Dynamic Resolution ADC/DAC Massive MIMO FD Relaying system over Correlated Rician Channel

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Abstract*—We consider a multi-pair two-way full-duplex (FD) massive multiple-input multiple-output hardware-impaired relay over spatially correlated Rician fading channels exchanging information with hardware-impaired FD users. The relay employs a dynamic resolution ADC/DAC architecture wherein each antenna is equipped with a different resolution ADC/DAC. We derive a novel linear minimum mean square error (LMMSE) channel estimator with correlated Rician fading channel, incorporating dynamic resolution ADCs/DACs and RF impairments. We examine the impact of relay and user hardware impairments on the normalized mean squared error (NMSE) and *show that the NMSE floors to a non-zero error floor even as pilot power goes to infinity.

I. Introduction

Full duplex (FD) relay, which can simultaneously transmit and receive on the same time-frequency resource, has higher spectral efficiency (SE) than a half-duplex relay. The primary challenge in FD relaying is the loop interference, which the recent FD literature has shown that it can be mitigated by applying various passive and active spatial mitigation techniques [1], [2]. Massive multi-input multi-output (MIMO) technology, is recently combined with FD relaying [1], [2].

Massive MIMO relay, with each antenna connected to a high-quality radio-frequency (RF) chain and high resolution analog-to-digital (ADC) and digital-to-analog (DAC) converters has high power consumption and hardware cost. A promising solution is to replace high-resolution ADC/DACs (e.g., 10-12 bits) with low-resolution ADC/DACs (e.g., 1-5 bits) and use low-quality RF chains. Reference [3] derived closed-form SE expressions for FD amplify and forward (AF) relaying system with low resolution ADCs and perfect channel state information (CSI) at the relay. Reference [4] derived closed-form SE expressions for an FD AF relaying with low-resolution ADCs in the presence of CSI errors.

Low resolution ADCs, due to quantization errors and signal distortion, degrade the capacity in high signal-to-noise ratio (SNR) regime [3], [4]. The mixed-ADC/DAC architecture, where a fraction of ADC/DAC are high-resolution, can compensate for this performance degradation. The authors in [5] derived closed-form SE expressions and power scaling laws for HD relaying with mixed ADC architecture. The current work generalizes the low and mixed ADC-DAC resolution frameworks in [3], [4] and [5] respectively, by proposing a dynamic-ADC/DAC framework for massive two-way FD MIMO relaying. The relay herein consists of multiple ADC/DAC pools, and each such pool, depending on the design requirements, can be configured for a different resolution.

The deployment cost and power consumption of massive MIMO systems are further reduced by using low-cost RF chains [6], [7]. The low-cost RF chains are prone to impairments like power amplifier non-linearities, I/Q imbalance and receiver noise. These hardware impairments can be mitigated using analog or digital compensation algorithms [6], [7], and the residual hardware impairments can be modelled using a generalized error vector magnitude (EVM) model [6], [7]. References [6] and [7] respectively provided approximate and closed form SE expressions for a hardware-impaired two-way FD relaying system, respectively. Reference [8] separately captured the impact of low-resolution ADCs and low-cost RF chains for single-hop massive MIMO system.

Due to the complexity of Rician channel model, only reference [9], has considered Rician fading for FD relaying with low resolution ADCs. Further, due to physical size constraints, the large number of antennas at the relay needs to be tightly packed together. These closely-packed antennas observe correlated channels. Reference [10] derived closed-form SE expression and joint power and hardware scaling laws for a hardware-impaired single-pair HD relaying with correlated Rayleigh channels. To the best of our knowledge, the existing literature has not studied the i) impact of hardware impairments for multi-pair relaying with *correlated Rician channel model*; and ii) *combined effect of low resolution ADC/DACs and residual hardware impairments*. The **main** contributions of the current work, takes a step in this direction by designing and investigating the performance of a linear minimum mean square error (LMMSE) channel estimator.

1) We propose a framework for the multi-pair two-way massive MIMO FD relay over spatially correlated Rician fading channels, with hardware-impaired relay and users. The current work, unlike previous studies [3]–[5], [9], considers a flexible architecture which enables each relay antenna to have a different resolution ADCs/DACs.
2) With correlated Rician fading channel, we derive a novel LMMSE channel estimator considering both dynamic resolution ADCs/DACs and RF impairments. We examine the impact of relay and user hardware impairments on the normalized mean squared error (NMSE) and show that *the NMSE floors to the non-zero error floor even when the pilot power goes to infinity*.

II. System Model

We consider, a two-way shared FD AF relay via which K FD user pairs exchange data on the same time-frequency resource. We assume that the user U_{2m-1} for $(m = 1, \cdots, K)$on one side of the relay exchanges information with the user U_{2m} on the other side of the relay. We consider each user to be equipped with one transmit and one receive antenna and the relay to be equipped with N transmit and N receive antennas.

We also assume that there is no direct path between the user pairs due to large path loss and heavy shadowing [1], [2].

Channel Model: The vectors $\mathbf{g}_k \in \mathbb{C}^{N\times 1}$ and $\mathbf{h}_k^T \in \mathbb{C}^{1\times N}$ represent the channels from the transmit antenna of the kth user to the relay receive antenna array and from the relay transmit antenna array to the receive antenna of the kth user respectively, and are given as

$$\mathbf{g}_k = \overline{\mathbf{g}}_{m_k} + \overline{\mathbf{R}}_{g_k}^{1/2}\mathbf{g}_{w_k} \text{ and } \mathbf{h}_k = \overline{\mathbf{h}}_{m_k} + \overline{\mathbf{R}}_{h_k}^{1/2}\mathbf{h}_{w_k}, \text{where} \quad (1)$$

$$\overline{\mathbf{g}}_{m_k} = \sqrt{(\sigma_{g_k}^2 K_{R_k})/(K_{R_k}+1)}\mathbf{g}_{m_k}, \overline{\mathbf{h}}_{m_k} = \sqrt{(\sigma_{h_k}^2 K_{R_k})/(K_{R_k}+1)}\mathbf{h}_{m_k},$$
$$\overline{\mathbf{R}}_{g_k} = \sigma_{g_k}^2/(K_{R_k}+1)\mathbf{R}_{g_k} \text{ and } \overline{\mathbf{R}}_{h_k} = \sigma_{h_k}^2/(K_{R_k}+1)\mathbf{R}_{h_k}. \quad (2)$$

