Online Switch-Based Hybrid Beamforming for Massive MIMO Systems

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***Abstract*—Switch-based hybrid beamforming is a low-cost solution for implementing the analog segment of a hybrid beamforming network. Although an analog beamformer comprising a network of switches allows low hardware complexity, designing such a network is computationally expensive. In this paper, we consider a single user massive multiple-input multiple-output (MIMO) system and propose a low computational complexity method for designing a switch-based hybrid precoder that maximizes the mutual information. We propose a method wherein the analog beamformer is approximated after solving a convex (concave) problem and employing low-rank matrix decomposition. Then, considering a sequence of channel realizations we frame the intermediate convex problem as an online convex optimization (OCO) and give the conditions under which the online version approaches the solution of the primary convex problem after some iterations by learning from previous steps. We finally study the performance through numerical results and demonstrate that the proposed online method offers a low complexity solution that tracks the spectral efficiency delivered by fully digital beamformer, and converges to the solution provided by direct maximization of the intermediate problem.**

***Index Terms*—Hybrid beamforming, Precoding, Massive MIMO, Online Convex Optimization.**

I. Introduction

Massive multiple-input multiple-output (MIMO) channels provide higher data rate per user by leveraging spatial diversity and multiplexing multiple data streams across multiple sub-channels [1]. Exploiting multiple parallel spatial sub-channels is accomplished by utilizing linear precoding and combining, or in a general sense, beamforming at the transmit and receive sides and capitalizing on the sparsity of the channel. Given the ever-increasing data-rate demand, massive MIMO networks utilizing spatial beamforming are pivotal components of future wireless communications networks [2], [3].

The implementation of the precoding and combining filters is achieved by multiplying the input symbol vector by a set of complex weights and sending (receiving) it through the transmit (receive) array of antennae via a set of dedicated radio frequency (RF) chains, digital to analog converters (DACs), and analog to digital converters (ADCs). Since antenna deployment is normally cheaper than that of a fully digital beamforming network, intermediate analog beamforming networks provide an affordable mapping from a lower dimension digital beamforming space to a higher dimension antenna space [4]. While hybrid networks are effective for scaling up wireless networks, enabling such beamforming and designing both baseband and RF beamformers is a complex problem [5].

Switch-based hybrid beamforming is a low complexity implementation for the analog beamformer that comprises only analog RF-switches [6], [7]. Employing RF switches, simplifies the RF chain design and offers very fast adjustment to rapid channel variations. However, due to the binary nature of switch-based beamforming matrices, the hybrid beamforming problem, which is already intractable, becomes even more challenging [8]. One way to take advantage of the low complexity hardware implementation and maintain low complexity computations is achieved by leveraging learning and casting the problem as an adaptation to the radio channel through consecutive trials. Learning-based hybrid beamforming design with phase-shifters has been proposed as supervised learning employing a deep neural network [9]. In this work, we propose unsupervised learning based on online convex optimization (OCO) for a switch-based hybrid beamforming design. OCO offers a learning strategy that adapts to the channel in an online fashion with known performance bounds.

OCO is an effective learning tool for convex problems [10]. In an OCO framework, the domain of the problem is represented as a convex decision set. In other words, each feasible point is a potential decision that can be made by the algorithm. After making a decision, the algorithm incurs a loss or regret value and the ultimate goal is to minimize the total loss after a given number of decision-making trials. In this context, we present the switch-based hybrid beamforming problem as a player and the communications channel as an adversary. In each step, the OCO algorithm makes a decision and a new channel realization is revealed along with its regret value. We propose a procedure that offers a sub-optimal solution for the switch-based design problem by solving a log-det maximization over a convex set, and low-rank matrix decomposition subsequently. We then model the intermediate log-det maximization as an online gradient ascent problem [11], and investigate the learning performance by extracting the regret bound. Finally, we show that the proposed method allows the switching network to be adjusted to a varying massive MIMO channel and converges to the intermediate

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maximization while enjoying low computational complexity.

The paper is organized as follows. Te system model and problem formulation are given in section II. In Section III, we describe the new method for switch-based hybrid beamforming design and then propose in Section IV an OCO algorithm to solve the problem. In Section V, we validate the effectiveness of the proposed method via numerical examples and give some conclusions in Section V.

