Explicit quaternion krawtchouk moment invariants for finger-spelling sign language recognition

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***Abstract*—Sign recognition is a difficult task due to the complexity of its composition which uses signs of different levels, words, facial expression, body posture and finger-spelling to convey meaning. With the development of recent technologies, such as Kinect sensor, new opportunities have emerged in the field of human computer interaction and sign language, allowing to capture both RGB and Depth (RGB-D) information. In the regard to feature extraction, the traditional methods process the RGB and Depth images independently. In this paper, we propose a robust static finger-spelling sign language recognition system adopting the Quaternion algebra that provide a more robust and holistical representation, based on fusing RGB images and Depth information simultaneously. Indeed, we propose, for the first time, a new sets of Quaternion Krawtchouk moments(QKMs) and Explicit Quaternion Krawtchouk Moment Invariants (EQKMIs). The proposed system is evaluated on three well-known finger-spelling datasets, demonstrate the performance of the novel method compared to other methods used in the literature, against geometrical distortion, noisy conditions and complex background, indicating that it could be highly effective for many other computer vision applications.**

***Index Terms*—Finger-spelling Recognition, Moment Invariants, Krawtchouk, Quaternion Algebra, RST Invariants.**

I. Introduction

Sign language recognition (SLR) is an important aspect in human computer interaction applications, used by deaf and hearing impaired [1]. In fact, SLR are gestural languages which uses signs for communication without speaking. This task is a challenging problem, mainly because of the nature of the hidden computer vision problem, such as the visual analogy of specific signs and the complex articulations presented by the hand. Typically, there are three components to constitute a sign gesture [2]: manual features which are gestures made with hands, non-manual features such as facial expressions, body posture, and finger-spelling when words are spelt into alphabet. In this context, the importance of finger-spelling can be noticed when a concept lacks a specific sign, such as names, technical terms, or foreign words. Therefore, several researches have been done to solve this issue, specially, with the development of RGB and RGB-D sensors, such as Microsoft Kinect [3], there has been an increasing interest in the research area related to finger-spelling recognition (F-SR) [4]–[7]. Despite the relevance of the proposed methods and their contribution to the field of F-SR, it is important to note that these studies failed to take into account two problems that are known to be highly challenging for SLR and more precisely for F-SR: (1) The robust description of hand pose under noisy conditions and (2) The invariability to Rotation, Scaling and Translation (RST) deformations.

The orthogonal moments and moment invariants are one of the most interesting research areas in the fields of image recognition and computer vision [9]. This is because of their attractive properties, such as the holistical description of the image components, the robustness to different kinds of noise and the invariability property against RST deformations [9]. In this connection, only few papers have been proposed for F-SR application using RGB-D images based on image moment [1], [10], [11]. Consequently, there are many issues that have not been sufficiently addressed. In the overmentioned works, only rotation invariance is achieved. In addition, most of the proposed approach fail to extract accurate and compact features for F-SR, since they trait depth and color information independently. Moreover, none of the existing approaches are able to recognize finger-spelling independently of hand size, orientation and position variations, in the presence of noise and complex background. Then, to our knowledge, this is the first study aimed at exploiting the power of quaternion for RGB-D F-SR using the Explicit Quaternion Krawtchouk Moment Invariants.

Motivated by the facts summarized above, we aim in this paper, to extend our previous work [8] on quaternion algebra and introduce a new method of feature extraction for static F-SR named EQKMI. This new descriptor can be used to extract features from RGB-D images, independently of geometric transformations. Moreover, our proposed descriptor can provide a holistical representation of RGB-D image components, using Quaternion algebra, in contrary to the previous research that trait color and depth channels independently. In addition, the choice of Krawtchouk moments, is justified by its discrete orthogonality, the simplicity of implementation and the high tolerance to different kinds of noise [12].

The main contribution of this paper relies on three points:(1) New sets of QKMs, defined by Krawtchouk polynomials, are introduced for the first time in this paper. (2) A direct approach to derive Rotation, Scale and Translation (RST) invariants of EQKMIs, based on Krawtchouk polynomials, is proposed. (3) An efficient and robust static RGB-D F-SR model, against various type of complex background and noises, is presented.

