Denoising ECG Signals Using Unbiased FIR Smoother and Harmonic State-Space Model

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***Abstract*—The electrocardiogram (ECG) signals provide information for making decisions about different kinds of heart diseases. During decades various approaches have been developed to denoise ECG data and extract useful features, although further increase in the accuracy is required. In this paper, we view the ECG signal as a quasi periodic process and employ the unbiased finite impulse response (UFIR) smoother on optimal horizons. It is shown that the UFIR smoother applied to a harmonic ECG model performs better than that recently developed for polynomial ECG models. Extensive investigation are provided for diverse ECG data. The results are compared in terms of the mean square error and signal-to-noise ratio.**

***Index Terms*—ECG signals, denoising , harmonic model, polynomial model, unbiased FIR smoothing.**

I. Introduction

Profound learning of the electrocardiogram (ECG) signal features plays a crucial role in medical sector, allowing finding cardiac disease patterns. The problem one meets here is in noise and artifacts, which are caused by the acquisition, breathe, and other body conditions That makes an automatic detection of ECG signal patterns unreliable and modern signal processing algorithms are required. During the last decades, many denoising algorithms were designed for ECG signals [1]–[3] to felicitate features extraction. The standard low-pass and band-pass filters were applied to ECG signals in [4] supposing that an ECG signal is stationary. The approach provides satisfactory denoising in the frequency domain, but overlooks the time resolution. This drawback was overcame in [5], [6] using the wavelet transform and a properly chosen wavelet. Yet another method based on the empirical mode decomposition was developed in [7].

In many cases, denoising of ECG signals require accurate (optimal and robust) methods due to the not well-known origin of the heart noise and data artifacts. In this regard, smoothing techniques are recognized as most powerful to remove noise while retaining fundamental properties of ECG signals. An example is the smoothing technique developed by Savitsky and Golay (SG) [8], which is widely applied to ECG signals [9]–[11]. However, as a fairly old polynomial technique, the SG smoother suffers from two drawbacks: its batch form is computationally inefficient and the output is related to the middle of the averaging horizon that does not hold for all polynomial degrees.

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A more general unbiased finite impulse response (UFIR) smoothing technique has been developed in [12] to generalize the SG smoother as a special case. The UFIR smoother operates in discrete-time state-space, has a fast iterative algorithm using recursions, and suggests that a suboptimal smoothing can be provided if to calculate an averaging horizons optimally as N_{opt} and choose an optimal lag q_{opt}. It was shown that an optimal lag q_{opt} corresponds to the middle of the averaging horizon only for odd-degree polynomials. Otherwise, q_{opt} must chosen individually for each even degree [12]–[14]. We notice it as an essential difference with the SG smoother.

In applications to ECG signals, the polynomial approach has been developed in [15], [16] for models employing the Taylor series expansion. It has been shown that the polynomial UFIR smoother produces more accuracy than other techniques. However, the polynomial model does not fit well with a quasi periodic ECG signal and further improvements can be achieved if we represent an ECG signal with the Fourier series. This deduction follows from the fact that the harmonic model is widely used in power systems, where signals are also quasi periodic [17]–[25]. Even though, we find only a few works applying a harmonic model to ECG signals [26], [27].

In this paper, we represent an ECG signal with a Fourier series, apply the UFIR smoother to real measurements of ECG signals, and provide a comparative analysis with the polynomial smoothers in terms of the denoising effect. We use the MIT-BIH Arrhythmia Database [28], [29], from which in this paper we take only the normal heartbeats.

A. Database

This investigation employs the MIT-BIH Arrhythmia Database as a benchmark. The data contain 48 ECG recordings applying two leads (e.g. MLII, V1) from 47 subjects. The recordings have been sampled to 360 Hz per channel with 11-bit resolution over a 10 mV range [29]. A part of the ECG signal record is given in Fig. 1. it is the lead MLII, because the ECG signal morphology is clearly recognized. All tests of synthetic data are provided using a special software designed on the MATLAB platform. The simulated signals employ the Fourier series assuming that data are corrupted at different levels by white Gaussian noise.

