tag{4}$$

where $rand_j$ is a uniformly random number in the range $[0, 1]$, ϵ is a small value and $R_{ij}[n]$ is the euclidean distance between the agents i and j, $R_{ij}[n] = ||X_i[n], X_j[n]||_2$, A_{best} is a set of the best A agents.

$$v_i[n+1] = rand_i \cdot v_i[n] + \mathrm{a}_i[n] \tag{5}$$

$$X_i[n+1] = X_i[n] + v_i[n+1] \tag{6}$$

where $rand_i$ is uniformly distributed random number in range $[0, 1]$.

$G[n]$ is the gravitational constant that varies over time. In our case, $G[n]$ linearly decrease, defining an initial and a final value (G_0 and G_f respectively).

$$G[n] = G_0 - (G_0 - G_f) \cdot \frac{n}{N} \tag{7}$$

where N is the number of iterations.

II. Direct Search Method

The direct search method (DSM) is a heuristic method that uses random variation Γ of the parameters c close to the initial values, where $-\gamma \leq \Gamma \leq \gamma$ [8], [18]. If the new value of the parameter achieves a better fitness function, it is kept and used as the initial value for the next iteration, otherwise it is updated using a random Γ value:

$$\hat{c}_i = c_i(1+\Gamma) \tag{8}$$

where $\hat{c}_i$ is the i-th parameter.

Compared to [8], where the DSM is used to design IIR filters optmizing gain level, filter quality factor and center frequency (thus, all SOS's are designed using closed-form equations), we use this method to design each coefficient of the SOS's. Another difference concerns the initialization: our experiments are performed by randomly initializing the coefficients and the gains, while in [8] parameters have been choosen a priori at the initialization.

III. IIR filter design

Audio equalization is performed using IIR filters composed by a cascade of $\mathcal{N}$ Biquad filters and defined as [19]:

$$H_s(z) = g_s \prod_{k=1}^{\mathcal{N}} \frac{b_{s,k,0} + b_{s,k,1}z^{-1} + b_{s,k,2}z^{-2}}{1 + a_{s,k,1}z^{-1} + a_{s,k,2}z^{-2}} \tag{9}$$

where $H_s(z)$ is the s-th IIR filter composed by $\mathcal{N}$ SOS's.

With the DSM, for each SOS, the algorithms designs the filter coefficients (numerator and denominator) and the gain g_s. Similarly, for the GSA, each agent A_i, determines the coefficients and gain of a SOS.

The fitness function for the algorithms is the average Mean Square Error ($\overline{MSE}$) between the equalized magnitude response $H_m(\omega)$ and the desired magnitude response $H_{des}(\omega)$, averaged among all microphones:

$$\overline{MSE} = \frac{1}{\mathcal{M}} \sum_{m=1}^{\mathcal{M}} \frac{\sum\limits_{\omega=\omega_l}^{\omega_h} (|H_m(\omega)| - |H_{des}(\omega)|)^2}{\omega_h - \omega_l} \tag{10}$$

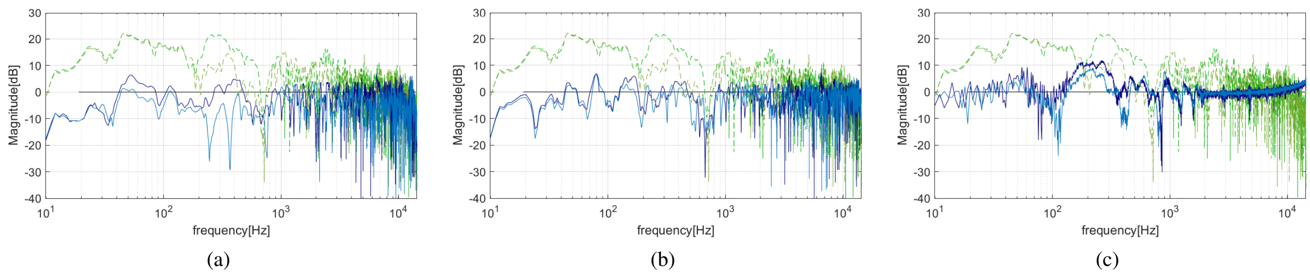

Fig. 1: Frequency responses of first and second microphone: (a) using DSM with 4 SOS's; (b) using GSA with 4 SOS's; (c) using GSA with 32 SOS's. Green lines are the non-equalized frequency responses, the blue lines are the equalized frequency responses

