A Multiple-Input Multiple-Output Extension of the Mueller and Müller Timing Error Detector

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***Abstract*—We present a multiple-input multiple-output (MIMO) extension of the timing error detector that was originally developed by Mueller and Müller for synchronization on pulse amplitude modulation over 1-D channels. This MIMO extension applies to joint detection of multiple signals with different timing offsets received over MIMO channels. We provide accurate theoretical expressions of the performance of the proposed scheme, and verify its applicability on a modern Two-Dimensional Magnetic Recording channel. Performance results show that the proposed timing error detector outperforms the conventional 1-D timing error detector that completely ignores the presence of crosstalk interference in MIMO channels.**

***Index Terms*—Timing recovery, multiple-input multiple-output (MIMO) channel, joint detection, crosstalk interference, two-dimensional magnetic recording (TDMR).**

I. Introduction

A timing error detector (TED), often used with a phase-locked loop (PLL) for timing recovery, is an essential building block of a communication system. A popular TED, widely known as M&M TED, was developed by Mueller and Müller in [1] in order to correct the sampling times of a single waveform received over a 1-D channel. A 2-D extension of the M&M TED was also proposed to synchronize a 2-D signal received over a 2-D channel [2]. Nevertheless, there are recent applications where a joint detection of *multiple* input signals, each modulated with different timings, from *multiple* received signals is desirable. For instance, applications such as data storage in two-dimensional magnetic recording (TDMR) [3]–[6], and asynchronous wireless networks [7] can be modeled by a MIMO channel and therefore can benefit from a MIMO TED. To our knowledge, there are no TEDs proposed so far that provide timing estimates in the presence of crosstalk interference over a MIMO channel, and where a joint detection of multiple asynchronous signals is desired. This prompts us to propose such a TED in this paper. Our MIMO extension departs from both of the previous TEDs of [1] and [2]: Unlike the 1-D channel, each of the received outputs includes crosstalk interference from other input signals that are modulated with different timings. Also, unlike the 2-D channel where usually a full scan of the 2-D signal is available for processing, here, only a handful of received signals are available. Further, unlike a 2-D channel where a 2-D model for timing offset is used, in our MIMO setting, similar to 1-D setting, we only consider timing offset in the temporal dimension.

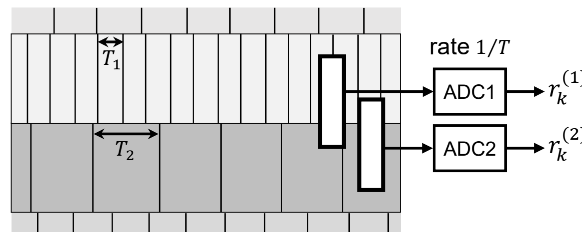

Fig. 1. An example of a TDMR channel with two tracks of interest whose timing differ in frequency and phase, and two readers with significant overlap.

Prior to a discussion on our proposed TED, let us closely look at the problem of timing recovery for a *joint detection* of multiple *asynchronous* signals from multiple received signals. The problem is illustrated in Fig. 1 for an application of TDMR and for the exemplary case of two data tracks of interest having different bit periods, and two overlapping readers. Assume that both of the analog-to-digital converters (ADCs) are free-running at rate $1/T$. The resulting two readback signals will carry contributions from both data tracks with different timings. In conventional detection strategies, we synchronize and detect each data track separately, one track at a time. This is the 1-D detection approach that is currently being implemented in TDMR industry [6]. For a joint detection of multiple asynchronous tracks, however, the synchronization changes drastically: It is impossible to synchronize two readback waveforms to the timings of the both tracks that are themselves asynchronous. Therefore, we must abandon the notion of a modular solution altogether. Rather, the synchronization and detection must be performed jointly. To this end, in a previous work we proposed the rotating-target (ROTAR) algorithm for jointly detecting multiple asynchronous tracks from one or more readback signals [5]. ROTAR modifies a Viterbi detector to internally account for the asynchrony of the tracks being detected.