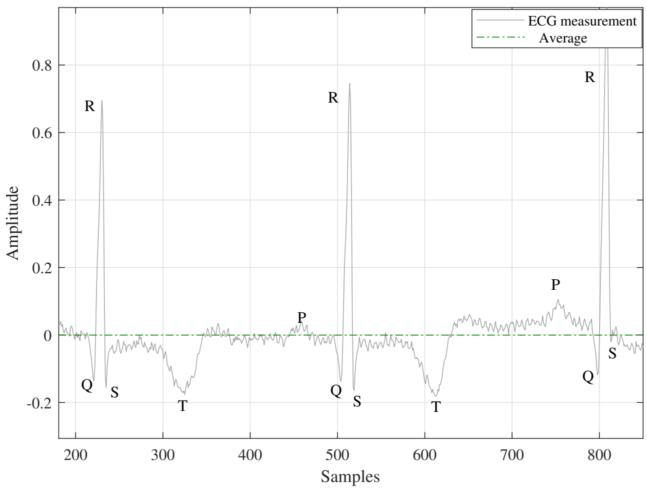

Fig. 1. An example of the centralized ECG record taken from MIT-BIH Arrhythmia Database. The record 100/MLII features are depicted as P, QRS complex and T waves. Applied a correction of baseline, the ECG measurement average is near to zero

II. State-Space Representation of ECG Signals using Harmonic Model

In view of a quasi-periodic nature of the heartbeats, we represent an ECG signal with a Fourier series corrupted by noise as

$$y(t) = A_{0,t} + \sum_{m=1}^{M} A_{m,t} \cos(m\omega t) + v(t)\,, \qquad (1)$$

where ω is the fundamental angular frequency, M is the number of harmonics, $A_{m,t}$ are time-varying amplitudes associated with the mth harmonic, and $v(t)$ is an additive zero mean white Gaussian noise. Before applying an estimator, we make efforts to centralize the ECG data about zero. Therefore, the DC offset component becomes zero, $E\{A_{0,t}\} = 0$, and we will further omit $A_{0,t}$. If we introduce the discrete time t_k, $k = 0, 1 \ldots$, a time step $\Delta t = t_k - t_{k-1}$, and set $A_{0,t} = 0$, then the model (1) can be represented as

$$y_k = \sum_{m=1}^{M} A_{m,k} \cos(m\omega\Delta tk) + v_k\,. \qquad (2)$$

Assuming that the mth harmonic components $\Delta A_{m,k}$ is random, the magnitude of the mth harmonic component at time index $k+1$ can be represented as

$$A_{m,k+1} = A_{m,k} + \Delta A_{m,k}\,. \qquad (3)$$

Referring to (2), we now introduce a $(2M)$-state vector $\mathbf{x}_k$,

$$\mathbf{x}_k = \begin{bmatrix} x_{1k} \\ x_{2k} \\ \\ \vdots \\ \\ x_{2M,k} \\ x_{2M+1,k} \end{bmatrix} = \begin{bmatrix} A_{1,k}\cos(w\Delta tk) \\ A_{1,k}\sin(w\Delta tk) \\ \vdots \\ A_{M,k}\cos(Mw\Delta tk) \\ A_{M,k}\sin(Mw\Delta tk) \end{bmatrix}, \qquad (4)$$

where the components $x_{i,k}$, $i \in [1, M]$, are specified by the model (2). Accordingly, the time-invariant state space model of an ECG signal can be written as

$$\mathbf{x}_k = \mathbf{A}\mathbf{x}_{k-1} + \mathbf{w}_k\,, \qquad (5)$$

$$y_k = \mathbf{C}\mathbf{x}_k + v_k\,, \qquad (6)$$

where matrix $\mathbf{A}$ is specified as

$$\mathbf{A} = [\text{blockdiag}(\Theta(m), m = 1, \cdots, M)]\,, \qquad (7)$$

where "blkdiag" in (7) means a block of matrices $\Theta(m)$ represented by

$$\Theta(m) = \begin{bmatrix} \cos(m\omega) & -\sin(m\omega) \\ \sin(m\omega) & \cos(m\omega) \end{bmatrix}. \qquad (8)$$

For this model, the observation vector becomes

$$\mathbf{C} = [1\ 0 \ldots 1\ 0]$$

and we think that the measurement noise v_k has zero mean, $E\{v_k\} = 0$, and unknown distribution and statistics, as it usually is in practice. Given the state-space model (5) and (6), the UFIR smoothing algorithm can be used as in the following.