The remainder of this paper is organized as follows: In Section 2, we present the proposed method for deriving QKM, with a brief introduction of Quaternion algebra. Then, in Section 3 and 4, we introduce the proposed EQKMI with the F-SR model. We provide the experimental results in Section 5. Finally, conclusions are made in Section 6.

II. Proposed Quaternion Krawtchouk Moments

This section is devoted to introduce new set of Quaternion Krawtchouk Moments(QKMs). Their basis function is obtained from Krawtchouk polynomials. Before presenting the method for deriving the proposed QKMs, let us introduce some necessary relations.

A. Quaternion algebra

In 1843, Hamilton formally introduced quaternions [13]. It is a type of hyper complex number. The quaternion, has four parts, one real part and three imaginary parts. The formula of a quaternion q is defined as follows:

$$q = q_r + q_i i + q_j j + q_k k, \tag{1}$$

where q_r, q_i, q_j, q_k are real numbers and i, j, k are complex operators, which obey the following rules:

$$i^2 = j^2 = k^2 = -1, ij = -ji = k, jk = -kj = i, ki = -ik = j. \tag{2}$$

A quaternion with zero real part is called a pure quaternion. For more information, see [13]. Let f(x,y) be an RGB-D image function. The quaternion representation for a RGB-D image is giving as follow:

$$f(x,y) = f_D(x,y) + f_R(x,y)i + f_G(x,y)j + f_B(x,y)k, \tag{3}$$

with $f_D(x,y), f_R(x,y), f_G(x,y)$ and $f_B(x,y)$ are respectively the depth, red, green and blue components of the pixel (x,y).

B. Quaternion Krawtchouk moments

The Krawtchouk polynomials of the n-th order, is defined in terms of the hypergeometric function ${}_2F_1()$ as follows [12]:

$$k_n(x;p,N) = {}_2F_1(-n,-x;-N;1/p), \tag{4}$$

Where $x, n = 0, 1, , N-1, N > 0, 0 < p < 1$.
The normalized Krawtchouk polynomials are defined as:

$$\tilde{k}_n(x;p,N) = \frac{k_n(x;p,N)}{\sqrt{\rho(n;p,N)}}, \tag{5}$$

and satisfy the orthogonality condition:

$$\sum_{x=0}^{N-1} \tilde{k}_n(x;p,N)\tilde{k}_m(x;p,N) = \omega(x;p,N)\delta_{nm}, \tag{6}$$

with respect to the weight function given by:

$$\omega(x;p,N) = \binom{N}{x} p^x (1-p)^{N-x}, \tag{7}$$

and the squared norm:

$$\rho(n;p,N) = (-1)^n (\frac{1-p}{p})^n \frac{n!}{(-N)_n}. \tag{8}$$

By expanding ${}_2F_1()$ into summation, we can define the $k_n(x;p,N)$ as finite power series by:

$$k_n(x;p,N) = \sum_{i=0}^{n} C_{n,i} x^i, \tag{9}$$

with $C_{n,i}$ is given by:

$$C_{n,i} = \sum_{k=i}^{n} \frac{(-1)^k n!(N-k)!}{\sqrt{\rho(n;p,N)}(p)^k N!(n-k)!k!} S_1(k,i), \tag{10}$$

Also, the invers polynomials formula, up to the order i is:

$$x^i = \sum_{s=0}^{i} D_{i,s}\tilde{k}_s(x;p,N), \tag{11}$$

where $D_{i,s}$ is defined by:

$$D_{i,s} = \sum_{m=s}^{i} \frac{\sqrt{\rho(n;p,N)}(-1)^s m!N!p^m}{(m-s)!(N-m)!s!} S_2(i,m), \tag{12}$$

and $S_2(i,m)$ are the Stirling numbers of the second kind. The right side of Quaternion Krawtchouk moment can be written as follow:

$$\begin{aligned}\widetilde{QKM}_{pq}^R = \sum_{p=0}^{N-1}\sum_{q=0}^{M-1} (f_D(x,y) + f_R(x,y)i + f_G(x,y)j \\ + f_B(x,y)k)\tilde{k}_p(x;N)\tilde{k}_q(y;M)\mu,\end{aligned} \tag{13}$$

$$\widetilde{QKM}_{pq}^R = A_0 + iA_1 + jA_2 + kA_3, \tag{14}$$

where

$$A_0 = \frac{1}{\sqrt{3}}[MK_R + KM_G + KM_B],$$
$$A_1 = -\frac{1}{\sqrt{3}}[KM_D + KM_G - KM_B],$$
$$A_2 = -\frac{1}{\sqrt{3}}[KM_D + KM_B - kM_R],$$
$$A_3 = -\frac{1}{\sqrt{3}}[KM_D + KM_R - KM_G].$$

KM_D, KM_R, KM_G and KM_B are the traditional Krawtchouk moments for the RGB-D components.