The synchronization module in 1-D detection schemes typically consists of a 1-D PLL and a 1-D TED. The joint detection approach, however, requires a TED that can provide simultaneous timing error estimates of multiple asynchronous input signals received over a MIMO channel with crosstalk interference. In this paper, we present, for the first time, a MIMO TED which can be used within any joint detection and synchronization scheme, including ROTAR. In the next section we introduce our MIMO channel model and assumptions. In Section III and IV we derive the MIMO TED and provide the variance as a measure of its performance. In Section IV we evaluate the performance of our MIMO TED using a modern TDMR channel.

II. Channel Model and Assumptions

We consider the problem of jointly detecting K input signals from N received channel outputs. We assume independent timing offsets for each of the K input signals, so that the i-th output is:

$$r_i(t) = \sum_{j=1}^{K} \sum_{n} a_n^{(j)} h_{i,j}(t - nT - \tau^{(j)}) + n_i(t), \quad (1)$$

where $a_n^{(j)}$ is the n-th symbol of input $j \in \{1, \ldots, K\}$, $h_{i,j}(t)$ is the symbol response from input j to output i, assumed to be bandlimited to half the symbol rate, and $\tau^{(j)} \geq 0$ is the timing offset for the n-th symbol of input j. T is the ADC sampling period, and $n_i(t)$ is the additive noise for the i-th output. We assume independent white and Gaussian noise with power-spectral density $N_0/2$ for each of the outputs. The assumption that the $\{\tau^{(j)}\}$ be nonnegative is equivalent to the assumption that the ADC sampling rate is large enough to avoid signal aliasing. The i-th output signal is filtered by a low-pass antialiasing filter and then sampled at the ADC rate $1/T$, yielding

$$r_k^{(i)} = \sum_{j=1}^{K} \sum_{n} a_n^{(j)} h_{i,j}(kT - nT - \tau^{(j)}) + n_k^{(i)}, \quad (2)$$

where $n_k^{(i)}$ is the k-th sample of the filtered noise $n_i(t)$, with zero mean and variance $\sigma^2 = N_0/(2T)$.

III. MIMO Timing Error Detector

The objective of our MIMO TED is to provide estimates of the actual $\{\tau^{(j)}\}$. To derive this TED, we follow the same process outlined in the original 1-D M&M TED [1]: The first step is to build an ideal timing function $f_{ideal}^{(j)}(\tau^{(j)})$ for each $j \in \{1, \ldots, N\}$ using a linear combination of samples of channel responses $\{h_{i,j}\}_{i \in \{1,\ldots,N\}}$ such that $f_{ideal}^{(j)}(\tau^{(j)}) = \tau^{(j)}/T$ holds ideally. Selecting type A class of timing functions described in [1], and for certain relevant types of $\{h_{i,j}(t)\}$ that will be described next, we build an approximation $f^{(j)}$ of $f_{ideal}^{(j)}$, $\forall j \in \{1, \ldots, N\}$, according to:

$$\begin{aligned} f^{(j)}(\tau^{(j)}) &= \frac{1}{2} \sum_{i=1}^{N} \beta_{i,j} \left(h_{i,j}(T - \tau^{(j)}) - h_{i,j}(-T - \tau^{(j)}) \right) \\ &\approx \frac{\tau^{(j)}}{T}, \end{aligned} \quad (3)$$

where $h_{i,j} = \alpha_{i,j} \operatorname{sinc}(t/T)$, $\{\alpha_{i,j}\} \in \mathbb{R}$, and $\{\beta_{i,j}\} \in \mathbb{R}$ such that $\sum_{i=1}^{N} \beta_{i,j} \alpha_{i,j} = 1$, for each j. Note that the approximation in (3) holds for $\{h_{i,j}(t)\}$ approximating an ideal intersymbol-intereference (ISI)-free channel that incorporates crosstalk interference between adjacent input signals. Therefore, the response from each input to each output would be a multiple of, for example, a sinc or a raised cosine function. Clearly, the values of $\{\alpha_{i,j}\}$ depend on the specific application. The values of $\{\beta_{i,j}\}$, however, are design parameters that will be discussed later in section IV. Also note that equation (3) is a weighted sum of N 1-D ideal functions in [1], and the 1-D case is a special case of the MIMO case of (3) where $N = K = 1$. Further, equation (3) states that the j-th timing offset is approximated by a weighted sum of the estimates that responses from input j to each output i, $\{h_{i,j}\}_{i \in \{1,\ldots,N\}}$, provide.