In designing coefficients with DSM and GSA, it is possible to incur in unstable SOS's. For this reason, we propose two methods to avoid unstable filters: the pole reflection technique [19] and a technique based on triangular stability [20]. For the first we calculate the poles of each second order filter at each iteration. If the pole is outside the unit circle, pole reflection is applied:

$$p_i = \frac{1}{p_i^* \cdot (1+\varepsilon)} \quad \text{if} \quad |p_i| \geq 1 \quad \text{for} \quad i = 1,2 \tag{11}$$

where ε is a small value to avoid conditional stability. Finally the coefficients are calculated and updated:

$$a_{s,k,1} = -(p_1 + p_2) \quad a_{s,k,2} = p_1 \cdot p_2 \tag{12}$$

With the second method, we check whether the denominator coefficients are inside the *stability triangle*, i.e.:

$$\begin{cases} |a_{s,k,2}| < 1 \\ |a_{s,k,1}| < 1 + a_{s,k,2}. \end{cases} \tag{13}$$

If this condition is not met, coefficients are updated according to $|a_{s,k,2}| = 1 - \epsilon$ and $|a_{s,k,1}| = 1 + a_{s,k,2} - \epsilon$.

IV. Experiments

Experiments were performed using binaural impulse responses ($\mathcal{M} = 2$) from an Alfa Romeo Giulia cockpit. The impulse responses have been measured using a Kemar mannequin type 45BA placed on the driver's seat and Audition 3.0 with Aurora plug-in in order to generate a Sine Sweep and to measure the impulse response [21], using a Roland Octa-Capture as audio interface. Impulse responses have been measured with a sampling rate of 28.8 kHz, later oversampled to 48 kHz. Since the original sampling rate was 28.8 kHz, the upper frequency bound ω_h was 14.4 kHz, while the lower frequency bound ω_l is equal to 20 Hz. The car is fitted with $\mathcal{S} = 7$ loudspeakers, positioned as follows: two on the front doors, two on the back doors, one the center part of the car, one near the driver headrest and a subwoofer within the trunk.

GSA is performed using *Python* as programming language and the *Tensorflow*[1] library as a framework for the experiments. DSM was implemented with *Matlab*. All experiments ran on a Linux-based Intel i7 server, with 32GB RAM and a GTX Titan GPU. The DSM and GSA have been implemented using tensors to allow the processing to run on the GPU.

In our experiments, the IIR filters have been designed by cascading $\mathcal{N} = 4, 8, 16$ or 32 SOS's. In the GSA case, the number of agents have been varied from 10 to 50 agents, while the number of best agents A_{best} was set from 2 to 10 agents. G has been varied setting the initial and final value: $1.0 \cdot 10^{-4} \leq G_0 \leq 0.1$ and $0.1 \leq G_f \leq 10.0$. On DSM, γ has been set to 0.05. The number of iterations is equal to 1000 for both GSA both for DSM.

For the initialization phase, agent and parameter values have been randomly set with an uniform distribution between -1.0 and 1.0.

V. Results

Experiments are compared in terms of $\overline{MSE}$, as in Eq. (10), and the average standard deviation $\overline{\sigma}$, calculated as:

$$\overline{\sigma} = \frac{1}{\mathcal{M}} \sum_{m=1}^{\mathcal{M}} \sigma_m \tag{14}$$

where σ_m is the standard deviation of m-th microphone:

$$\sigma_m = \sqrt{\tfrac{1}{\omega_h - \omega_l + 1} \textstyle\sum_{\omega=\omega_l}^{\omega_h} (10 \cdot log_{10}|H_m(\omega)| - D)^2} \tag{15}$$

$$D = \frac{1}{\omega_h - \omega_l + 1} \sum_{\omega=\omega_l}^{\omega_h} (10 \cdot log_{10}|H_m(\omega)|). \tag{16}$$