The vectors $\mathbf{g}_{w_k}$ and $\mathbf{h}_{w_k}$ contains i.i.d $\mathcal{CN}(0,1)$ elements, the scalar K_{R_k} represents the Rician K-factor of the kth user and the scalars $\sigma_{g_k}^2$ and $\sigma_{h_k}^2$ represent the large scale fading coefficients. The vectors $\mathbf{g}_{m_k}$ and $\mathbf{h}_{m_k}$ denotes the deterministic line of sight (LoS) components whose elements are

$$[\mathbf{g}_{m_k}]_n = \exp^{-j(n-1)(2\pi d/\lambda)\sin(\theta_k)} \text{ and}$$
$$[\mathbf{h}_{m_k}]_n = \exp^{-j(n-1)(2\pi d/\lambda)\sin(\theta_k)}, \quad (3)$$

where θ_k is the nominal angle for the kth user, λ is the wavelength and d is the antenna spacing. The matrices $\mathbf{R}_{g_k}$ and $\mathbf{R}_{h_k}$ characterize the spatial correlation of the channels $\mathbf{g}_k$ and $\mathbf{h}_k$ respectively. We concatenate the channel vectors $\mathbf{g}_k$ and $\mathbf{h}_k$ to obtain the matrices $\mathbf{G} = [\mathbf{g}_1, \cdots, \mathbf{g}_{2K}] \in \mathbb{C}^{N\times 2K}$ and $\mathbf{H} = [\mathbf{h}_1, \cdots, \mathbf{h}_{2K}] \in \mathbb{C}^{N\times 2K}$.

The relay self-loop interference channel for closely placed transmit and receive antenna arrays have a dominating LoS component and can be modeled as Rician channel. However references [1], [2] have shown that the LoS component of the relay self-loop interference channel can be mitigated by passive suppression techniques. We therefore similar to [1], [2] consider the relay self-loop interference channel $\mathbf{G}_R$ to be spatially-correlated Rayleigh faded which is modeled as

$$\mathbf{G}_R = \sigma_R \mathbf{R}_{RR}^{1/2}\mathbf{S}_R\mathbf{R}_{TR}^{1/2}, \quad (4)$$

where $\mathbf{S}_R$ with i.i.d $\mathcal{CN}(0,1)$ elements represent the small scale fading and σ_R represents the large scale fading coefficient. The matrices $\mathbf{R}_{RR}$ and $\mathbf{R}_{TR}$ denotes the receive and transmit spatial correlation of the relay loop interference channel, respectively. The user self-loop interference channel Ω_{kk} for $k = 1, \cdots, 2K$, similar to [1], [2], is assumed to be distributed as $\mathcal{CN}(0, \sigma_{kk}^2)$ and the inter-user interference channel between the ith and kth user $\Omega_{i,k} (k, i \in \mathcal{U}_k, i \neq k)$ is distributed as $\mathcal{CN}(0, \sigma_{ik}^2)$.

Transmission Protocol: We consider T-symbol channel coherence interval, with τ_p symbol training interval and $(T-\tau_p)$ symbol data transmission interval. At the nth instant of the data transmission interval, all the users simultaneously transmit their respective signals $\sqrt{p_k}x_k^{(n)}$ for $k = 1, \cdots, 2K$. We consider the users to be equipped with low-cost RF chains which are prone to residual hardware impairments [6], [7]. We model the residual hardware impairments using the EVM model, where the transmit/receive signal is affected by an additive independent distortion term, which is proportional to the respective transmit/receive signal power [6], [7]. The effective user transmit signal, therefore is given by $\sqrt{p_k}x_k^{(n)} + \eta_{tu_k}$,

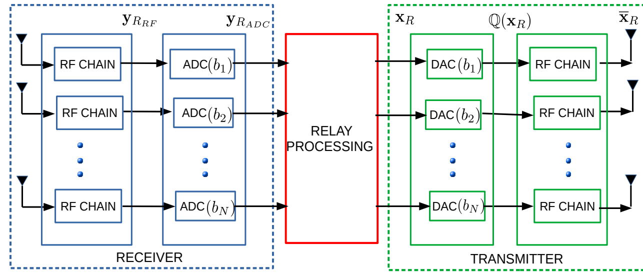

Fig. 1: Block diagram of massive MIMO FD-relay.

with $\eta_{tu_k} \sim \mathcal{CN}(0, \kappa_{tu}^2 p_k)$ where κ_{tu} characterizes the level of transmit residual hardware impairment at the user, and can be interpreted as the user transmit EVM [6], [7].

We also assume, as depicted in Fig. 1, that the relay employs low cost receive/transmit RF chains and dynamic resolution ADC/DAC architecture. The received signal at the output of the receive low-cost RF chains given by

$$\mathbf{y}_{R_{RF}}^{(n)} = \sum_{i=1}^{2K}\mathbf{g}_i(\sqrt{p_i}x_i^{(n)} + \eta_{tu_i}^{(n)}) + \mathbf{G}_R\overline{\mathbf{x}}_R^{(n)} + \boldsymbol{\eta}_{rr}^{(n)} + \boldsymbol{\zeta}_r^{(n)}$$
$$= \mathbf{G}(\mathbf{P}_u^{1/2}\mathbf{x}^{(n)} + \boldsymbol{\eta}_{tu}^{(n)}) + \mathbf{G}_R\overline{\mathbf{x}}_R^{(n)} + \boldsymbol{\eta}_{rr}^{(n)} + \boldsymbol{\zeta}_r^{(n)}, \quad (5)$$