II. Problem Formulation

The problem formulation below follows from that in [6]. A switch-based hybrid beamforming architecture for applying transmit precoding in a single-user mmWave MIMO system is depicted in Fig. 1. The receive side mirrors that of the transmit side, where the roles of combiners and splitters are interchanged and power amplifiers (PAs) replaced by low-noise amplifiers (LNAs). The transmitter comprises N_t antennas and L_t RF transmit chains, and sends N_s data streams to the receiver, where $N_\mathrm{s} \leq L_\mathrm{t} \leq N_\mathrm{t}$. The transmit digital beamforming matrix, $\mathbf{F}_\mathrm{BB}$, is of size $L_\mathrm{t} \times N_\mathrm{s}$, and the RF precoder matrix, $\mathbf{F}_\mathrm{RF}$, of size $N_\mathrm{t} \times L_\mathrm{t}$. RF switches or analog phase-shifters are used to implement $\mathbf{F}_\mathrm{RF}$. The transmit signal in discrete-time is then $\mathbf{x} = \mathbf{F}\mathbf{s}$, where $\mathbf{F} = \mathbf{F}_\mathrm{RF}\mathbf{F}_\mathrm{BB}$, $\mathbf{s}$ is the $N_\mathrm{s} \times 1$ symbol vector, such that $\mathbb{E}[\mathbf{s}\mathbf{s}^H] = \frac{1}{N_\mathrm{s}}\mathbf{I}_{N_\mathrm{s}}$ with $\mathbb{E}$ being the expected value, and $\|\mathbf{F}_\mathrm{RF}\mathbf{F}_\mathrm{BB}\|_F^2 = N_\mathrm{s}$. At the receiver, N_r antennas are connected to L_r RF receive chains to recover the transmit symbol $\mathbf{s}$. The receive beamformer $\mathbf{W} = \mathbf{W}_\mathrm{RF}\mathbf{W}_\mathrm{BB}$ is composed of the $N_\mathrm{r} \times L_\mathrm{r}$ RF combining matrix $\mathbf{W}_\mathrm{RF}$ and $L_\mathrm{r} \times N_\mathrm{s}$ baseband beamforming matrix $\mathbf{W}_\mathrm{BB}$.

Introducing a $N_\mathrm{r} \times N_\mathrm{t}$ channel matrix $\mathbf{H}$ representing the narrowband frequency-flat channel model, with $\mathbb{E}\left[\|\mathbf{H}\|_F^2\right] = N_\mathrm{t}N_\mathrm{r}$, the received signal becomes

$$\mathbf{y} = \sqrt{\rho}\mathbf{W}_\mathrm{BB}^H\mathbf{W}_\mathrm{RF}^H\mathbf{H}\mathbf{F}_\mathrm{RF}\mathbf{F}_\mathrm{BB}\mathbf{s} + \mathbf{W}_\mathrm{BB}^H\mathbf{W}_\mathrm{RF}^H\mathbf{n},$$

where ρ is average receive power, and $\mathbf{n}$ additive i.i.d. noise with zero-mean and variance σ_n^2. Also, $\mathbf{W}_\mathrm{BB}^H$ denotes the complex conjugate transpose of $\mathbf{W}_\mathrm{BB}$.

We divide the transmit power equally among all the data streams, so that the mutual information is

$$\mathcal{I} = \log_2\left(\left|\mathbf{I}_{N_\mathrm{s}} + \frac{\rho}{N_\mathrm{s}}\mathbf{R}_n^{-1}\mathbf{W}_\mathrm{BB}^H\mathbf{W}_\mathrm{RF}^H\mathbf{H}\mathbf{F}_\mathrm{RF}\mathbf{F}_\mathrm{BB} \times \mathbf{F}_\mathrm{BB}^H\mathbf{F}_\mathrm{RF}^H\mathbf{H}^H\mathbf{W}_\mathrm{RF}\mathbf{W}_\mathrm{BB}\right|\right), \quad (1)$$

where $\mathbf{R}_n = \sigma^2\mathbf{W}_\mathrm{BB}^H\mathbf{W}_\mathrm{RF}^H\mathbf{W}_\mathrm{RF}\mathbf{W}_\mathrm{BB}$ is the covariance matrix of the receiver noise. The optimal beamformer is given by the precoding and combining matrices ($\mathbf{F}_\mathrm{BB}, \mathbf{F}_\mathrm{RF}, \mathbf{W}_\mathrm{BB}, \mathbf{W}_\mathrm{RF}$) that maximise the mutual information. However, such joint non-convex optimization is intractable. Thus, we decompose the design into separate transmit and receive subproblems [12], with the transmit-side mutual information being,

$$\mathcal{I} = \log_2\left(\left|\mathbf{I}_{N_\mathrm{s}} + \frac{\rho}{N_\mathrm{s}}\mathbf{H}\mathbf{F}_\mathrm{RF}\mathbf{F}_\mathrm{BB}\mathbf{F}_\mathrm{BB}^H\mathbf{F}_\mathrm{RF}^H\mathbf{H}^H\right|\right). \quad (2)$$

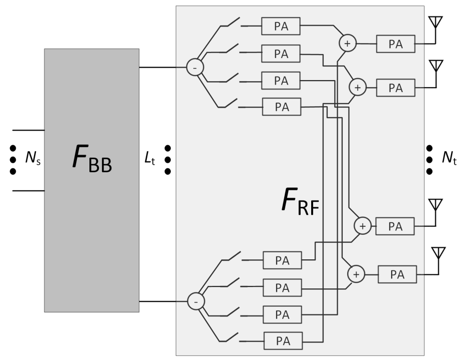

Fig. 1. Simplified architecture for Hybrid MIMO beamforming with analog switches, combiners, and splitters.