Following the process in [1], since the *actual* sampled channel responses in (3) are not available, we will instead estimate each $f^{(j)}(\tau^{(j)})$ from a group of past samples of output signals such that the expected values of the estimate is equal to $f^{(j)}(\tau^{(j)})$.

A. Extraction of Timing Information from Received Signals

Duo to the linear character of (3) and adopting the procedure of the 1-D setting in [1] to our MIMO setting, we estimate each timing offset $\tau^{(j)}$ at time k using a linear combination of past samples of $\{r_k^{(i)}\}_{i \in \{1,\ldots,N\}}$ in the form

$$\hat{\tau}_k^{(j)} = \sum_{i=1}^{N} {\mathbf{g}_k^{(i,j)}}^T \mathbf{r}_k^{(i)}, \forall j, \quad (4)$$

where $\mathbf{r}_k^{(i)} = [r_{k-m+1}^{(i)}, \ldots, r_k^{(i)}]^T$ is the $m \times 1$ vector formed by the past m samples of the received signal i at time k, and where we denoted the $m \times 1$ vector of weights (later defined in Section IV) from input j to output i at time k as functions of data symbols of input j according to

$$\mathbf{g}_k^{(i,j)} = [g_1(\mathbf{a}_k^{(j)}), g_2(\mathbf{a}_k^{(j)}), \ldots, g_m(\mathbf{a}_k^{(j)})]^T, \quad (5)$$

where $\mathbf{a}_k^{(j)} = [a_{k-m+1}^{(j)}, \ldots, a_k^{(j)}]^T$. Note that, in accordance with the timing function in (3), equation (4) states that the estimated timing offset of each input j, $\hat{\tau}_k^{(j)}$, is a weighted sum over all the estimates that the N output signals provide for that input j.

To be unbiased, the estimate in (4) must satisfy $E\{\hat{\tau}_k^{(j)}\} = f^{(j)}(\tau^{(j)}) \approx \tau^{(j)}/T$, for each j. This condition helps us to determine the linear combination weights of (5) as follows.

$$\begin{aligned} E\{\hat{\tau}_k^{(j)}\} &= E\{\sum_{i=1}^{N} {\mathbf{g}_k^{(i,j)}}^T \mathbf{r}_k^{(i)}\} \\ &= E\{E\{\sum_{i=1}^{N} {\mathbf{g}_k^{(i,j)}}^T \mathbf{r}_k^{(i)} | \mathbf{a}_k^{(1)}, \ldots, \mathbf{a}_k^{(K)}\}\}, \forall j. \end{aligned} \quad (6)$$

Because $\{\mathbf{g}_k^{(i,j)}\}$ only depend on the input symbols that are contained within $[\mathbf{a}_k^{(1)}, \ldots, \mathbf{a}_k^{(K)}]$, the inner conditional expected value can be written as

$$\sum_{i=1}^{N} {\mathbf{g}_k^{(i,j)}}^T E\{\mathbf{r}_k^{(i)} | \mathbf{a}_k^{(1)}, \ldots, \mathbf{a}_k^{(K)}\} = \sum_{i=1}^{N} {\mathbf{g}_k^{(i,j)}}^T \mathbf{v}_k^{(i)}, \forall j, \quad (7)$$

where we have introduced a new $m \times 1$ vector $\mathbf{v}_k^{(i)} = E\{\mathbf{r}_k^{(i)}|\mathbf{a}_k^{(1)}, \ldots, \mathbf{a}_k^{(K)}\}$ whose components are given by

$$[\mathbf{v}_k^{(i)}]_l = E\{r_{k-m+l}^{(i)}|\mathbf{a}_k^{(1)}, \ldots, \mathbf{a}_k^{(K)}\}$$
$$= \sum_{t=1}^{K}\sum_{x=1}^{K} a_{k-m+x}^{(t)} h_{i,t}\left((l-x)T - \tau^{(t)}\right), \forall i. \quad (8)$$