where the tranmit signal vector $\mathbf{x}^{(n)} = [x_1^{(n)}, \cdots, x_{2K}^{(n)}]^T$ with $\mathbb{E}\{\mathbf{x}\mathbf{x}^H\} = \mathbf{I}_{2K}$, the vector $\boldsymbol{\eta}_{tu} = [\eta_{tu_1}, \cdots, \eta_{tu_{2K}}]^T$ and the matrix $\mathbf{P}_u = \text{diag}\{p_1, \cdots, p_{2K}\}$. The vector $\overline{\mathbf{x}}_R^{(n)}$ (refer Fig. 1) is the effective relay transmit signal after incorporating the impairments due to low-quality relay transmit hardware. The additive distortion term $\boldsymbol{\eta}_{rr} \sim \mathcal{CN}\left(\mathbf{0}, \kappa_{rr}^2\mathbf{W}_d\right)$ represents the receive residual hardware impairments at the relay, where $\mathbf{W}_d = \text{diag}\big\{\mathbb{E}\big\{\big(\mathbf{G}(\mathbf{P}_u^{\frac{1}{2}}\mathbf{x} + \boldsymbol{\eta}_{tu}) + \mathbf{G}_R\overline{\mathbf{x}}_R\big)\big(\mathbf{G}(\mathbf{P}_u^{\frac{1}{2}}\mathbf{x} + \boldsymbol{\eta}_{tu}) + \mathbf{G}_R\overline{\mathbf{x}}_R\big)^H\big\}\big\}$, and κ_{rr} is the relay receive EVM [6], [7]. The term $\boldsymbol{\zeta}_r \sim \mathcal{CN}(\mathbf{0}, \zeta\mathbf{I}_N)$ denotes the receiver noise at the relay.

The signal at the output of the dynamic resolution ADCs is affected by quantization noise which is modeled using AQNM [5], and is given as

$$\mathbf{y}_{R_{ADC}}^{(n)} = \mathbb{Q}(\mathbf{y}_{R_{RF}}^{(n)}) = \mathbf{A}_a\mathbf{y}_{R_{RF}}^{(n)} + \mathbf{n}_{qa}^{(n)}$$
$$= \mathbf{A}_a\big(\mathbf{G}(\mathbf{P}_u^{\frac{1}{2}}\mathbf{x}^{(n)} + \boldsymbol{\eta}_{tu}^{(n)}) + \mathbf{G}_R\overline{\mathbf{x}}_R^{(n)} + \boldsymbol{\eta}_{rr}^{(n)} + \boldsymbol{\zeta}_r^{(n)}\big) + \mathbf{n}_{qa}^{(n)}. \quad (6)$$

Here the matrix $\mathbf{A}_a = \text{diag}\{\alpha_{a1}, \cdots, \alpha_{aN}\}$, where $\alpha_{ai} = 1 - \rho_{ai}$ with ρ_{ai} being the ADC distortion factor for the ith relay antenna. The vector $\mathbf{n}_{qa} \sim \mathcal{CN}(\mathbf{0}, \mathbf{B}_a\mathbf{S}_d)$ is the additive ADC quantization noise with $\mathbf{S}_d = \text{diag}\left\{\mathbb{E}\left\{\mathbf{y}_{R_{RF}}\mathbf{y}_{R_{RF}}^H\right\}\right\}$ and $\mathbf{B}_a = \mathbf{A}_a(\mathbf{I}_N - \mathbf{A}_a)$.

At time instant $n > 1$, the relay generates its transmit signal $\mathbf{x}_R^{(n)}$ by multiplying $\mathbf{y}_{R_{ADC}}$ with the precoder matrix $\widetilde{\mathbf{F}}$, such that

$$\mathbf{x}_R^{(n)} = \widetilde{\mathbf{F}}\mathbf{y}_{R_{ADC}}^{(n-1)}. \quad (7)$$

The output of the dynamic-resolution DACs, which is modeled using AQNM [5], therefore is expressed as

$$\mathbb{Q}(\mathbf{x}_R^{(n)}) = \mathbf{A}_d\mathbf{x}_R^{(n)} + \mathbf{n}_{qd}^{(n)}, \quad (8)$$

where the matrix $\mathbf{A}_d = \text{diag}\{\alpha_{d1}, \cdots, \alpha_{dN}\}$ with $\alpha_{di} = 1 - \rho_{di}$, and ρ_{di} denotes the DAC distortion factor of the ith relay antenna. The term $\mathbf{n}_{qd} \sim \mathcal{CN}\left(\mathbf{0}, \frac{P_R}{N}\mathbf{B}_d\right)$ is the additive DAC quantization noise with $\mathbf{B}_d = \mathbf{A}_d(\mathbf{I}_N - \mathbf{A}_d)$. The signal at the output of the low-cost relay transmit RF chains is expressed as

$$\overline{\mathbf{x}}_R^{(n)} = \mathbb{Q}(\mathbf{x}_R^{(n)}) + \boldsymbol{\eta}_{tr}^{(n)} = \mathbf{A}_d\mathbf{x}_R^{(n)} + \mathbf{n}_{qd}^{(n)} + \boldsymbol{\eta}_{tr}^{(n)}. \quad (9)$$

The additive distortion $\boldsymbol{\eta}_{tr} \sim \mathcal{CN}\left(\mathbf{0}, \kappa_{tr}^2\frac{P_R}{N}\mathbf{A}_d\right)$, where the scalar κ_{tr} is interpreted as the relay transmit EVM [6], [7]. The signal $\overline{\mathbf{x}}_R^{(n)}$ in (9) is the effective relay transmit signal after incorporating the impairments due to the dynamic resolution DACs and the low-cost transmit RF chains.

We assume similar to [1], [2], that the relay self-loop interference can be significantly suppressed after employing LIC techniques [1], [2]. The residual loop interference is modelled as an additive Gaussian noise denoted as $\boldsymbol{\Lambda}^{(n)}$[1], [2].

Proposition 1. The elements of RLI i.e., $\boldsymbol{\Lambda}^{(n)}$, are i.i.d. random variable and are distributed as $\mathcal{CN}\left(\mathbf{0}, \sigma_R^4 \frac{P_R}{N}\left(1+\kappa_{tr}^2\right) \operatorname{Tr}\left(\mathbf{R}_{TR}^{\frac{1}{2}} \mathbf{A}_d \mathbf{R}_{TR}^{\frac{1}{2}}\right) \mathbf{R}_{RR}\right)$.

Proof: Derivations are omitted due to space constraints.