Moreover, to assess the perceptual impact of the equalization, we weight the $\overline{MSE}$ with the normalized A-curve $\psi_A(\omega)$ [22], as follows:

$$\overline{MSE}_A = \frac{1}{\mathcal{M}} \sum_{m=1}^{\mathcal{M}} \frac{\sum_{\omega=\omega_l}^{\omega_h} \psi_a(\omega)(|H_m(\omega)| - |H_{des}(\omega)|)^2}{\omega_h - \omega_l} \tag{17}$$

Results are presented in Table I. Very low $\overline{MSE}$ are obtained by the GSA using both pole reflection and triangular stability. However, the solutions obtained by the triangular stability constraint with 16 and 32 SOS's are trivial. Indeed, the solutions found by the algorithm in these two cases applies

[1] https://www.tensorflow.org/

Method	Number of SOS's	Pole reflection			Triangular stability		
		$\overline{MSE}$	$\overline{MSE_A}$	$\overline{\sigma}$	$\overline{MSE}$	$\overline{MSE_A}$	$\overline{\sigma}$
GSA	4	0.15	0.11	2.27	0.16	0.12	2.34
	8	0.17	0.13	2.29	0.16	0.12	2.45
	16	0.20	0.15	2.62	0.25	0.18	2.86
	32	**0.09**	**0.04**	**1.06**	0.28	0.22	0.63
DSM	4	*0.22*	*0.16*	*2.72*	0.25	0.19	2.82
	8	0.23	0.17	2.72	0.24	0.18	2.94
	16	0.26	0.20	3.00	0.31	0.24	3.11
	32	0.30	0.19	3.20	0.25	0.23	2.96

TABLE I: Best results of GSA and DSM with 4, 8, 16, 32 SOS's IIR filters using pole reflection and stability constraint. The experiments indicated in italic provide a trivial solution to the problem, as detailed in Section V.

Method	Filter (Order)	$\overline{MSE}$	$\overline{MSE_A}$	$\overline{\sigma}$
No EQ	-	2.14	1.23	3.57
SD	FIR (640)	0.98	0.72	7.13
PD	FIR (512)	0.80	0.52	5.6
GSA	FIR (768)	0.13	0.10	2.07
GSA	IIR (32 SOS's)	**0.09**	**0.04**	**1.06**

TABLE II: Best results from different methods, from [16].

a flat equalization to one of the loudspeakers and silences all other loudspeakers by selecting extremely low gains in the filters transfer function. The frequency response of all filters but one stays always below -90 dB, thus the overall magnitude frequency response, plotted in Figure 2, is mainly given by the energy contribution of a single loudspeaker. These solutions, although valid for the optimization algorithm, are not of interest for a practical application, thus these two results have been discarded. The best result not presenting such an issue, is the 32 SOS experiment using pole reflection, which achieves a 0.09 $\overline{MSE}$. Its frequency response is shown in Figure 1 where it is compared to the 4 SOS pole reflection case and the best DSM case. As can be seen, the frequency responses achieved with the 4-SOS's GSA IIR presents a deep notch below 700 Hz, several peaks and notches at low frequencies and many notches at high frequency. The 32-SOS's GSA IIR is able to reduce these, resulting in a quite flat response over 1 kHz, and in the low range only a large bell around 200 Hz is present. Regarding the DSM (Figure 1(c)), its frequency responses present more notches and peaks than GSA, especially at high frequency.

Figure 1 shows the frequency responses of GSA and DSM. The frequency responses achieved with the 4-SOS's GSA IIR presents a deep notch below 700 Hz, several peaks and notches at low frequencies and many notches at high frequency. The 32-SOS's GSA IIR is able to reduce these, resulting in a quite flat response over 1 kHz, and in the low range only a large bell around 200 Hz is present. Regarding the DSM (Figure 1(a)), its frequency responses present more notches and peaks than GSA, especially at high frequency.