The analog precoder and combining matrices, $\mathbf{F}_\mathrm{RF}$ and $\mathbf{W}_\mathrm{RF}$, are generally implemented by using analog phase shifters or analog switches and RF combiners/splitters. In this work, we consider the switch-based implementation.

III. Switch Based Hybrid Precoder Design

Let us consider the singular value decomposition (SVD) of the channel $\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H$ such that $\mathbf{U}$ is a $N_\mathrm{r} \times \mathrm{rank}(\mathbf{H})$ unitary matrix, $\mathbf{\Sigma}$ is a $\mathrm{rank}(\mathbf{H}) \times \mathrm{rank}(\mathbf{H})$ diagonal matrix of descending singular values, and $\mathbf{V}$ is a $N_\mathrm{t} \times \mathrm{rank}(\mathbf{H})$ unitary matrix. Then, the first N_s right singular vectors, $\mathbf{V}_{N_\mathrm{s}}$, provide the unconstrained optimum precoder $\mathbf{F}_\mathrm{opt} = \mathbf{V}_{N_\mathrm{s}}$.

In order to decouple the design of the analog, and digital beamformers, we first embed $\mathbf{F}_\mathrm{RF}$ into the channel by defining a new virtual channel matrix $\tilde{\mathbf{H}} = \mathbf{H}\mathbf{F}_\mathrm{RF}$ of size $N_\mathrm{r} \times L_\mathrm{t}$. Using SVD, we write the new channel as $\tilde{\mathbf{H}} = \tilde{\mathbf{U}}\tilde{\mathbf{\Sigma}}\tilde{\mathbf{V}}^H$, and proceed to maximize (2) by putting $\mathbf{F}_\mathrm{BB} = \tilde{\mathbf{V}}_{N_\mathrm{s}}$, yielding

$$\mathcal{I} = \log_2\left(\left|\mathbf{I} + \frac{\rho}{N_\mathrm{s}\sigma^2}\tilde{\mathbf{H}}_{N_\mathrm{s}}\tilde{\mathbf{V}}_{N_\mathrm{s}}\tilde{\mathbf{V}}_{N_\mathrm{s}}^H\tilde{\mathbf{H}}_{N_\mathrm{s}}^H\right|\right) \quad (3)$$

where $\tilde{\mathbf{V}}_{N_\mathrm{s}}$ is the first N_s right singular vectors, and $\tilde{\mathbf{H}}_{N_\mathrm{s}}$ denotes the new virtual channel representation comprising the first N_s singular values, e.g., $\tilde{\mathbf{H}}_{N_\mathrm{s}} = \tilde{\mathbf{U}}_{N_\mathrm{s}}\tilde{\mathbf{\Sigma}}_{N_\mathrm{s}}\tilde{\mathbf{V}}_{N_\mathrm{s}}^H$. It is important to note that the digital beamformer $\mathbf{F}_\mathrm{BB}$, can realize $\tilde{\mathbf{V}}_{N_\mathrm{s}}$ accurately. Now, in order to find the best $\mathbf{F}_\mathrm{RF}$, we restrict the rank of $\tilde{\mathbf{H}}$ as $\mathrm{rank}(\tilde{\mathbf{H}}) = N_\mathrm{s}$. Therefore, we have that $\tilde{\mathbf{H}} = \tilde{\mathbf{H}}_{N_\mathrm{s}}$, and we can specify a maximization problem as

$$\max_{\mathbf{F}_\mathrm{RF}} \ \log_2\left(\left|\mathbf{I}_{N_\mathrm{s}} + \frac{\rho}{N_\mathrm{s}\sigma^2}\mathbf{H}\mathbf{F}_\mathrm{RF}\mathbf{F}_\mathrm{RF}^H\mathbf{H}^H\right|\right)$$

$$\text{s.t} \ [\mathbf{F}_\mathrm{RF}]_{i,j} \in \{0,1\} \quad (4a)$$

$$\mathrm{rank}(\mathbf{H}\mathbf{F}_\mathrm{RF}) = N_\mathrm{s} \quad (4b)$$

$$\|\mathbf{F}_\mathrm{RF}\mathbf{F}_\mathrm{BB}\|^2 = N_\mathrm{s}. \quad (4c)$$