Using Proposition 1, we can rewrite (5), (6) and (7) as

$$\widehat{\mathbf{y}}_{R_{RF}}^{(n)} = \mathbf{G}(\mathbf{P}_u^{\frac{1}{2}} \mathbf{x}^{(n)} + \boldsymbol{\eta}_{tu}^{(n)}) + \boldsymbol{\Lambda}^{(n)} + \boldsymbol{\eta}_{rr}^{(n)} + \boldsymbol{\zeta}_r^{(n)}, \tag{17}$$

$$\widehat{\mathbf{y}}_{R_{ADC}}^{(n)} = \mathbf{A}_a\big(\mathbf{G}\big(\mathbf{P}_u^{\frac{1}{2}}\mathbf{x}^{(n)} + \boldsymbol{\eta}_{tu}^{(n)}\big) + \boldsymbol{\Lambda}^{(n)} + \boldsymbol{\eta}_{rr}^{(n)} + \boldsymbol{\zeta}_r^{(n)}\big) + \mathbf{n}_{qa}^{(n)}, \tag{18}$$

$$\text{and } \mathbf{x}_R^{(n)} = \widetilde{\mathbf{F}}\, \widehat{\mathbf{y}}_{R_{ADC}}^{(n-1)}. \tag{19}$$

We notice from (19) that $\mathbf{x}_R^{(n)}$ depends only on the signal received by the relay at the $(n-1)$th time instant i.e., $\widehat{\mathbf{y}}_{R_{ADC}}^{(n-1)}$. We therefore drop the time labels from (5), (6) and (7) for the sake of brevity. The relay transmit signal, by substituting (18) in (9), is now given as

$$\begin{aligned}\overline{\mathbf{x}}_R &= \mathbf{A}_d \widetilde{\mathbf{F}} \mathbf{A}_a \mathbf{G}(\mathbf{P}_u^{\frac{1}{2}} \mathbf{x} + \boldsymbol{\eta}_{tu}) + \mathbf{A}_d \widetilde{\mathbf{F}} \mathbf{A}_a \boldsymbol{\Lambda} + \mathbf{A}_d \widetilde{\mathbf{F}} \mathbf{A}_a \boldsymbol{\eta}_{rr} \\ &+ \mathbf{A}_d \widetilde{\mathbf{F}} \mathbf{A}_a \boldsymbol{\zeta}_r + \mathbf{A}_d \widetilde{\mathbf{F}} \mathbf{n}_{qa} + \mathbf{n}_{qd} + \boldsymbol{\eta}_{tr}.\end{aligned} \tag{20}$$

The received signal at the k'th user after incorporating the user receive hardware impairments is given by

$$\begin{aligned} y_{k'} &= \mathbf{h}_{k'}^T \overline{\mathbf{x}}_R + \sum_{i \in \mathcal{U}_{k'}} \Omega_{ik'}\left(\sqrt{p_i} x_i + \eta_{tu_i}\right) + \eta_{ru_{k'}} + n_{k'} \\ &= \underbrace{\sqrt{p_k} \mathbf{h}_{k'}^T \mathbf{A}_d \widetilde{\mathbf{F}} \mathbf{A}_a \mathbf{g}_k x_k}_{\text{desired signal}} + \underbrace{\sqrt{p_{k'}} \mathbf{h}_{k'}^T \mathbf{A}_d \widetilde{\mathbf{F}} \mathbf{A}_a \mathbf{g}_{k'} x_{k'}}_{\text{self-interference}} \\ &+ \underbrace{\sum_{i \neq k, k'} \sqrt{p_i} \mathbf{h}_{k'}^T \mathbf{A}_d \widetilde{\mathbf{F}} \mathbf{A}_a \mathbf{g}_i x_i}_{\text{inter-pair interference}} + \underbrace{\sum_{i=1}^{2K} \mathbf{h}_{k'}^T \mathbf{A}_d \widetilde{\mathbf{F}} \mathbf{A}_a \mathbf{g}_i \eta_{tu_i} + \eta_{ru_{k'}}}_{\text{user residual impairments}} \\ &+ \underbrace{\mathbf{h}_{k'}^T \mathbf{A}_d \widetilde{\mathbf{F}} \mathbf{A}_a \boldsymbol{\Lambda}}_{\text{relay self-loop interference}} + \underbrace{\mathbf{h}_{k'}^T \mathbf{A}_d \widetilde{\mathbf{F}} \mathbf{A}_a \boldsymbol{\zeta}_r}_{\text{receiver noise}} + \underbrace{\mathbf{h}_{k'}^T \mathbf{A}_d \widetilde{\mathbf{F}} \mathbf{n}_{qa} + \mathbf{h}_{k'}^T \mathbf{n}_{qd}}_{\text{low resolution converters}} \\ &+ \underbrace{\mathbf{h}_{k'}^T \mathbf{A}_d \widetilde{\mathbf{F}} \mathbf{A}_a \boldsymbol{\eta}_{rr} + \mathbf{h}_{k'}^T \boldsymbol{\eta}_{tr}}_{\text{relay residual impairments}} + \underbrace{\sum_{i \in \mathcal{U}_{k'}} \Omega_{ik'}(\sqrt{p_i} x_i + \eta_{tu_i})}_{\text{inter-user+user self interference}} + \underbrace{n_{k'}}_{\text{user noise}}. \end{aligned} \tag{21}$$

Here $\eta_{ru_{k'}}$ is the user receive residual hardware impairment which is distributed as

$$\begin{aligned}\eta_{ru_{k'}} \sim \mathcal{CN}\Big(0, \kappa_{ru}^2 \mathbb{E}\big\{ \big(\mathbf{h}_{k'}^T \overline{\mathbf{x}}_R + \sum_{i \in \mathcal{U}_{k'}} \Omega_{ik'}(\sqrt{p_i} x_i + \eta_{tu_i})\big) \\ \big(\mathbf{h}_{k'}^T \overline{\mathbf{x}}_R + \sum_{i \in \mathcal{U}_{k'}} \Omega_{ik'}(\sqrt{p_i} x_i + \eta_{tu_i})\big)^H \big\}\Big).\end{aligned} \tag{22}$$

with κ_{ru} being the user receive EVM [6], [7].