III. Proposed Explicit Quaternion Krawtchouk Moment Invariants

In this section, we introduce new sets of Explicit Quaternion Krawtchouk Moment Invariants, which are directly derived from their corresponding orthogonal moments defined in the previous subsection II. The Explicit Quaternion Krawtchouk translation invariants are shown in Eq. (15) by evaluating their central moments:

$$\begin{aligned}\widetilde{EQKM}_{nm}^t = \sum_{x=0}^{N-1}\sum_{y=0}^{M-1} \tilde{k}_n(x-x_0)\tilde{k}_m(y-y_0)(f_D(x,y) \\ + f_R(x,y)i + f_G(x,y)j + f_B(x,y)k)\mu.\end{aligned} \tag{15}$$

Proposition 1: The Explicit Quaternion Krawtchouk Moments $\widetilde{EQKM}^{t}_{nm}$ of a translated image $f^t(x,y)$ can be written in terms of $\widetilde{QKM}_{nm}$ of the original image $f(x,y)$ as:

$$\begin{aligned}\widetilde{EQKM}^{t}_{n,m} = &\sum_{i=0}^{n}\sum_{j=0}^{m}\sum_{s=0}^{i}\sum_{t=0}^{j}\sum_{u=0}^{s}\sum_{v=0}^{t}\binom{i}{s}\binom{j}{t} \\ &\times C_{n,i}C_{m,j}D_{s,u}D_{t,v}(-1)^{i-s+j-t} \\ &\times \bar{x}^{i-s}\bar{y}^{j-t}\widetilde{QKM}_{u,v}.\end{aligned} \tag{16}$$

With $\bar{x} = \frac{C_{0,0}KM^{RGBD}_{1,0} - C_{0,0}KM^{RGBD}_{0,0}}{C_{1,1}KM^{RGBD}_{0,0}}$, $\bar{y} = \frac{C_{0,0}KM^{RGBD}_{0,1} - C_{0,0}KM^{RGBD}_{0,0}}{C_{1,1}KM^{RGBD}_{0,0}}$.
Where $KM^{RGBD}_{i,j} = KM^{R}_{i,j} + KM^{G}_{i,j} + KM^{B}_{i,j} + KM^{D}_{i,j}$.
The Explicit Quaternion Krawtchouk Moments of the deformed image $f^d(x,y)$ is defined as:

$$\begin{aligned}\widetilde{EQKM}^{d}_{n,m} = &\sum_{x=0}^{N-1}\sum_{y=0}^{M-1} k_n(a_{1,1}x + a_{1,2}y; N) \\ &k_m(a_{2,1}x + a_{2,2}y; M)(f_D(x,y) + f_R(x,y)i \\ &+ f_G(x,y)j + f_B(x,y)k)\mu.\end{aligned} \tag{17}$$

Proposition 2: The Explicit Quaternion Krawtchouk Moments $(\widetilde{EQKM})^{d}_{nm}$ of a deformed image $f^d(x,y)$ can be written in terms of $(QKM)_{nm}$ of the image $f(x,y)$ as:

$$\begin{aligned}\widetilde{EQKM}^{d}_{n,m} = &\sum_{i=0}^{n}\sum_{j=0}^{m}\sum_{s=0}^{i}\sum_{t=0}^{j}\sum_{u=0}^{i+j-s-t}\sum_{v=0}^{s+t}\binom{i}{s}\binom{j}{t} \\ &\times C_{n,i}C_{m,j}D_{i+j-s-t,u}D_{s+t,v}(a_{1,1})^{i-s} \\ &(a_{1,2})^{s}(a_{2,1})^{j-t}(a_{2,2})^{t}\widetilde{QKM}_{u,v},\end{aligned} \tag{18}$$