Although the maximization of log-det function is a concave problem, the multiplication of variables in the objective function, and the constraints make the problem non-convex. Thus, we lift the norm constraint in (4c), and after finding $\mathbf{F}_\mathrm{RF}$, we update $\mathbf{F}_\mathrm{BB}$ such that this constraint is met, by using the QR-based update method described in [8]. In order to tackle the remaining sources of non-convexity, we introduce a new matrix variable, $\mathcal{F}_{\mathrm{RF}}$, such that $\mathcal{F}_{\mathrm{RF}} = \mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{RF}}^H$. Due to the binary constraint (4a), we have $[\mathcal{F}_{\mathrm{RF}}]_{i,j} \in \{0, 1, \ldots, N_{\mathrm{s}}\}$. Now relaxing constraint (4b) to $\mathrm{rank}(\mathbf{H}\mathbf{F}_{\mathrm{RF}}) \leq N_{\mathrm{s}}$ and noting that $\mathrm{rank}(\mathbf{H}) = \min(N_{\mathrm{r}}, N_{\mathrm{t}})$, the only condition that we need to meet is $\mathrm{rank}(\mathbf{F}_{\mathrm{RF}}) = N_{\mathrm{s}}$, and hence $\mathrm{rank}(\mathcal{F}_{\mathrm{RF}}) = N_{\mathrm{s}}$. We employ the trace function, denoted by $\mathrm{Tr}(\cdot)$, as a surrogate for the rank constraint. Finally by relaxing the integer constraint on $[\mathcal{F}_{\mathrm{RF}}]_{i,j}$, the relaxed problem becomes

$$\begin{aligned} \max_{\mathcal{F}_{\mathrm{RF}}} \quad & \log_2 \left(\left| \mathbf{I}_{N_{\mathrm{s}}} + \frac{\rho}{N_{\mathrm{s}}\sigma^2} \mathbf{H}\mathcal{F}_{\mathrm{RF}}\mathbf{H}^H \right| \right) \\ \text{s.t} \quad & 0 \leq [\mathcal{F}_{\mathrm{RF}}]_{i,j} \leq N_{\mathrm{s}} \quad i, j = 1, \ldots, N_{\mathrm{t}} \qquad (5a) \\ & \mathrm{Tr}(\mathcal{F}_{\mathrm{RF}}) \leq N_{\mathrm{t}}(N_{\mathrm{s}} - 1), \qquad (5b) \end{aligned}$$

where in (5b), we chose the upper-bound by noting that if $\mathrm{Tr}(\mathcal{F}_{\mathrm{RF}}) = N_{\mathrm{t}}N_{\mathrm{s}}$, then $[\mathcal{F}_{\mathrm{RF}}]_{i,i} = N_{\mathrm{s}}, i = 1, ..., N_{\mathrm{t}}$. Given that $\mathcal{F}_{\mathrm{RF}} = \mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{RF}}^H$, the latter condition may lead to a decomposition such that $[\mathbf{F}_{\mathrm{RF}}]_{i,j} = 1, i(j) = 1, ..., N_{\mathrm{t}}(N_{\mathrm{s}})$ and $\mathrm{rank}(\mathbf{F}_{\mathrm{RF}}) = 1$. Therefore, when $N_{\mathrm{t}} >> N_{\mathrm{s}}$, (5b) ensures $\mathbf{F}_{\mathrm{RF}}$ is not singular. After solving (5), we employ a low-rank alternating norm minimization procedure [13] in order to decompose $\mathcal{F}_{\mathrm{RF}}$ and find $\mathbf{F}_{\mathrm{RF}}$. It is worth noting that the log-det maximization in (5) can be solved efficiently when approximated by barrier-generated path-following interior-point method, and a problem with $n = N_{\mathrm{t}}^2$ variables and $m = N_{\mathrm{t}}(N_{\mathrm{t}} + 1)$ constraints has complexity of $\mathcal{O}(m^{2.5}(n^2 + m))$ [14]. We approximate this complexity by $\mathcal{O}(N_{\mathrm{t}}^9)$.

IV. Online Convex Optimization

In switch-based hybrid beamforming we aim to devise a low-complexity precoder (combiner) that adapts to the eigen-channels. The channel, however, is typically represented by a non-deterministic matrix that changes rapidly in time. Therefore, the beamforming problem needs to be solved for each new channel realization. OCO is an effective framework to take advantage of learning for convex problems with input parameters that appear in an online fashion [10]. Inspired by game theory, a decision is made in each OCO iteration and a subsequent loss is incurred. The main reason why online learning can be incorporated into convex optimization problems is that in the OCO models the set of possible decisions, which we denote by $\mathcal{K}$, as a convex set in Euclidean space. In each step t, the OCO player picks $\mathbf{x}_t \in \mathcal{K}$. Let $f_t(\mathbf{x}_t)$ be the cost associated with this choice, where $f_t \in \mathcal{S}$ and $\mathcal{S}$ is the set of convex functions. Then a regret (loss) value is expressed in terms of the difference between the cost of the chosen decision and that of the best fixed decision in hindsight, i.e., $\min_{\mathbf{x} \in \mathcal{K}} f_t(\mathbf{x})$. The objective of an OCO algorithm is to minimize the total incurred regret after T iterations:

$$\mathrm{regret}_T := \sum_{t=1}^{T} f_t(\mathbf{x}_t) - \min_{\mathbf{x} \in \mathcal{K}} \sum_{t=1}^{T} f_t(\mathbf{x}). \qquad (6)$$