The ergodic sum-SE expression with MRC/MRT processing for the considered two-way FD relaying system is given as

$$R = \left(1 - \frac{\tau_p}{T}\right) \sum_{k'=1}^{2K} \mathbb{E}\left\{\log_2\left(1 + \text{SINR}_{k'}\right)\right\}, \tag{23}$$

where the term $\text{SINR}_{k'}$ is expressed using (21) in (24) (at the top of the next page).

Relay Precoder Design:We next design the relay precoder matrix $\widetilde{\mathbf{F}}$ considering MRC/MRT processing at the relay using the channel estimates $\widehat{\mathbf{G}}$ and $\widehat{\mathbf{H}}$, which is given by

$$\widetilde{\mathbf{F}} = \beta \widehat{\mathbf{H}}^* \mathbf{T} \widehat{\mathbf{G}}^H \triangleq \beta \mathbf{F}. \tag{25}$$

Here the block-diagonal matrix $\mathbf{T} = \text{diag}\{\mathbf{T}_1, \cdots, \mathbf{T}_{2K}\}$ with $\mathbf{T}_k = [0, 1; 1, 0]$ is the permutation matrix designed to enable the exchange of information between the desired user pairs. The scalar β is the amplification factor designed to satisfy the relay transmit power constraint P_R. We therefore have

$$\begin{aligned} P_R = \mathbb{E}\{\|\mathbf{x}_R\|^2\} = \beta^2 \mathbb{E}\Big\{(1+\kappa_{tu}^2)\|\mathbf{F}\mathbf{A}_a \mathbf{G} \mathbf{P}_u^{\frac{1}{2}}\|^2 + \|\mathbf{F}\mathbf{A}_a \boldsymbol{\Lambda}\|^2 \\ + \zeta^2 \|\mathbf{F}\mathbf{A}_a\|^2 + \kappa_{rr}^2 \|\mathbf{F}\mathbf{A}_a \mathbf{W}_d^{\frac{1}{2}}\|^2 + \|\mathbf{F}(\mathbf{B}_a \mathbf{S}_d)^{\frac{1}{2}})\|^2\Big\}. \end{aligned} \tag{26}$$

III. Channel Estimation

The relay estimates the CSI to design the precoder matrix $\widetilde{\mathbf{F}}$. We derive a novel LMMSE channel estimator that incorporates user and relay residual hardware impairments and dynamic ADC/DAC architecture. We divide the pilot interval $\tau_p = \tau_{p_1} + \tau_{p_2}$ such that in the first training period of length τ_{p_1} symbols, all the $2K$ users simultaneously transmit their respective pilots $\sqrt{P_p}\boldsymbol{\Phi}_{1k} = \sqrt{P_p}[\phi_{1k}^{(1)}, \cdots, \phi_{1k}^{(\tau_{p_1})}] \in \mathbb{C}^{1 \times \tau_{p_1}}$ from their transmit antenna to the relay receive antenna array. We assume that the pilots are orthogonal i.e., $\boldsymbol{\Phi}_{1i}\boldsymbol{\Phi}_{1j}^H = \tau_{p_1}$ for $i = j$ and 0 for $i \neq j$, we require $\tau_{p1} \geq 2K$ [11].

The received pilot signals at the receive antenna array of the relay $\mathbf{Y}_{rg}^p \in \mathbb{C}^{N \times \tau_{p_1}}$ is

$$\mathbf{Y}_{rg}^p = \mathbf{A}_a\Big(\sum_{i=1}^{2K} \mathbf{g}_i\big(\sqrt{P_p}\boldsymbol{\Phi}_{1i} + \boldsymbol{\Upsilon}_{TUi}\big) + \boldsymbol{\Upsilon}_{RR} + \boldsymbol{\Gamma}_R\Big) + \mathbf{N}_{QA}. \tag{27}$$

Here $\boldsymbol{\Upsilon}_{TUi} \in \mathbb{C}^{1 \times \tau_{p_1}}$ and $\boldsymbol{\Upsilon}_{RR} \in \mathbb{C}^{N \times \tau_{p_1}}$ represent the transmit hardware impairment of the ith user and relay receive hardware impairment over the pilot training interval τ_{p_1}. The matrices $\boldsymbol{\Gamma}_R \in \mathbb{C}^{N \times \tau_{p_1}}$ and $\mathbf{N}_{QA} \in \mathbb{C}^{N \times \tau_{p_1}}$ represent receiver noise and additive quantization noise due to dynamic resolution ADCs at the relay over the pilot training interval τ_{p_1}. The received pilot signal at the relay $\mathbf{Y}_{rg}^p$ is now multiplied with $\boldsymbol{\Phi}_{1k}^*$ to estimate the channel vector $\mathbf{g}_k$ such that

$$\begin{aligned}\mathbf{y}_{rgk}^p = \mathbf{Y}_{rg}^p \boldsymbol{\Phi}_{1k}^* = \sqrt{P_p}\tau_{p_1}\mathbf{A}_a \mathbf{g}_k + \sum_{i=1}^{2K} \mathbf{A}_a \mathbf{g}_i \boldsymbol{\Upsilon}_{TUi}^T \boldsymbol{\Phi}_{1k}^* \\ + \mathbf{A}_a \boldsymbol{\Upsilon}_{RR} \boldsymbol{\Phi}_{1k}^* + \mathbf{A}_a \boldsymbol{\Gamma}_r \boldsymbol{\Phi}_{1k}^* + \mathbf{N}_{QA} \boldsymbol{\Phi}_{1k}^*.\end{aligned} \tag{28}$$

The vector $\mathbf{y}_{rgk}^p \in \mathbb{C}^{N \times 1}$ is a sufficient statistics to estimate the channel $\mathbf{g}_k$. We assume that the relay applies LMMSE estimator to estimate the channel $\mathbf{g}_k$.