Then, we can construct a set of RST invariants $\widetilde{EQKMI}^{RST}_{n,m}$:

$$\begin{aligned}\widetilde{EQKMI}^{RST}_{n,m} = &\sum_{i=0}^{n}\sum_{j=0}^{m}\sum_{s=0}^{i}\sum_{t=0}^{j}\sum_{u=0}^{i+j-s-t}\sum_{v=0}^{s+t}(-1)^{j-t} \\ &\binom{i}{s}\binom{j}{t} \times C_{n,i}C_{m,j}D_{i+j-s-t,u}D_{s+t,v} \\ &\times(\lambda_f)^{-\frac{i+j+2}{2}}(\cos\theta_f)^{i+t-s}(\sin\theta_f)^{j-t+s}\widetilde{EQKM}^{t}_{u,v},\end{aligned} \tag{19}$$

with

$$\lambda_f = KM^{RGBD}_{0,0}, \theta_f = \frac{1}{2}\arctan\left[\frac{(rKM^{RGBD}_{1,1} - lKM^{RGBD}_{0,0})}{(KM^{RGBD}_{2,0} - KM^{RGBD}_{0,2})}\right]. \tag{20}$$

IV. Proposed finger-spelling recognition model

Figure 1 gives the block diagram of the proposed model for static F-SR using the RGB-D components. The system is composed of three steps. The first one is Detection and segmentation, then feature extraction using the proposed EQKMI. Finally, hand gesture classification is performed by KNN.

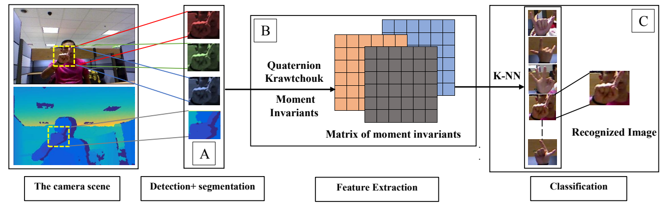

Fig. 1. Proposed finger-spelling recognition model.

A. Step 1

Preprocessing, including hand detection and segmentation, is the first step of our proposed model. Using the depth image, the hand can be detected, and the feature extraction method can be applied to only the segmented area. The three used datasets are captured using the Kinect Sensor. In this respect, and supposing that the hand in a depth image is the closest object to camera [6] [14] [15], and there are no obstacles between the sensor and the hands, we remove other objects and background. Then, the target region can be obtained marking pixels with depth value less than a certain threshold.

B. Step 2

The Feature extraction is very essential and one of the crucial steps in the F-SR models, which highly affects recognition accuracy. For this reason, we introduce direct approach to derive RST invariants of the proposed EQKMI. These new set of invariants were calculated from both color and depth images. The Quaternion representation of RGB-D information treat holistically the four channels: Red, Green, Blue and Depth, hence, in our model this representation have been effectively employed.

C. Step 3

Afterwards, we retrieve the feature vector that is extracted by using the proposed EQKMI. In this context, we use the KNN classifier to recognize finger-spelling signs based on the trained set, with the parameters: k=5 for KNN. It is important to note that the choice of KNN classifier is motivated by three reasons [18]: the simplicity of implementation, high classification performance even with a small number of training samples. Also, the necessity to evaluate the discrimination capability of the extracted features by using our proposed EQKMI. 5-folder cross-validation approach is conducted to evaluate the recognition performance in all experiments.

V. Performance evaluation

In this section, experimental results are conducted to validate the proposed approach numerically. In this paper we will use three famous static finger-spelling databases, namely HKU EEE DSP Kinect Gesture Dataset V3 (HKU) [14], NTU Hand Digit Dataset (NTU) [15] and American Sign Language (ASL) Finger-spelling dataset [6], acquired with the Kinect, for testing the robustness of our proposed method. In the first experiment, the influence of the neighborhood parameter k on the classification accuracy has been evaluated. In the second experiment, noise robustness has been tested on the color and depth images. Finally, we choose to use different complex background in the last experiment. In this study, we consider the HOG, LBP, SIFT, SURF, and Gabor methods used in the literature, for comparaison purpose. Note that, all our numerical experiments are performed in matlab 8.5 on a PC with Intel(R) core i7 2.1 GHz and 4GB of memory.