A. Unbiased FIR Smoothing

The p-shift UFIR filtering approach [30] suggests that 1) UFIR filtering must be provided as $\hat{\mathbf{x}}_k$ and 2) the q-lag UFIR smoothing organized by projecting the filtering estimate $\hat{\mathbf{x}}_k$ to $k-q$ as $\hat{\mathbf{x}}_{k-q} = \mathbf{A}^{-q}\hat{\mathbf{x}}_k$.

On a horizon $[m, n]$ of N points, from $m = n - N + 1$ to n, the UFIR filter processes N past ECG data y_n. To provide a near optimal output, the horizon length must be set optimally as N_{opt}. For the sake of best denoising with the minimum mean square error (MSE), it is also required to make the horizon adaptive around the QRS complex (Fig. 1).

The batch UFIR filtering estimate can be represented as follow [30],

$$\hat{\mathbf{x}}_k = \mathbf{H}_{m,k}\mathbf{Y}_{m,k} \qquad (9a)$$

$$= (\mathbf{W}_{m,k}^T\mathbf{W}_{m,k}^T)^{-1}\mathbf{W}_{m,k}^T\mathbf{Y}_{m,k}\,, \qquad (9b)$$

where $\mathbf{Y}_{m,k}$ represents the observation vector, $\mathbf{H}_{m,k}$ is the UFIR filter gain, and $\mathbf{W}_{m,k}$ is an auxiliary matrix,

$$\mathbf{Y}_{m,k} = [\,y_m^T\ y_{m+1}^T \ldots y_k^T\,]^T\,, \qquad (10)$$

$$\mathbf{W}_{m,k} = \begin{bmatrix} \mathbf{C}(\mathcal{A}^{m+1})^{-1} \\ \mathbf{C}(\mathcal{A}^{m+2})^{-1} \\ \vdots \\ \mathbf{C}A^{-1} \\ \mathbf{C} \end{bmatrix}, \qquad (11)$$

where the product $\mathcal{A}$ is specified as

$$\mathcal{A}_r^g = \begin{cases} \mathbf{A}^{r-g+1}\,, & g < r+1\,, \\ \mathbf{I}\,, & g = r+1\,, \\ \mathbf{0}\,, & g > r+1\,. \end{cases} \qquad (12)$$

The discrete convolution-based batch UFIR filter estimate thus appears as $\hat{\mathbf{x}}_k = \mathbf{H}_{m,k}\mathbf{Y}_{m,k}$, where the UFIR filter gain is computed by $\mathbf{H}_{m,k} = (\mathbf{W}_{m,k}^T \mathbf{W}_{m,k}^T)^{-1} \mathbf{W}_{m,k}^T = \mathbf{G}_{m,k} \mathbf{W}_{m,k}^T$, where $\mathbf{G}_{m,k}$ is the generalized noise power gain (GNPG)

$$\mathbf{G}_k = \mathbf{H}_{m,k} \mathbf{H}_{m,k}^T = (\mathbf{W}_{m,k} \mathbf{W}_{m,k})^{-1} , \qquad (13)$$

which is responsible for an optimal balance between the regular (bias) and random errors. Provided $\hat{\mathbf{x}}_k$, the q-lag UFIR smoothed estimate appears by projecting the filtering estimate to $k-q$ as $\hat{\mathbf{x}}_{k-q} = \mathbf{A}^{-q} \hat{\mathbf{x}}_k$.

The batch estimate (9a) can also be computed iteratively using recursions [15], [30], [31], like in the Kalman filter. Since our concern in this work is to reduce smoothing errors, we will not consider the computational complexity of (9a) and postpone it to the next stage.

III. Optimal Horizon for UFIR Smoother Employing Harmonic Model

To achieve the best denoising effect in ECG signals, the UFIR smoother must operate on optimal averaging horizons of N_{opt} points, which can be found for the UFIR filter following a methodology developed in [15], [16].

To specify N_{opt}, we select 10000 samples of healthy heartbeats and consider several harmonics of the Fourier series (2). Following [32], we compute the measurement residual as a difference between the ECG data and the filter output, compute the mean square value (MSV) of the residual as a function of $N_{\min} \leqslant N \leqslant 10^3$, and approximate this function with a cubic polynomial as V_N. We next apply the derivative $\partial V_N / \partial N$, find its minimum, and find N_{opt} at this point for several harmonics ($m = 1, 2, 3, 5$) as shown in Fig. III–Fig. 5. As can be seen, V_n behaves similarly for all of the harmonics selected, although the value of N_{opt} decreases from 14 for $m = 1$ to 7 for $m = 5$.