Theorem 1. The LMMSE estimate of the channel from the transmit antenna of the kth user to the relay receive antenna array i.e. $\mathbf{g}_k$ is given as

$$\widehat{\mathbf{g}}_k = \overline{\mathbf{g}}_{mk} + \sqrt{P_p}\mathbf{A}_a \overline{\mathbf{R}}_{g_k} \boldsymbol{\Psi}_{g_k}\left(\mathbf{y}_{rgk}^p - \overline{\mathbf{y}}_{rgk}^p\right), \text{ where} \tag{29}$$

$$\begin{aligned}\boldsymbol{\Psi}_{g_k} = \Big(&\tau_{p_1} P_p \mathbf{A}_a \overline{\mathbf{R}}_{g_k} \mathbf{A}_a + \sum_{i=1}^{2K} \kappa_{tu}^2 P_p \mathbf{A}_a\left(\overline{\mathbf{g}}_{mi}\overline{\mathbf{g}}_{mi}^H + \overline{\mathbf{R}}_{g_i}\right)\mathbf{A}_a \\ &+ \zeta^2 \mathbf{A}_a^2 + P_p \kappa_{rr}^2 (1+\kappa_{tu}^2) \mathbf{A}_a \operatorname{diag}\Big(\sum_{i=1}^{2K}\big(\overline{\mathbf{g}}_{m_i}\overline{\mathbf{g}}_{m_i}^H + \overline{\mathbf{R}}_{g_i}\big)\Big)\mathbf{A}_a \\ &+ \mathbf{B}_a\Big(P_p(1+\kappa_{tu}^2)(1+\kappa_{rr}^2)\operatorname{diag}\Big(\sum_{i=1}^{2K}\big(\overline{\mathbf{g}}_{m_i}\overline{\mathbf{g}}_{m_i}^H + \overline{\mathbf{R}}_{g_i}\big)\Big) + \zeta^2 \mathbf{I}_N\Big)\Big)^{-1}, \\ \overline{\mathbf{y}}_{rgk} = \mathbb{E}&\left\{\mathbf{y}_{rgk}\right\} = \sqrt{P_p}\tau_{p_1}\mathbf{A}_a \overline{\mathbf{g}}_{mk}.\end{aligned} \tag{30}$$

$$\mathrm{SINR}_{k'}=\frac{p_k\left|\mathbf{h}_{k'}^T\mathbf{A}_d\widetilde{\mathbf{F}}\mathbf{A}_a\mathbf{g}_k\right|^2}{\left\{\begin{array}{l}\sum\limits_{i\neq k}p_i\left|\mathbf{h}_{k'}^T\mathbf{A}_d\widetilde{\mathbf{F}}\mathbf{A}_a\mathbf{g}_i\right|^2+\sum\limits_{i=1}^{2K}\kappa_{tu}^2p_i\left|\mathbf{h}_{k'}^T\mathbf{A}_d\widetilde{\mathbf{F}}\mathbf{A}_a\mathbf{g}_i\right|^2+\left\|\mathbf{h}_{k'}^T\mathbf{A}_d\widetilde{\mathbf{F}}\mathbf{A}_a\mathbf{\Lambda}\right\|^2+\kappa_{rr}^2\|\mathbf{h}_{k'}^T\mathbf{A}_d\widetilde{\mathbf{F}}\mathbf{A}_a\mathbf{W}_d^{\frac{1}{2}}\|^2+\zeta^2\left\|\mathbf{h}_{k'}^T\mathbf{A}_d\widetilde{\mathbf{F}}\mathbf{A}_a\right\|^2\\ +\|\mathbf{h}_{k'}^T\mathbf{A}_d\widetilde{\mathbf{F}}(\mathbf{B}_a\mathbf{S}_d)^{\frac{1}{2}}\|^2+\frac{P_R}{N}\|\mathbf{h}_{k'}^T\mathbf{B}_d^{\frac{1}{2}}\|^2+\frac{P_R}{N}\kappa_{tr}^2\|\mathbf{h}_{k'}^T\mathbf{A}_d^{\frac{1}{2}}\|^2+\sum_{i\in\mathcal{U}_{k'}}\sigma_{ik'}^2p_i(1+\kappa_{tu}^2)+\mathbb{E}\left\{|\eta_{ru_{k'}}|^2\right\}+\sigma_{nu}^2\end{array}\right\}}. \quad (24)$$

Proof: Derivations are omitted due to space constraints.

The normalized mean square error (NMSE) for the channel vector $\mathbf{g}_k$, using the channel estimate in (29) is given by [11]

$$\mathrm{NMSE}_{g_k}=\frac{\mathbb{E}\{\|\mathbf{g}_k-\widehat{\mathbf{g}}_k\|^2\}}{\mathbb{E}\{\|\mathbf{g}_k\|^2\}}=\frac{\mathrm{Tr}(\mathbf{C}_{g_k})}{\mathrm{Tr}(\overline{\mathbf{g}}_{m_k}\overline{\mathbf{g}}_{m_k}^H+\overline{\mathbf{R}}_{g_k})}, \quad (31)$$

where the channel error covariance matrix $\mathbf{C}_{g_k}$ is

$$\mathbf{C}_{g_k}=\overline{\mathbf{R}}_{g_k}-P_p\tau_{p_1}\mathbf{A}_a\overline{\mathbf{R}}_{g_k}\mathbf{\Psi}_{g_k}\overline{\mathbf{R}}_{g_k}\mathbf{A}_a. \quad (32)$$

Based on Theorem 1, we represent the channel $\mathbf{g}_k$ as the sum of its LMMSE channel estimate $\widehat{\mathbf{g}}_k$ and the unknown estimation error $\mathbf{e}_{g_k}$. We note that, unlike LMMSE channel estimation for conventional massive MIMO systems with ideal hardware, the channel estimate $\widehat{\mathbf{g}}_k$ and the estimation error $\mathbf{e}_{g_k}$ are not independent but only uncorrelated, with $\mathbb{E}\{\mathbf{g}_k\mathbf{g}_k^H\}=\overline{\mathbf{R}}_{g_k}-\mathbf{C}_{g_k}$ and $\mathbb{E}\{\mathbf{e}_{g_k}\mathbf{e}_{g_k}^H\}=\mathbf{C}_{g_k}$.

Corollary 1. Substituting $\overline{\mathbf{g}}_{m_k}=\sqrt{(\sigma_{g_k}^2K_{R_k})/(K_{R_k}+1)}\mathbf{g}_{m_k}$ and $\overline{\mathbf{R}}_{g_k}=\sigma_{g_k}^2/(K_{R_k}+1)\mathbf{R}_{g_k}$ for $k=1,\cdots,2K$ from (2) in (32) we observe that, as the Rician factor $K_{R_k}\to\infty$ the error covariance matrix becomes zero and therefore the channel is perfectly estimated.