A. Experiment 1

The aim of this experiment is to test the performance of the proposed method without noisy effects in the task of F-SR. We evaluate the recognition accuracy of the proposed EQKMI, with a chosen number of coefficients in the feature vector (16 coefficients), and using KNN as classifier with different values of the neighborhood parameter k, with 5-folds cross validation.

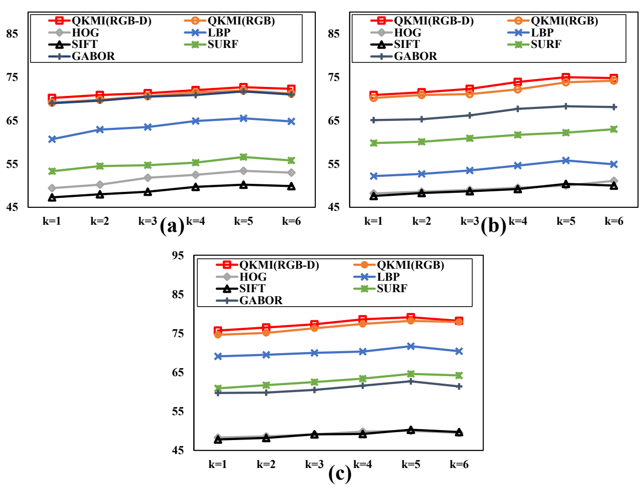

Fig. 2. Comparative analysis of recognition accuracy (%) of different methods, for datasets: (a) NTU, (b) HKU and (c) ASL, with a varying number of parameter k.

In order to choose the relatively optimal parameter k for KNN classifier, we compared the recognition rates of different parameters with the proposed EQKMI and the methods used in the literature. The obtained results are shown in Fig. 2. As one can see from the obtained results, the proposed EQKMI(RGB-D) demonstrate a superiority in terms of image classification accuracy for the three databases compared with the other methods used in the literature, approving the discrimination capability even though with the simplest classifier. In addition, the new invariants proved their good robustness without depth information, however, EQKMI(RGB-D) outperform EQKMI(RGB) slightly. From the figures, it can be observed that the maximum recognition rates for the NTU, HKU and ASL datasets has been observed in case of k=5. Therefore, in our work we choose to use for the rest of this paper k=5 as parameter for KNN classifier.

B. Experiment 2

It is well known that F-SR is a highly challenging task in the fields of pattern recognition. This is due to the various sources of noise. consequently, we aim in this experiment to inspect the capability of our descriptor to remain robust against this type of problems. Precisely, we explore our proposed approach in the presence of other kinds of noise, salt and pepper and Gaussian noises. Each image of the three datasets is corrupted by salt-and-pepper noise with noise densities varying from 1% to 5% with 1% increments, as well as Gaussian noise with noise densities changing from 0.1 to 0.5 with 0.1 increments. The performance of this F-SR model on the datasets is presented on Fig. 3.
As one can see, the plotted curves show the same trend, with a values of recognition rate decreased by increasing the density of noise and noise variance for Salt-and-pepper and Gaussian noise respectively. Moreover, it is clear that the results of the proposed EQKMI(RGB-D)-based approach are superior to all the compared methods This is probably due to the ability of our proposed EQKMI(RGB-D) to effectively represent global information of finger-spelling image, and also due to their robustness against noise effects [12]. The obtained results indicate that the proposed method is more informative shape descriptor than the traditional methods used in the literature: HOG, SIFT, SURF, LBP and Gabor, even in the presence of noise and absence of depth information. Finally, we can conclude that our new descriptor can be used in real-world applications such as computer vision and remote sensing.