One fundamental algorithm that effectively minimizes the regret is online gradient descent [11]. In each iteration the algorithm takes a gradient step towards the solution. Since the gradient step may land outside the convex decision set, $\mathcal{K}$, the algorithm applies a projection to navigate it to the closest point inside $\mathcal{K}$. If

$$\forall \mathbf{x}, \mathbf{y} \in \mathcal{K}, \|\mathbf{x} - \mathbf{y}\| \leq D \qquad (7)$$

$$\text{and } \|\nabla f_t(\mathbf{x})\| \leq G, \qquad (8)$$

with D and G denoting upper-bounds on the diameter of $\mathcal{K}$ and the norm of gradient vector respectively, it has been shown that an online gradient descent algorithm with step sizes $\left\{\eta_t = \frac{D}{G\sqrt{t}}, t \in [T]\right\}$ guarantees the following bound on the total regret after T steps:

$$\mathrm{regret}_T \leq \frac{3}{2} GD\sqrt{T}. \qquad (9)$$

Let $\mathcal{F}_{\mathrm{RF}}$ be the decision variable, and define the convex decision set

$$\mathcal{K} = \{\mathcal{F}_{\mathrm{RF}} \mid \mathcal{F}_{\mathrm{RF}} \in \mathbb{R},\ 0 \leq [\mathcal{F}_{\mathrm{RF}}]_{i,j} \leq N_{\mathrm{s}}\ i, j = 1, \ldots, N_{\mathrm{t}}\}.$$

Specifying the set of convex (concave) functions

$$\mathcal{S} = \left\{ f_t \mid f_t(\mathcal{F}_{\mathrm{RF}}) = \log_2 \left(\left| \mathbf{I}_{N_{\mathrm{s}}} + \frac{\rho}{N_{\mathrm{s}}\sigma^2} \mathbf{H}_t\mathcal{F}_{\mathrm{RF}}\mathbf{H}_t^H \right| \right) \right\},$$

we can employ OCO to adapt a low-complexity switch-based hybrid beamformer to the time-varying channel. In particular, for online gradient ascent, the gradient of $f_t(\mathcal{F}_{\mathrm{RF}})$ is

$$\nabla f_t(\mathcal{F}_{\mathrm{RF}}) = \mathbf{H}_t \left(\mathbf{I}_{N_{\mathrm{s}}} + \frac{\rho}{N_{\mathrm{s}}\sigma^2} \mathbf{H}_t\mathcal{F}_{\mathrm{RF}}\mathbf{H}_t^H \right)^{(-1)} \mathbf{H}_t^H. \qquad (10)$$

An upper-bound on the decision set diameter is given by

$$D = \|\mathcal{F}_{\mathrm{RF}}\| \ \text{ s.t. } \ [\mathcal{F}_{\mathrm{RF}}]_{i,j} = N_{\mathrm{s}}\ i, j = 1, \ldots, N_{\mathrm{t}}$$

while an upper-bound on the norm of the gradient vector is

$$\begin{aligned} G &= \|\nabla f_t(\mathcal{F}_{\mathrm{RF}})\| \ \text{ s.t. } \ [\mathcal{F}_{\mathrm{RF}}]_{i,j} = 0\ i, j = 1, \ldots, N_{\mathrm{t}} \\ &= \|\mathbf{H}_t\mathbf{H}_t^H\| = \sqrt{\sum_{i=1}^{N_{\mathrm{r}}} \lambda_i^2(\mathbf{H}_t\mathbf{H}_t^H)}, \qquad (11) \end{aligned}$$

where $\lambda_i(\mathbf{H}_t\mathbf{H}_t^H)$ denotes the i-th eigenvalue of $\mathbf{H}_t\mathbf{H}_t^H$.

The proposed online switch-based hybrid design procedure is outlined in Algorithm 1. This algorithm comprises four main steps: (1) an online gradient step, (2) decomposition by alternating minimization, (3) rounding $\mathbf{F}_{\mathrm{RF}}$, and (4) updating $\mathbf{F}_{\mathrm{BB}}$. The online gradient ascent which is equivalent to (5) has complexity of $\mathcal{O}(n^3)$ (where $n = N_{\mathrm{t}}$) for computing matrix inversion. Also, implementing the projection by linear programming (LP) requires a complexity of $\mathcal{O}(mn^2)$ for $n = N_{\mathrm{t}}^2$ variables and $m = N_{\mathrm{t}}(N_{\mathrm{t}} + 1)$ constraints. Thus, the total complexity is found to be $\mathcal{O}(N_{\mathrm{t}}^6)$.