We next state a corollary where we analyze the NMSE for high pilot power P_p.

Corollary 2. Using (31) and (32), with $P_p\to\infty$ the NMSE_{g_k} is given by

$$\lim_{P_p\to\infty}\mathrm{NMSE}_{g_k}=\frac{\mathrm{Tr}(\overline{\mathbf{R}}_{g_k}-\tau_{p_1}\mathbf{A}_a\overline{\mathbf{R}}_{g_k}\widehat{\mathbf{\Psi}}_{g_k}\overline{\mathbf{R}}_{g_k}\mathbf{A}_a)}{\mathrm{Tr}(\overline{\mathbf{g}}_{m_k}\overline{\mathbf{g}}_{m_k}^H+\overline{\mathbf{R}}_{g_k})}, \quad (33)$$

$$\text{where } \widehat{\mathbf{\Psi}}_{g_k}=\Big(\tau_{p_1}\mathbf{A}_a\overline{\mathbf{R}}_{g_k}\mathbf{A}_a+\sum_{i=1}^{2K}\kappa_{tu}^2\mathbf{A}_a(\overline{\mathbf{g}}_{mi}\overline{\mathbf{g}}_{mi}^H+\overline{\mathbf{R}}_{g_i})\mathbf{A}_a +\kappa_{rr}^2(1+\kappa_{tu}^2)\mathbf{A}_a\,\mathrm{diag}\Big(\sum_{i=1}^{2K}\big(\overline{\mathbf{g}}_{m_i}\overline{\mathbf{g}}_{m_i}^H+\overline{\mathbf{R}}_{g_i}\big)\Big)\mathbf{A}_a +\mathbf{B}_a\Big((1+\kappa_{tu}^2)(1+\kappa_{rr}^2)\mathrm{diag}\Big(\sum_{i=1}^{2K}\big(\overline{\mathbf{g}}_{m_i}\overline{\mathbf{g}}_{m_i}^H+\overline{\mathbf{R}}_{g_i}\big)\Big)\Big)\Big)^{-1}.$$

We observe from (33) that, as $P_p\to\infty$: i) NMSE goes to zero for ideal hardware i.e., $\kappa_{tu}=\kappa_{rr}=0$ and $\mathbf{A}_a=\mathbf{I}_N$; and ii) NMSE with hardware impairments saturates to a positive error floor given by (33). This shows that perfect channel estimation cannot be achieved with hardware impairments even with high pilot power. This is because, with the increase in the pilot power P_p, along with the desired signal, the receive residual and dynamic resolution ADC impairments at the relay and the transmit residual impairments at the user also increases. We also notice from (33) that the error floor increases with the increase in hardware impairments.

We next investigate the impact of the pilot-length (τ_{p1}) on the error floor derived in Corollary 2 from which we observe that for ideal hardware, error floor is independent of pilot length τ_{p1}. With non-ideal hardware we, however, note from Corollary 2 that the error floor depends on pilot length τ_{p1}. To further investigate the impact of τ_{p1} on the the error floor derived in Corollary 2, we consider a special case with $\mathbf{R}_{g_k}=\sigma_{g_k}^2\mathbf{I}_N$, $K_R=0$ and $\mathbf{A}_a=\alpha_a\mathbf{I}_N$. The error floor in Corollary 2 hence becomes

$$\lim_{P_p\to\infty}\mathrm{NMSE}_{g_k}=\sigma_{g_k}^2\left(1-\frac{a\tau_{p1}}{a\tau_{p1}+b}\right)\mathbf{I}_N, \quad (34)$$

where $a=\alpha_a^2\sigma_{g_k}^2$ and $b=\sum\limits_{i=1}^{2K}\sigma_{g_i}^2(\alpha_a^2(\kappa_{tu}^2+\kappa_{rr}^2(1+\kappa_{tu}^2))+\alpha_a(1-\alpha_a)(1+\kappa_{tu}^2)(1+\kappa_{rr}^2))$. We notice from (34) that with ideal hardware (i.e. $\kappa_{tu}=\kappa_{rr}=0$ and $\alpha_a=1$), the term $b=0$. The error floor, therefore becomes independent of τ_{p1} and goes to zero for all values of τ_{p1}. With non-ideal hardware we, however, have a non-zero positive error floor which is a decreasing function of τ_{p1}. We can therefore decrease the NMSE by increasing τ_{p1}. Similar to theorem 1 we can estimate channel $\mathbf{H}$.

IV. Simulations Results

We now investigate the performance of the LMMSE channel estimator derived in Section III, and study its impact on the NMSE and the ergodic sum SE (defined in (23)) for different hardware impairment levels and Rician factors.

System Settings We model the FD massive MIMO relay system assuming the relay coverage area as a circular disc of radius D m. We assume that the relay is placed at the geometric center of the disc and the users are randomly distributed at distances larger than $d_0=75$m from the relay.

1) LoS Component: We consider that the relay is equipped with an uniform linear array of antennas with half wavelength antenna spacing. The LoS component of the channel vectors $\mathbf{g}_k$ and $\mathbf{h}_k$ is given by (3) and the Rician factor is set to $K_{R_k}=13-0.03d_k$ dB [11], where d_k is the distance between the relay and the kth user.

2) Large scale fading model: The large scale fading coefficients $\sigma_{g_k}^2$ and $\sigma_{h_k}^2$ are modelled as

$\sigma_{g_k}^2=\gamma-10\xi\log_{10}(d_k)+F_{g_k}$ and $\sigma_{h_k}^2=\gamma-10\xi\log_{10}(d_k)+F_{h_k}$.

Here γ is the median channel gain at a reference distance of 1m, ξ is the path loss component and $F_{g_k}(F_{h_k})\sim\mathcal{N}(0,\sigma_{sf}^2)$ is the shadow fading [11].

3) Covariance matrices: We similar to [11], model the channel covariance matrices $\overline{\mathbf{R}}_{g_k}$ and $\overline{\mathbf{R}}_{h_k}$ and the relay transmit and receive loop interference covariance matrices $\mathbf{R}_{RR}$ and $\mathbf{R}_{TR}$ using the uniform local scattering model. The level of spatial correlation is characterized by the angular standard deviation (ASD), which determines the range of deviation from the nominal angle (θ_k). As the ASD increases the system becomes more uncorrelated [11].