Collecting all the components from (8) into a vector gives

$$\mathbf{v}_k^{(i)} = \sum_{t=1}^{K} {A_k^{(t)}}^T \mathbf{h}_{i,t}, \forall i, \quad (9)$$

where $\mathbf{h}_{i,t} = [h_{i,t}\left((1-m)T - \tau^{(t)}\right), \ldots, h_{i,t}\left(-\tau^{(t)}\right), \ldots, h_{i,t}\left((m-1)T - \tau^{(t)}\right)]^T$ is a $(2m-1) \times 1$ vector of sampled channel responses from input t to output i, and $A_k^{(t)}$ is a $(2m-1) \times m$ matrix of the symbols of input t

$$A_k^{(t)} = \begin{bmatrix} a_k^{(t)} & 0 & 0 & \ldots & 0 \\ a_{k-1}^{(t)} & a_k^{(t)} & 0 & \ldots & 0 \\ \vdots & & & & \vdots \\ a_{k-m+1}^{(t)} & & & \ldots & a_k^{(t)} \\ 0 & a_{k-m+1}^{(t)} & & \ldots & a_{k-1}^{(t)} \\ \vdots & & & & \vdots \\ 0 & 0 & 0 & \ldots & a_{k-m+1}^{(t)} \end{bmatrix}. \quad (10)$$

By combining (6), (7), and (9) we obtain

$$E\{\hat{\tau}_k^{(j)}\} = \sum_{i=1}^{N}\sum_{t=1}^{K} \mathbf{h}_{i,t}^T E\{A_k^{(t)}\mathbf{g}_k^{(i,j)}\}, \forall j. \quad (11)$$

Note that the timing function in (3) can be written as

$$f^{(j)}(\tau^{(j)}) = \sum_{i=1}^{N} \mathbf{h}_{i,j}^T \mathbf{u}_{i,j}, \forall j, \quad (12)$$

where the vector $\mathbf{u}_{i,j}$ is the vector of linear combining weights and can be deducted from (3). Our objective is to choose the weights $\{\mathbf{g}_k^{(i,j)}\}_{i\in\{1,\ldots,N\}}$, $\forall j$ such that the estimate (4) is unbiased. By comparing equations (11) and (12) we see that this requires

$$E\{A_k^{(j)}\mathbf{g}_k^{(i,j)}\} = \mathbf{u}_{i,j}, \forall i, \forall j. \quad (13)$$

(13) can be used to determine the data dependent weights $\{\mathbf{g}_k^{(i,j)}\}_{i\in\{1,\ldots,N\}}$ for each $j \in \{1, \ldots, K\}$ as functions of the data symbols on the j-th input. Before we do so in section IV, we first calculate the variance of the estimators in (4).

B. Variance of the Estimators

The variance of (4) gives a linear measure of the mean square error involved in estimating the timing functions through (4). First we evaluate

$$E\left\{\left(\hat{\tau}_k^{(j)}\right)^2\right\}$$
$$= E\{\sum_{i=1}^{N}\sum_{l=1}^{N} {\mathbf{g}_k^{(i,j)}}^T \mathbf{r}_k^{(i)} {\mathbf{r}_k^{(l)}}^T \mathbf{g}_k^{(l,j)}\}$$
$$= E\{\sum_{i=1}^{N}\sum_{l=1}^{N} {\mathbf{g}_k^{(i,j)}}^T E\{\mathbf{r}_k^{(i)}{\mathbf{r}_k^{(l)}}^T|\mathbf{a}_k^{(1)}, \ldots, \mathbf{a}_k^{(K)}\}\mathbf{g}_k^{(l,j)}\}, \forall j. \quad (14)$$