V. Numerical Results

In this section, we evaluate the performance of the proposed method through simulations. To this end, we adopt a clustered channel model and maximize the spectral efficiency by maximizing the mutual information at the transmit side. The clustered channel model is

$$\mathbf{H} = \gamma \sum_{i,\ell} \alpha_{i\ell} \Lambda_{\mathrm{r}}(\phi_{i\ell}^{\mathrm{r}}, \theta_{i\ell}^{\mathrm{r}}) \Lambda_{\mathrm{t}}(\phi_{i\ell}^{\mathrm{t}}, \theta_{i\ell}^{\mathrm{t}}) \mathbf{a}_{\mathrm{r}}(\phi_{i\ell}^{\mathrm{r}}, \theta_{i\ell}^{\mathrm{r}}) \mathbf{a}_{\mathrm{t}}(\phi_{i\ell}^{\mathrm{t}}, \theta_{i\ell}^{\mathrm{t}})^*,$$

Algorithm 1 Switch-based Hybrid Design by Online Convex Optimization (SHD-OCO)

1: Input: $\mathcal{K}, \mathcal{F}_{\mathrm{RF}}^{(1)} \in \mathcal{K}, \eta_t$
2: **for** $t = 1, ..., T$ **do**
3: $\quad \mathcal{G}^{(t+1)} = \mathcal{F}_{\mathrm{RF}}^{(t)} + \eta_t \nabla f_t(\mathcal{F}_{\mathrm{RF}}^{(t)})$
4: $\quad \mathcal{F}_{\mathrm{RF}}^{(t+1)} = \min_{\mathcal{F}_{\mathrm{RF}}^{(t+1)}} \|\mathcal{F}_{\mathrm{RF}}^{(t+1)} - \mathcal{G}^{(t+1)}\|_F^2$
5: $\quad$ Decompose $\mathcal{F}_{\mathrm{RF}}^{(t+1)}$ as $\mathcal{F}_{\mathrm{RF}}^{(t)} = \mathbf{F}_{\mathrm{RF}}^{(t)} \mathbf{F}_{\mathrm{RF}}^{(t)^H}$ using alternating minimization:
6: $\quad$ Initialize $\tilde{\mathbf{F}}_{\mathrm{RF}}^{(t)}$ at random
7: $\quad$ **for** $l = 1, ..., L$ **do**
8: $\quad\quad \min_{\mathbf{F}_{\mathrm{RF}}^{(t)}} \|\mathcal{F}_{\mathrm{RF}}^{(t)} - \mathbf{F}_{\mathrm{RF}}^{(t)} \tilde{\mathbf{F}}_{\mathrm{RF}}^{(t)^H}\| \quad$ s.t. $0 \le [\mathbf{F}_{\mathrm{RF}}]_{i,j} \le 1$
9: $\quad\quad \min_{\tilde{\mathbf{F}}_{\mathrm{RF}}^{(t)}} \|\mathcal{F}_{\mathrm{RF}}^{(t)} - \mathbf{F}_{\mathrm{RF}}^{(t)} \tilde{\mathbf{F}}_{\mathrm{RF}}^{(t)^H}\| \quad$ s.t. $0 \le [\tilde{\mathbf{F}}_{\mathrm{RF}}]_{i,j} \le 1$
10: $\quad$ **end for**
11: $\quad$ Round $\mathbf{F}_{\mathrm{RF}}^{(t)}$
12: $\quad$ Update $\mathbf{F}_{\mathrm{BB}}^{(t)}$
13: **end for**

where $\gamma = \sqrt{\frac{N_{\mathrm{t}} N_{\mathrm{r}}}{N_{\mathrm{cl}} N_{\mathrm{ray}}}}$ is a normalization factor and $\alpha_{i\ell}$ is the complex amplitude attributed to the ℓ-th ray in the i-th cluster. The antenna gains at the direction of departure (DoD) azimuth and elevation angles $(\phi_{i\ell}^{\mathrm{t}}, \theta_{i\ell}^{\mathrm{t}})$, and direction of arrival (DoA)$(\phi_{i\ell}^{\mathrm{r}}, \theta_{i\ell}^{\mathrm{r}})$, are denoted by $\Lambda_{\mathrm{r}}(\phi_{i\ell}^{\mathrm{r}}, \theta_{i\ell}^{\mathrm{r}})$, and $\Lambda_{\mathrm{t}}(\phi_{i\ell}^{\mathrm{r}}, \theta_{i\ell}^{\mathrm{r}})$, respectively. The DoDs and DoAs of the scatterers are assumed to be randomly distributed with a Laplacian distribution. The vectors, $\mathbf{a}_{\mathrm{r}}(\phi_{i\ell}^{\mathrm{r}}, \theta_{i\ell}^{\mathrm{r}})$ and $\mathbf{a}_{\mathrm{t}}(\phi_{i\ell}^{\mathrm{r}}, \theta_{i\ell}^{\mathrm{r}})$ are respectively the receive and transmit array steering vectors. We employ a model with $N_{\mathrm{cl}} = 8$ clusters and $N_{\mathrm{ray}} = 10$ rays in each cluster. The complex amplitudes of the rays are sampled from a complex normal distribution with an average power of unity in each cluster. The transmit and receive antenna arrays are uniform planar arrays (UPA) with half-wavelength inter-element spacing. We assume an angluar sector of 60° in azimuth and 30° in elevation at the transmit side, while at the receive side we use omni-directional antennas.