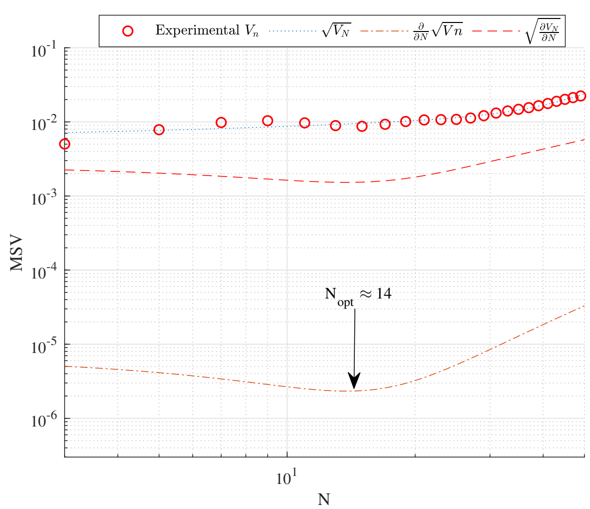

Fig. 2. Effect of N on the MSV (circled) for $m = 1$. A cubic approximation of the MSV is $\sqrt{V_N}$ and the optimal horizon $N_{\text{opt}} = 14$ corresponds to the minimum of $\sqrt{\frac{\partial}{\partial N} V_N}$.

An analysis of the smoothing errors [33] produced by the 1st and 3rd harmonics reveals no significant differences, except

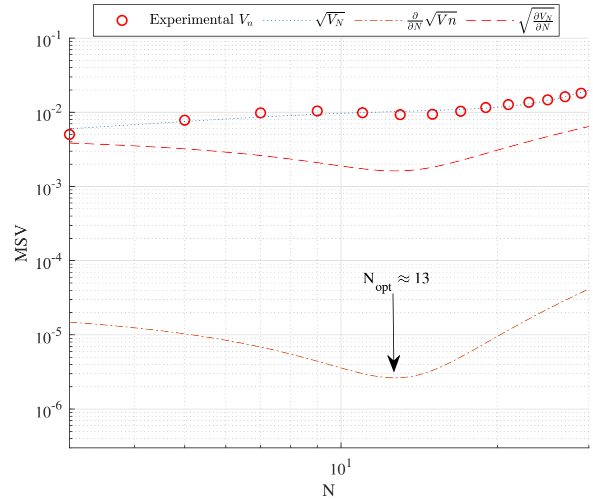

Fig. 3. Effect of N on the MSV (circled) for $m = 2$. A cubic approximation of the MSV is $\sqrt{V_N}$ and the optimal horizon $N_{\text{opt}} = 14$ corresponds to the minimum of $\sqrt{\frac{\partial}{\partial N} V_N}$.

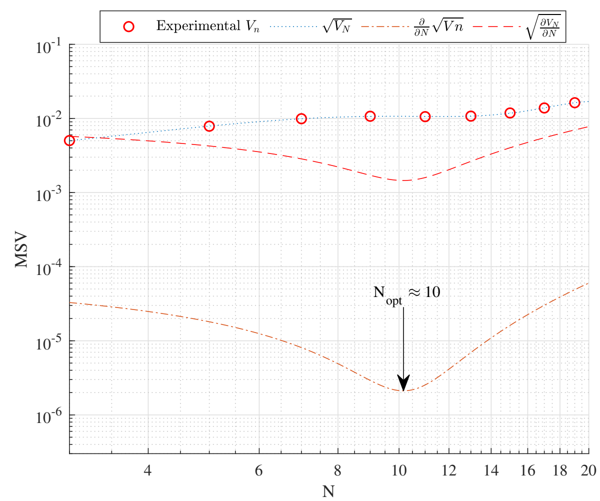

Fig. 4. Effect of N on the MSV (circled) for $m = 3$. A cubic approximation of the MSV is $\sqrt{V_N}$ and the optimal horizon $N_{\text{opt}} = 14$ corresponds to the minimum of $\sqrt{\frac{\partial}{\partial N} V_N}$.

for the horizon length, which inherently grows with m. This fact can be explained by the observation that a lag $q = \frac{N-1}{2}$ makes the noise power gain (NPG) in both smoothers equal [12]. The role of q-lag on the NPG of the p-shift UFIR filter, $q = -p$, has been studied in [12]. The optimal shifts were implemented in [15], [16] as the p-shift 1 and p-shift 2.