Let $m \times m$ matrix $M_k^{(i,l)} = E\{\mathbf{r}_k^{(i)}{\mathbf{r}_k^{(l)}}^T|\mathbf{a}_k^{(1)}, \ldots, \mathbf{a}_k^{(K)}\}$ for each i, $l \in \{1, \ldots, N\}$, the elements of which are given by

$$m_{p,q}^{(i,l)} = E\{\mathbf{r}_{k-m+p}^{(i)}\mathbf{r}_{k-m+q}^{(l)}|\mathbf{a}_k^{(1)}, \ldots, \mathbf{a}_k^{(K)}\}$$
$$= \sum_{j=1}^{K}\sum_{j'=1}^{K}\sum_{x=1}^{m}\sum_{y=1}^{m} a_{k-m+x}^{(j)} a_{k-m+y}^{(j')}$$
$$\times h_{i,j}\left((p-x)T - \tau^{(j)}\right) h_{l,j'}\left((q-y)T - \tau^{(j')}\right)$$
$$+ E\{a^2\} \sum_{x\notin(1:m)}\sum_{j=1}^{K} h_{i,j}\left((p-x)T - \tau^{(j)}\right)$$
$$\times h_{l,j}\left((q-x)T - \tau^{(j)}\right). \quad (15)$$

Observing equation (8), the matrix in (15) can be more conveniently expressed as

$$M_k^{(i,l)} = \mathbf{v}_k^{(i)}{\mathbf{v}_k^{(l)}}^T + Q_k^{(i,l)}, \forall i, \forall l, \quad (16)$$

where the elements of Q are given by the second term in (15). Therefore, the second moment of $\hat{\tau}_k^{(j)}$ in (14) can be written as

$$E\left\{\left(\hat{\tau}_k^{(j)}\right)^2\right\}$$
$$= E\left\{\sum_{i=1}^{N}\sum_{l=1}^{N} {\mathbf{g}_k^{(i,j)}}^T \mathbf{v}_k^{(i)}{\mathbf{v}_k^{(l)}}^T \mathbf{g}_k^{(l,j)} + {\mathbf{g}_k^{(i,j)}}^T Q_k^{(i,l)} \mathbf{g}_k^{(l,j)}\right\}, \forall j. \quad (17)$$

Finally, the variance of each $\hat{\tau}^{(j)}$ is given by

$$S^{(j)} = E\left\{\sum_{i=1}^{N}\sum_{l=1}^{N} {\mathbf{g}_k^{(i,j)}}^T \mathbf{v}_k^{(i)}{\mathbf{v}_k^{(l)}}^T \mathbf{g}_k^{(l,j)} + {\mathbf{g}_k^{(i,j)}}^T Q_k^{(i,l)} \mathbf{g}_k^{(l,j)}\right\}$$
$$- E^2\left\{\sum_{i=1}^{N} {\mathbf{g}_k^{(i,j)}}^T \mathbf{v}_k^{(i)}\right\}. \quad (18)$$

So far in our variance calculations we have neglected the noise, following [1]. The effect of adding the noise, however, can be taken into account by only replacing $Q_k^{(i,l)}$ by $Q_k^{(i,l)} + \sigma^2 I$ where I is the $m \times m$ identity matrix.

The results show that the variances of the estimators depend strongly on the chosen weights. Also, note that if the channel responses are ideal and there are no timing offsets, the matrix Q will reduce to zero and the variances will only depend on the weights and the data symbols.