We consider a 25×9 UPA MIMO system ($k_{\mathrm{t}} = 25$ and $k_{\mathrm{r}} = 9$) transmitting $N_{\mathrm{s}} = 4$ streams using $L_{\mathrm{t}} = L_{\mathrm{r}} = 4$ RF chains. Assuming an ideal combiner at the receiver, we examine a scenario comprising $T = 1000$ consecutive steps and employ the SHD-OCO method (Algorithm 1) to design a switch-based precoder. In each step the channel realization is generated randomly, e.g., a convex cost function is drawn by the adversary. As the algorithm involves random initiation, and heuristic approaches for relaxation and decomposition, we average the performance of 300 Monte Carlo (MC) runs. It is worth noting that we generate the channel realizations once, and in each experiment the objective is to adapt to this model.

We first investigate the performance of the OCO method when maximizing (5) and show the results in Fig. 2. We employ a direct log-det maximization using CVX package [15], and compare it with that of OCO (i.e. the value of f_t after projection at line 4 of Algorithm 1). For clarity of presentation,

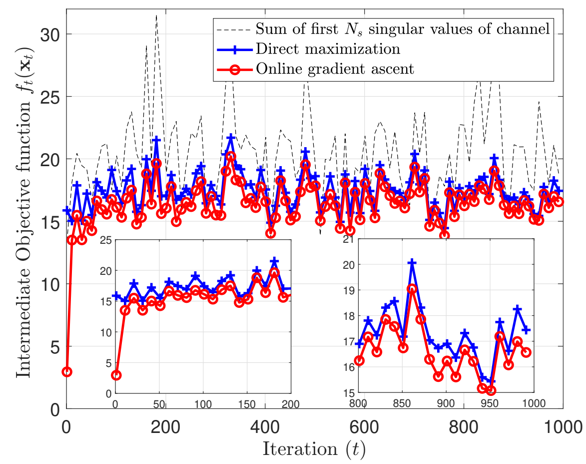

Fig. 2. Maximum value of (5) achieved by direct maximization and online gradient ascent.

we plot the curves for every 10 steps. Also, to show the channel variations we plot the sum of the first N_s singular values of the channel. Fig. 2 illustrates that the presented online gradient ascent maximizes the objective function with only a small gap to that achieved by direct maximization but with lower complexity ($\mathcal{O}(N_{\mathrm{t}}^6)$ versus $\mathcal{O}(N_{\mathrm{t}}^9)$).

In Fig. 3, we study the performance of the SHD-OCO in terms of the achieved spectral efficiency of the MIMO communications channel. We first determine the spectral efficiency of an unconstrained beamformer comprising only a digital beamformer of size $N_{\mathrm{s}} \times N_{\mathrm{t}}$ and adopt the optimal solution, i.e. the first N_{s} singular vectors of the channel. Then, we design a switch-based system via direct maximization of (5), and follow steps 5 to 12 in Algorithm 1. We call this solution, switch-based hybrid design by direct maximization or SHD-DM. Finally we employ the SHD-OCO algorithm to design the precoding network and maximize the spectral efficiency. It is important to note that the spectral efficiency of the unconstrained beamformer is given as a reference and that the gap between the SHD-DM and optimal is expected since we are using a switch-based hybrid design. Observe that the SHD-OCO initially displays a worse efficieny than SHD-DM. However, the gap between the two approaches decreases as the number of iterations increases and the algorithm learns to make better choices and minimize the loss. The learning performance is further demonstrated in Fig. 4. Given N MC runs, we define two mean-squared errors (MSEs) as

$$\mathrm{MSE}^f = \frac{1}{N} \sum_i^N (f_t^i(\mathcal{F}_{\mathrm{RF}}^{\mathrm{DM}}) - f_t^i(\mathcal{F}_{\mathrm{RF}}^{\mathrm{OCO}}))^2 \tag{12}$$

$$\mathrm{MSE}^{\mathrm{SE}} = \frac{1}{N} \sum_i^N \mathcal{I}_t^i(\mathcal{F}_{\mathrm{RF}}^{\mathrm{SHD-DM}}) - \mathcal{I}_t^i(\mathcal{I}_{\mathrm{RF}}^{\mathrm{SHD-OCO}})^2, \tag{13}$$

to quantify the learning performance in terms of the difference of the values achieved by direct maximization (desired) and OCO (achieved) for the intermediate log-det expression as well

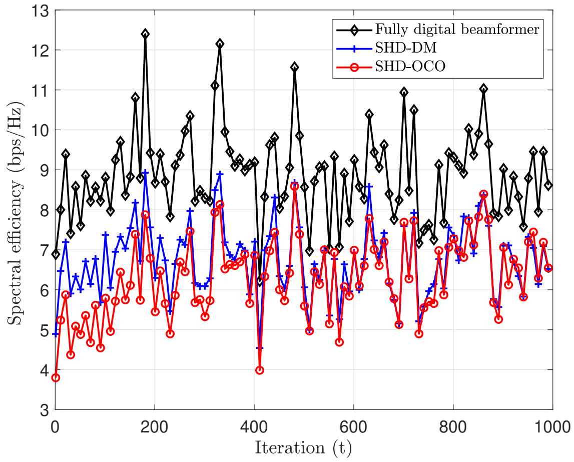

Fig. 3. Spectral efficiency achieved by fully digital beamformer, SHD-DM, and SHD-OCO for a 25 MIMO system.