Given $\mathbf{A}$ by (7) and $\mathbf{C}$ (see Section II), Fig. 6 sketches smoothed estimates provided using the 1st, 3rd, and 5th harmonics of an ECG signal. It follows that the estimates are accurate in the slow part. To smooth a fast excursion around the QRS complex, an adaptive algorithm developed in [15], [16] is applied to avoid bias errors.

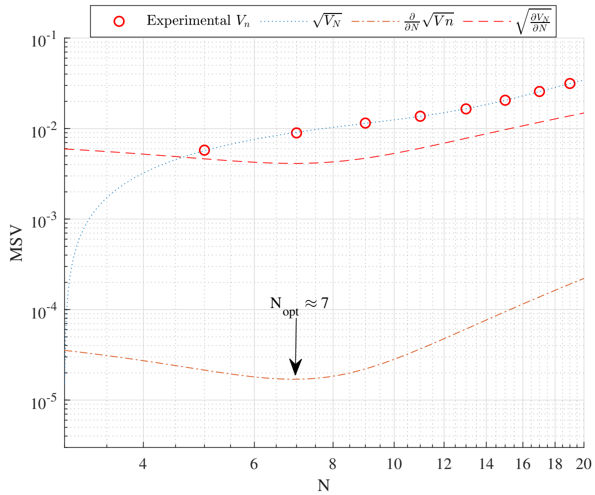

Fig. 5. Effect of N on the MSV (circled) for $m = 5$. A cubic approximation of the MSV is $\sqrt{V_N}$ and the optimal horizon $N_{\text{opt}} = 14$ corresponds to the minimum of $\sqrt{\frac{\partial}{\partial N} V_N}$.

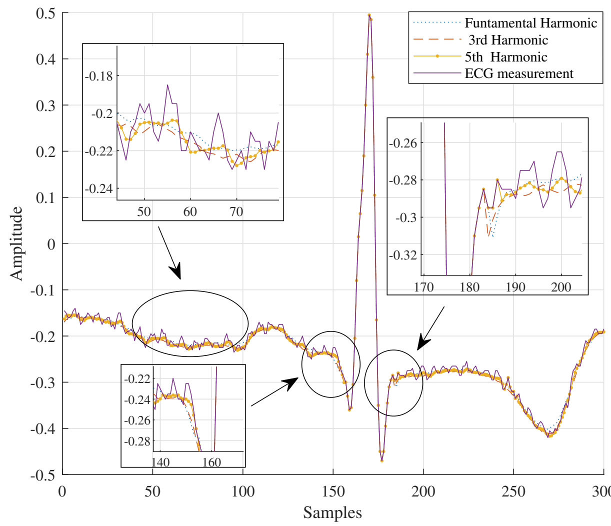

Fig. 6. UFIR smoothing of an ECG signal using the 1st harmonic (dotted), 3rd harmonic (dashed), and 5th harmonic (solid-dotted). The ECG data are depicted with a solid line.

IV. Testing UFIR Smoother by Harmonic Model

We now provide an experimental test of the UFIR smoother performance by a harmonic model with the following system matrix

$$\mathbf{A} = \begin{bmatrix} \cos(\omega_1 \Delta tk) & -\sin(\omega_1 \Delta tk) \\ \sin(\omega_1 \Delta tk) & \cos(\omega_1 \Delta tk) \end{bmatrix} \quad (14)$$

and observation matrix $\mathbf{C} = [1\,, 0]$, where ω_1 is a chosen angular fundamental frequency. For a periodic signal $y = \cos\theta + \sin\theta$ corrupted by an additive white Gaussian noise (AWGN) with mean zero and the variance $\sigma^2 = 0.0625$, the results are sketched in Fig. 7. A comparison is provided with respect to the polynomial model discussed in [15], [16]. As can be seen, both UFIR smoothers are most successful in accuracy, since their estimates range most close to the generated signal. To support this conclusion, the smoother RMSEs computed over 1000 iterations are sketched in Fig. 8.