IV. CHOOSING THE WEIGHTING VECTORS

The objective now is to choose $\{\mathbf{g}_k^{(i,j)}\}$ such that equation (13) holds. The optimum solution that simultaneously minimizes the variance of (18) depends on the channel responses. A slightly sub-optimum but extremely practical solution, according to [1], that does not depend on the channel responses, and therefore does not require a knowledge of the actual channel, is achieved by setting

$$A_k^{(j)} \mathbf{g}_k^{(i,j)} = \mathbf{u}_{i,j} + \mathbf{d}_k^{(i,j)}, \forall i, \forall j, \tag{19}$$

such that $\mathbf{d}_k^{(i,j)}$ is a zero mean random vector and $\mathbf{u}_{i,j}$ is given by (12). The choice of the random vector $\mathbf{d}_k^{(i,j)}$ affects the variance according to

$$S^{(j)} = E\left\{\sum_{i=1}^{N}\sum_{l=1}^{N} \mathbf{d}_k^{(i,j)T} \mathbf{h}_{i,j} \mathbf{h}_{l,j}{}^T \mathbf{d}_k^{(l,j)} + \mathbf{g}_k^{(i,j)T} Q_k^{(i,l)} \mathbf{g}_k^{(l,j)}\right\}. \tag{20}$$

Therefore, we should choose the components of $\mathbf{d}_k^{(i,j)}$ to be as small as possible. Further, the component of $\mathbf{d}_k^{(i,j)}$ associated with the main sample of the channel response $h_{i,j}(-\tau^{(j)})$ should be zero so that the variance in the case of a Nyquist channel with no timing offset is zero.

Next, we adopt the 1-D solution given in [1] for our MIMO problem in (19) for an exemplary channel with $K = 2$ input and $N = 2$ outputs, and where the memory $m = 2$. Adaptations for other channels with different K and N parameters are straightforward.

Using (3) and (12), equation (19) becomes

$$\begin{bmatrix} a_k^{(j)} & 0 \\ a_{k-1}^{(j)} & a_k^{(j)} \\ 0 & a_{k-1}^{(j)} \end{bmatrix} \begin{bmatrix} g_{k-1}^{(i,j)} \\ g_k^{(i,j)} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}\beta_{i,j} \\ 0 \\ \frac{1}{2}\beta_{i,j} \end{bmatrix} + \begin{bmatrix} d_{k-2}^{(i,j)} \\ 0 \\ d_k^{(i,j)} \end{bmatrix}, \forall j, \forall i. \tag{21}$$

Equation (21) resembles the 1-D equation in [1]. The important result here is that the MIMO equation (19) breaks down into several 1-D problems with the following solution for the timing offset of input j

$$\mathbf{g}_k^{(i,j)} = \frac{\beta_{i,j}}{2E\{a_k^2\}} \begin{bmatrix} -a_k^{(j)} \\ a_{k-1}^{(j)} \end{bmatrix},$$
$$\mathbf{d}_k^{(i,j)} = \frac{\beta_{i,j}}{2E\{a_k^2\}} \begin{bmatrix} E\{a_k^2\} - \left(a_k^{(j)}\right)^2 \\ 0 \\ \left(a_{k-1}^{(j)}\right)^2 - E\{a_k^2\} \end{bmatrix}, \forall i, \tag{22}$$

yielding the estimate

$$\hat{\tau}_k^{(j)} = \frac{1}{2E\{a_k^2\}} \sum_{i=1}^{N} \beta_{i,j} \left(r_k^{(i)} a_{k-1}^{(j)} - r_{k-1}^{(i)} a_k^{(j)}\right). \tag{23}$$

We observe that the estimated timing offset of the input signal j is a weighted sum of the estimates that all output signals provide for that parameter. This is expected based on our selection of the ideal timing functions in (3). Here, in contrast to the solutions in [1] and [2], the solution to (21) does additionally depend on the values of $\{\alpha_{i,j}\}$ and $\{\beta_{i,j}\}$. These values, however, can be deduced from target responses that are available after equalization, at no extra cost. As explained before, the estimates $\{\hat{\tau}_k^{(j)}\}$ can only be used for timing recovery along with the detection of the data symbols where the target responses are known to the detector [4], [5]. This is in contrast to the conventional modular schemes where the timing recovery is performed separately and often times prior to the detection [6], [8]. Note that the coefficients $\{\beta_{i,j}\}$ are design parameters. In general, the parameters $\{\beta_{i,j}\}$ should be selected to satisfy two conditions: 1) according to (3), $\sum_{i=1}^{N} \beta_{i,j}\alpha_{i,j} = 1$, for each j, and 2) to suppress the crosstalk interference of other input signals as much as possible. The first condition guarantees unbiased estimates of the timing offset parameters $\{\tau^{(j)}\}$, while the second condition improves the performance of the MIMO TED by decreasing the variance of the estimators. Based on our results, the minimum variance is achieved when $\{\beta_{i,j}\}$ are selected such that for each $\tau_k^{(j)}$ of interest, the interference cause from of all other inputs $j' \neq j$ are removed from the channel outputs in (23). Such selection of $\{\beta_{i,j}\}$ reduces the MIMO setting of (23) to a 1-D setting of [1]. Therefore, the optimum selection of $\{\beta_{i,j}\}$ does depend on the specific application and the target responses as will be discussed in the following section.