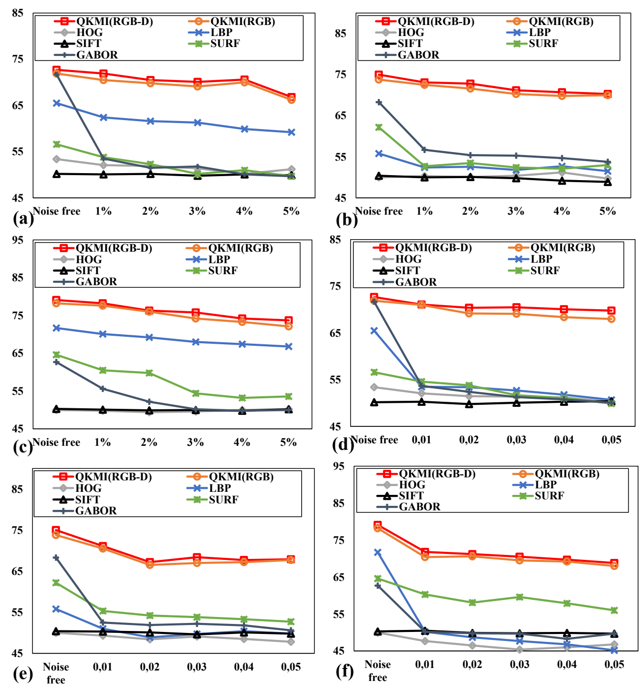

Fig. 3. Comparative results of recognitiona accuracy (%) for datasets: a,d NTU, b,e HKU and c,f ASL, affected by a-c salt-and pepper noise, and d-f Gaussian noise densities.

C. Experiment 3

The independence to geometric deformations, the robustness to different kinds of noise, and the ability to deal with complex backgrounds are three crucial criteria that a descriptor must satisfy [17]. In this direction, the purpose of this section is to inspect the capability of our descriptor to deal with complex background. For this purpose, we have used VisTex texture database [16]. Indeed, we have chosen 50 images from VisTex. These selected images have been randomly combined with each sign image as complex background. Therefore, we have created additional testing cases for every finger-spelling database. Few examples of the created images are shown in Fig. 4. For a similar parameters settings as the previous experiments. Table I summarize respectively the obtained recognition results on the three testing datasets, HKU, NTU and ASL, for different texture background, by using the proposed EQKMI, in comparison with the traditional HOG, LBP, SIFT SURF and GABOR descriptors. It is observed from Table I that the recognition of finger-spelling sign on the three datasets submitted to complex backgrounds was largely in favor of EQKMI(RGB-D) then EQKMI(RGB) indicating the improvement when using depth information, with an average recognition rate more than 70%, followed by Gabor and LBP descriptors. This is probably justified by the holistical representation using the four components of the RGB-D image. Finally, we can conclude the importance of fusing depth and RGB images in feature extraction process and we can deduce that our method could be very effective for many applications.

Fig. 4. Examples from the new dataset using Vistex database.

TABLE I
INFLUENCE OF COMPLEX BACKGROUNDS ON THE F-SR ACCURACY (%) FOR HKU, NTU AND ASL, BY USING DIFFERENT DESCRIPTORS.

Database	Methods	KNN	
		Uniforme background	**vistex**
NTU	EQKMI(RGB-D)	**72.7**	**70.5**
	EQKMI(RGB)	**71,9**	**69.8**
	HOG	**53.4**	**50.7**
	LBP	**65.5**	**63.1**
	SIFT	**50.2**	**50.4**
	SURF	**56.6**	**52.6**
	GABOR	**72.2**	**69.8**
HKU	EQKMI(RGB-D)	**75**	**72.2**
	EQKMI(RGB)	**73.8**	**70**
	HOG	**50**	**49.5**
	LBP	**55.8**	**53.7**
	SIFT	**50.4**	**49.2**
	SURF	**62.2**	**59.6**
	GABOR	**68.3**	**63.7**
ASL	EQKMI(RGB-D)	**79.1**	**77.1**
	EQKMI(RGB)	**78.2**	**76.6**
	HOG	**50**	**48.8**
	LBP	**71.7**	**69.7**
	SIFT	**50.3**	**50**
	SURF	**64.6**	**61.9**
	GABOR	**62.7**	**60.5**

VI. CONCLUSION

In this paper, we define a new descriptor named EQKMI. A RGB-D F-SR system was modeled by using this new method. Experiments indicate a better classification rate of the proposed EQKMI than some other existing algorithms for the following reasons: it process four components of RGB-D image holistically, based on the quaternion approach; Moreover the proposed set of EQKMI have proved their robustness in the presence of noisy conditions, and complex background. In the future work, the research of other existent orthogonal polynomials will be focused. In addition, we will try to apply the proposed method for real time application like dynamic sign language. And we plan to evaluate our descriptor by replacing KNN, with new techniques such as deep learning.

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