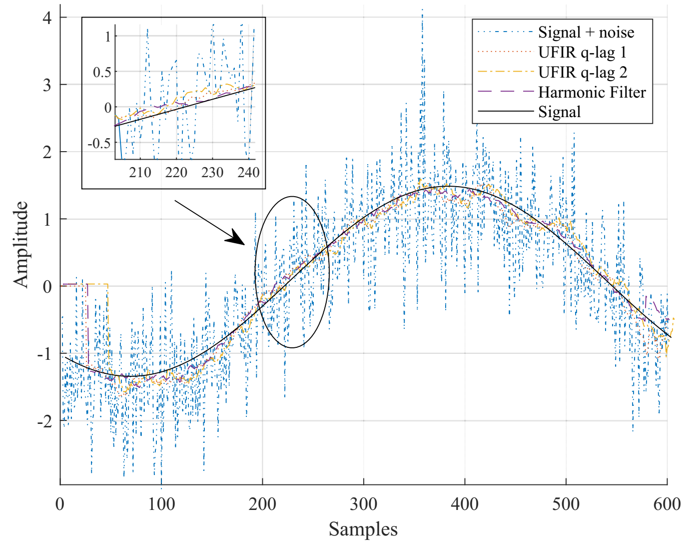

Fig. 7. Denoising of a test harmonic signal (solid) corrupted by the AWGN using a harmonic filter (double dash-triple dots), UFIR smoother with q-lag 1 (dotted) [15] and with q-lag 2 (one dash-dotted) [16].

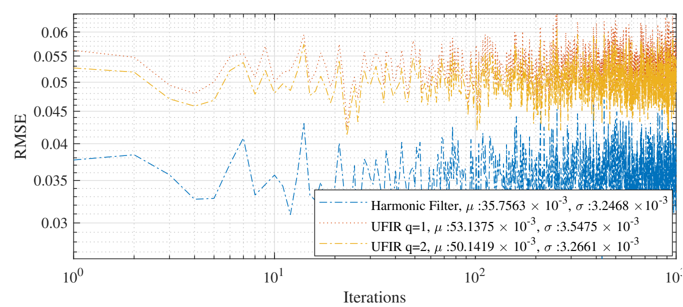

Fig. 8. RMSEs corresponding to Fig. 7 as computed over 1000 iterations for the harmonic filter, UFIR q-lag 1 smoother, and UFIR q-lag 2 smoother.

A. Signal-to-Noise Ratio (SNR) Analysis

We next provide an analysis of the signal-to-noise ratios (SNRs) at the filter outputs in terms of root MSE (RMSE). A synthetic ECG signal having characteristics similar to a real ECG is considered with known noise. For a comparison, we consider as well a polynomial model. As can be seen in Fig. 9, the harmonic model-based filter outperforms both the polynomial model-based UFIR smoothers.

V. Conclusion

A modified UFIR smoother based on the harmonic model has been developed for ECG signals and an optimal horizon determined for the first, third, and fifth harmonics. It has been demonstrated that the UFIR smoothers relying of the first and third harmonics produce the best denoising effect in

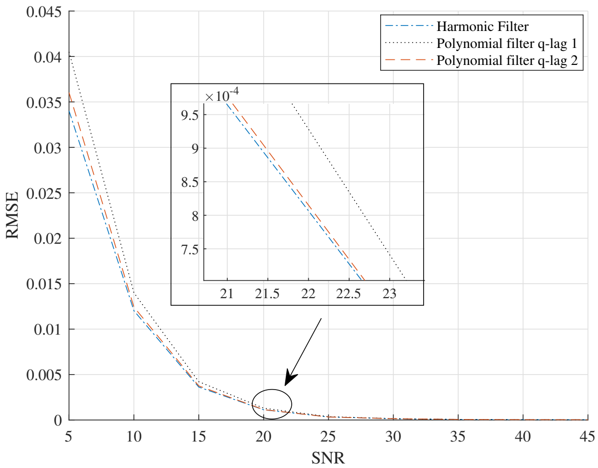

Fig. 9. RMSEs of the UFIR smoother compared to the polynomial and harmonic model and standard filters.

the ECG signal. The harmonic model-based UFIR smoother also outperforms the polynomial model-based UFIR smoothers in terms of the SNR and MSE. As future work, we plan modifying the UFIR smoother approach for the high-order harmonics to increase an accuracy in the ECG signal features extraction.

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