4) Other Parameters: We fix the AWGN variances at the relay and the users as $\sigma_{nr}^2=\sigma_{nu}^2=\sigma_n^2=-94$ dBm [11]. We set the relay and user loop interference and the inter-user interference

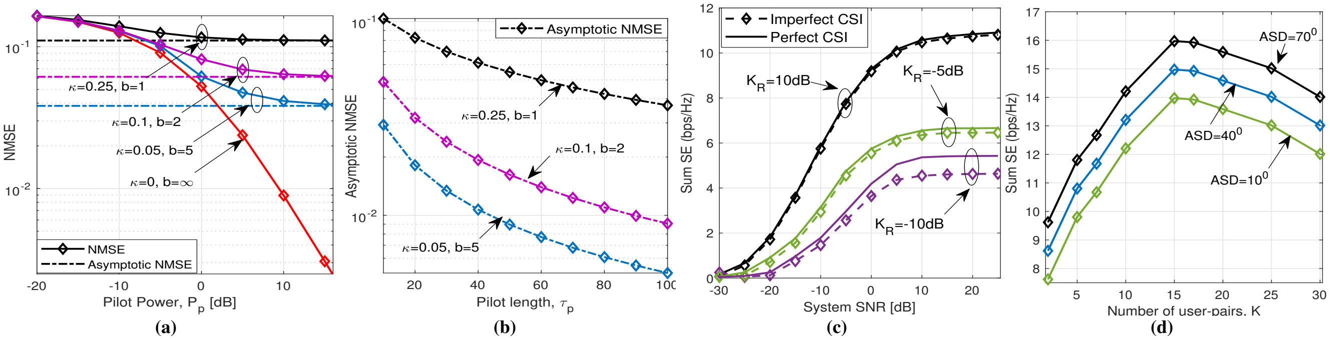

Fig. 2: (a) NMSE vs Pilot power (P_p), (b) NMSE vs Pilot length (τ_p), (c) Ergodic sum-SE vs SNR and (d) Ergodic sum-SE vs K

Table I: System Parameters

Parameter	BW	D	γ	ξ	σ_{sf}
Values	20MHz	0.300km	-30.18	2.6	3dB

as $\sigma_R^2 = \sigma_{k'k'}^2 = 0$ dB and $\sigma_{ik'}^2 = 0$ dB with respect to σ_n^2. The large scale fading coefficients are also normalized with respect to the AWGN variance σ_n^2. For this study we set $\kappa_{rr} = \kappa_{tr} = \kappa_{tu} = \kappa_{ru} = \kappa$ and $\mathbf{A}_a = \mathbf{A}_d = \alpha \mathbf{I}_N$. The exact values of $\alpha = 1 - \rho$ where ρ is the distortion factor corresponding to b-bit ADC/DAC are given in [Table I, [5]]. We define SNR $= (P_R + 2KP_S)/\sigma_n^2$, where we set $P_R = \text{SNR}/2$ and allocate equal power $P_S = \text{SNR}/4K$ to all the users. We set $K = 5$, SNR$= 20$ dB and $N = 64$ and the remaining system parameters are specified in Table I.

Impact on NMSE: We first plot in Fig. 2a, NMSE versus pilot power P_p for different hardware impairment levels, with NMSE $\triangleq \frac{1}{2K}\sum_{k=1}^{2K}\left(\frac{\mathbb{E}\{\|\mathbf{g}_k - \widehat{\mathbf{g}}_k\|^2\}}{\mathbb{E}\{\|\mathbf{g}_k\|^2\}} + \frac{\mathbb{E}\{\|\mathbf{h}_k - \widehat{\mathbf{h}}_k\|^2\}}{\mathbb{E}\{\|\mathbf{h}_k\|^2\}}\right)$. We observe that the NMSE decreases monotonously with increasing pilot power and for ideal hardware the NMSE goes to zero as $P_p \to \infty$. However, with hardware impairments the NMSE saturates to a non-zero error floor as $P_p \to \infty$ (Corollary 2) and its values increase with hardware impairment levels. This is because, as we increase P_p, the receive residual and dynamic resolution ADC impairments at the relay and the transmit residual impairments at the user also increases. We next plot in Fig. 2b, the error floor (derived in Corllary 2) versus the pilot length τ_p and observe that the error floor decreases monotonously with increasing τ_p. This is because the error floor derived in Corollary 2 is a decreasing function of the pilot length τ_p.

Impact on sum-SE: We next plot in Fig. 2d, the ergodic sum-SE versus SNR for $\kappa = 0.1$, $b = 3$ and Rician factor $K_R = \{10, -5, -10\}$ dB. We note that the impact of imperfect CSI on the SE reduces with the increase in Rician factor. For $K_R = 10$ dB the sum-SE of both imperfect CSI and perfect CSI overlaps. This is because for higher K_R values the channel becomes deterministic and the error covariance matrix in (32) approaches zero (as stated in Corollary 1).

We next investigate in Fig. 2c the impact of spatial correlation on the sum-SE by varying the number of user pairs K. We consider the following set of ASDs $\{10°, 40°, 70°\}$ which characterizes the spatial correlation and $\kappa = 0.1$ and $b = 3$. We notice that as ASD decreases (i.e spatial correlation increases) the SE falls for all K values. This is because, with the increase in spatial correlation, both spatial diversity and the channel rank decreases. This significantly lowers the array gain provided by multiple relay antennas. We also observe that the sum-SE initially increases with K, reaches a maximum value and then falls beyond a particular K value. This is because of i) the increase in the inter-user interference and ii) the decrease in power allocated per user.

V. Conclusion

We considered a multi-pair massive MIMO FD relaying system with hardware-impaired relay and users over spatially correlated Rician fading channels. The relay employs a dynamic ADC/DAC architecture, which allows us to investigate the performance for extreme scenarios where each antenna can have different resolution ADC/DAC. With correlated Rician fading channel, we derived a novel MMSE channel estimator considering both dynamic resolution ADCs/DACs and RF impairments.

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