V. Simulation Results

To the best of our knowledge the only detector that jointly detects multiple asynchronous data tracks from one or a few readback waveforms is the ROTAR algorithm of [5]. Therefore, we present the performance of the proposed MIMO TED when used within the ROTAR algorithm for a joint detection of $K = 2$ asynchronous data tracks from $N = 2$ readback signals, as illustrated in Fig. 1. For a complete description of the algorithm please refer to [4]. We also compare the performance of the MIMO TED within ROTAR with the conventional 1-D M&M TED applied within a conventional 1-D receiver that follows the current industry standard for single-track detection [6], [9].

We adopt the channel model of (2) where the unknown timing offset parameters are frequency offsets with $\tau_k^{(1)}/T = 2k \times 10^{-5}$ and $\tau_k^{(2)}/T = 2k \times 10^{-4}$ for track 1 and 2, respectively. The sector length is $L = 40$ kbits, which results in a maximum slip of 0.8 and 8 bit periods, respectively, for track 1 and track 2 at the end of the sector. The target responses are $\mathbf{h}_{1,1}^* = \mathbf{h}_{2,2}^* = [1, 0.5]$, $\mathbf{h}_{1,2}^* = \mathbf{h}_{2,1}^* = [0.4, 0.16]$. We use the proposed MIMO TED of (23) within a per-survivor processing (PSP) algorithm to estimate the unknown timing offsets of each of the tracks and for each state in the trellis [10]. For simplicity in (23), we can mitigate, but not completely remove, the intertrack interference by using only one set of $\{\beta_{i,j}\}$ parameters. From equation (23), it is straightforward to show that

$$\beta_{1,1} = \frac{\alpha_{2,2}}{\alpha_{2,2}\alpha_{1,1} - \alpha_{1,2}\alpha_{2,1}}, \beta_{2,1} = \frac{-\alpha_{1,2}}{\alpha_{2,2}\alpha_{1,1} - \alpha_{1,2}\alpha_{2,1}}$$
$$\beta_{1,2} = \frac{-\alpha_{2,1}}{\alpha_{2,2}\alpha_{1,1} - \alpha_{1,2}\alpha_{2,1}}, \beta_{2,2} = \frac{\alpha_{1,1}}{\alpha_{2,2}\alpha_{1,1} - \alpha_{1,2}\alpha_{2,1}}, \tag{24}$$

where $\alpha_{1,1} = \alpha_{2,2} = 1$ and $\alpha_{1,2} = \alpha_{2,1} = 0.4$, removes the interference associated with the main components of the target responses. We use the resulted MIMO TED with a second-