A. Considerations

We first compare the GSA and DSM IIR design methods with results from [16], based on the following FIR design methods: prototype design (PD) [2], stochastic descent (SD) [23] and GSA. Table II clearly shows that all methods improve

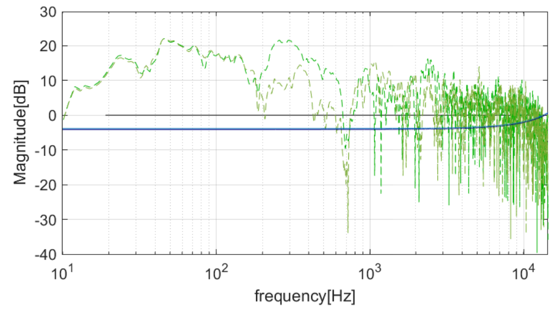

Fig. 2: Frequency response of the experiment with GSA, triangular stability, 32 SOS. The equalized curve is almost flat, however, the solution to the equalization problem is trivial, as all designed filters but one have extremely low gains.

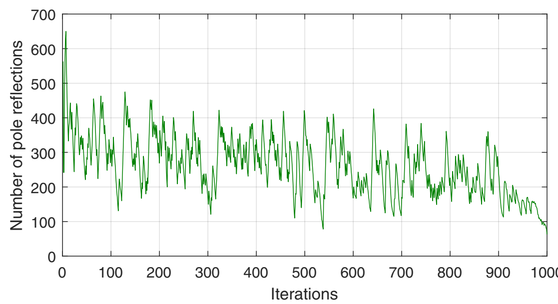

Fig. 3: Overall number of pole reflection occurrences

significantly with respect to the non-equalized case, but the GSA method outperforms the other ones. In particular the proposed IIR GSA design slightly improves over the previous method based on FIR filters, at a much lower computational cost. Indeed, considering the cost of a SOS in terms of sums and products, the cascade of 7 filters composed by 32 SOS's requires 2016 floating point operations per sample, while 7 FIR filters with 768 taps requires 10745 operations. On the other hand, the PD method, as implemented in [16], employs only one FIR filter for all seven loudspeakers. Since the best results is for the FIR length of 512 taps, only 1023 operations are required. Its use may be considered in low-computational cost settings, however, the GSA method allows to achieve superior results even with 4 SOS's, i.e. 252 floating point operations.

For what concerns the two stability constraints, after each iteration, the SOS's are checked for instability and, if it is the case, either pole reflection or triangular stability is enforced, depending on the chosen method. In Figure 3 we show the number of occurrences of pole reflection for all agents at a given iteration. At initialization, a large number of pole reflections occur, due to initial random values having high probability of instability. As the iterations increase, there is a decrease of pole reflections. This means that the algorithm is able to update the coefficients to provide stable filters.

VI. Conclusion

In this paper we proposed the use of an evolutionary algorithm, the GSA for IIR filter design in the audio equalization scenario. In order to prevent filter instabilities we proposed two methods: pole reflection and triangular stability. Similarly, we compare GSA to a modified version of the DSM, previously used for computing the gains, here modified to directly compute the filter coefficients. We pick the in-car equalization scenario as a use case, where multiple loudspeakers are associated to one IIR filter each, and a binaural head is used to record the individual impulse responses.

The experiments show that the GSA with pole reflection technique is superior to state of the art methods, including the GSA algorithm for FIR filters. With respect to the latter, a computational cost reduction is also achieved, which may prove noteworthy for embedded applications.

In this work, IIR filter stability was introduced as a constraint. Future works may, instead, reformulate the problem to include stability as a penalty term within the fitness function. Furthermore, additional constraints or penalties may be required to avoid trivial solutions, such as those found in this work. Finally, since the proposed methods are heuristic, the hyperparameters search may be further enlarged by launching a large number of batch experiments in order to seek for better performance.

Other experiments will be carried out in other acoustic scenes (i.e. multi-point equalization) to improve the study of machine learning techniques for audio equalization and to approximate linear phase on output responses. Moreover, deep learning techniques will be investigated for the design of audio equalization filters (both FIR and IIR) and perceptual tests will be performed.

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