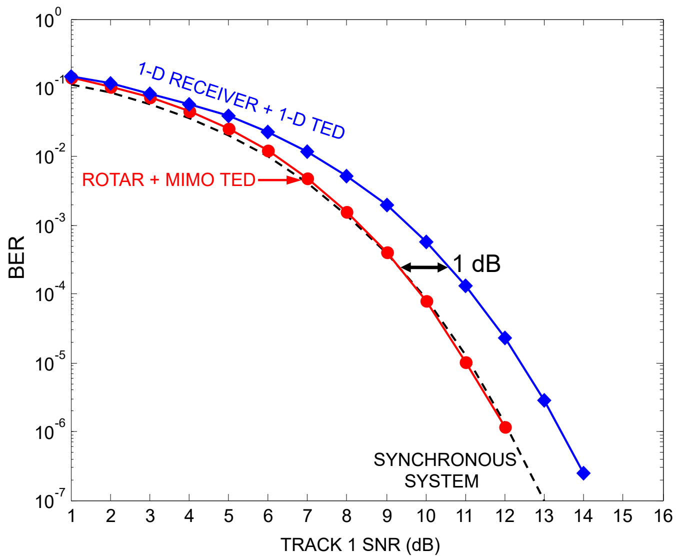

Fig. 2. BER performance of the ROTAR with the proposed TED used in the PSP algorithm for timing estimations.

order PLL that updates its timing estimates according to the recursion

$$\tau_{k+1}^{(j)} = \epsilon\tau_k^{(j)} + \gamma(1-\epsilon)\hat{\tau}_k^{(j)} + \eta(1-\epsilon)\sum_{l=1}^{k-1}\hat{\tau}_l^{(j)}, j \in \{1,2\}, \tag{25}$$

where $\epsilon = 0.6$ is the leakage rate and $\gamma = 0.001$ and $\eta = \gamma^2/4$ are PLL gains. The MIMO TED is operated in the decision-directed mode where estimates of the bits, $\{\hat{a}_k^{(j)}\}$, are first extracted using the received signals by the ROTAR detector and used in place of their actual values in (23).

Fig. 2 compares the bit-error rate performance of the ROTAR detector with PSP, where the proposed MIMO TED is used, against two other detectors. Here due to the symmetry of the setting, the individual error rates of the two tracks of interest are very similar and therefore the average of the two bit-error probabilities is plotted instead of the individual error probabilities.

The curve labeled "ROTAR + MIMO TED" shows the performance of the ROTAR algorithm with the aide of the MIMO TED. The curve labeled "1-D RECEIVER + 1-D TED" shows the performance of a conventional 1-D receiver with the aide of the 1-D M&M TED. This conventional receiver uses two independent two-input single-output MISO equalizers followed by two independent 2-state Viterbi detectors with PSP to separately detect the two tracks of interest. This 1-D receiver simulates similar approaches taken by the industry implementations today [3], [6]. We observe a gain of 1 dB is achieved when the 1-D detection is replaced by the joint detection with ROTAR. This performance gain is expected due to the fact that, in the presence of intertrack interference, joint detection is superior to one-dimensional detection [5].

Fig. 2 also shows the performance of a fabricated system, labeled as "SYNCHRONOUS SYSTEM", where the two tracks are written synchronously with each other and also with the sampling rate of the two ADCs. This system simulates an ideal setting where the tracks of interest are written synchronously and both ADCs are sampling exactly at correct timings. The dashed curve presents the performance of a standard 4-state joint Viterbi detector for this synchronous case. We see that the ROTAR detector closely matches the performance of the synchronous system, despite the presence of the frequency offsets.

These results attest to the superiority of a joint detector performance where a MIMO TED is implemented over a conventional 1-D detector where a 1-D TED is implemented. Thereby, these results also attest to the fact that a MIMO TED that provides estimates of timing errors of multiple input signals is needed to implement a joint detector that internally accounts for the asynchrony of the signals in a MIMO channel. The proposed TED is exactly that.

VI. Conclusion

We have proposed, for the first time, a MIMO extension of the well-known M&M TED to be used in joint detection of multiple asynchronous input signals from one or more received signals over a MIMO channel with crosstalk interference. We evaluated the performance of the proposed MIMO TED when used in a previously published ROTAR detector [5]. The results show the applicability of the MIMO TED within a joint detector that outperforms a conventional 1-D receiver where the 1-D M&M TED is often employed. Such a MIMO TED is needed because in the absence of a MIMO TED, the only alternative would be to apply the 1-D TED that completely ignores the presence of crosstalk interference in the MIMO channels. This is regardless of the specific assumptions made in our simulation environment.

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