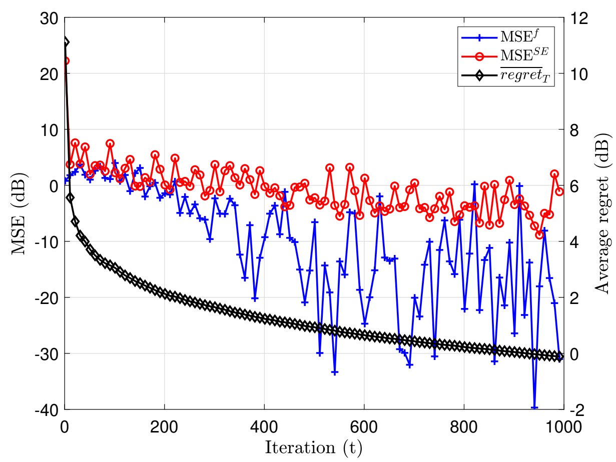

Fig. 4. Left axes: Online learning performance in terms of MSE of maximums of direct maximization and online gradient ascent, as well as that of the spectral efficiencies. Right axes: Online learning performance in terms of average regret.

as the mutual information (spectral efficiency). We also define the average regret as

$$\overline{\text{regret}_T} = \frac{1}{N}\sum_{i}^{N}\frac{\text{regret}_T^i}{T}. \quad (14)$$

As can be seen on the left axes of Fig. 4, the MSEs of both log-det maximums and mutual information decrease as the number of iterations increases and new realizations are revealed, confirming the ability of the SHD-OCO to learn from past experiences in an unsupervised fashion. It is worth noting that MSE^{SE} exhibits more fluctuations due to random initialization of the alternating minimization as well as rounding which is a non-linear operator. The average regret (right axes of Fig. 4) which is the equivalent of convergence in OCO, also decreases and proves that the SHD-OCO, as a player, makes fewer mistakes and minimizes the total regret.

VI. Conclusion

We proposed an online switch-based hybrid design algorithm that offers a low computational complexity procedure for switch-based hybrid beamforming design as a solution, featuring low hardware complexity. We first proposed a log-det maximization in order to approximate a sub-optimal solution for the analog beamformer. We then cast an online method to maximize the proposed convex problem based on online gradient ascent method. We showed that the maximum achieved by the proposed online method approaches that of the direct maximization as the number of trials increases. Also, the maximum spectral efficiency offered by the proposed method converges to the efficiency provided by direct maximization but with lower computational cost.

References

[1] X. Gao, O. Edfors, F. Rusek, and F. Tufvesson, "Massive MIMO performance evaluation based on measured propagation data," *IEEE Trans. Wireless Commun.*, vol. 14, no. 7, pp. 3899–3911, 2015.

[2] G. Interdonato, E. Björnson, H. Q. Ngo, P. Frenger, and E. G. Larsson, "Ubiquitous cell-free massive MIMO communications," *EURASIP J. Wirel. Comm.*, vol. 2019, no. 1, 2019.

[3] S. Han, C. lin I, Z. Xu, and C. Rowell, "Large-scale antenna systems with hybrid analog and digital beamforming for millimeter wave 5G," *IEEE Commun. Mag.*, vol. 53, no. 1, pp. 186–194, 2015.

[4] A. Alkhateeb, O. E. Ayach, G. Leus, and R. W. Heath, "Channel estimation and hybrid precoding for millimeter wave cellular systems," *IEEE J. Sel. Topics Signal Process.*, vol. 8, no. 5, pp. 831–846, 2014.

[5] C. Rusu, R. Mendez-Rial, N. Gonzalez-Prelcic, and R. W. Heath, "Low complexity hybrid precoding strategies for millimeter wave communication systems," *IEEE Trans. Wireless Commun.*, vol. 15, no. 12, pp. 8380–8393, 2016.

[6] H. Nosrati, E. Aboutanios, D. Smith, and X. Wang, "Switch-based hybrid precoding in mmWave massive MIMO systems," in *2019 27th European Signal Processing Conference (EUSIPCO)*, 2019.

[7] R. Mendez-Rial, C. Rusu, A. Alkhateeb, N. Gonzalez-Prelcic, and R. W. Heath, "Channel estimation and hybrid combining for mmWave: Phase shifters or switches?" in *2015 Information Theory and Applications Workshop (ITA)*. IEEE, 2015.

[8] H. Nosrati, E. Aboutanios, X. Wang, and D. Smith, "Switch-based hybrid beamforming for massive mimo communications in mmwave bands," *arXiv:1908.10500 [eess.SP]*, 2019.

[9] H. Huang, J. Yang, H. Huang, Y. Song, and G. Gui, "Deep learning for super-resolution channel estimation and DOA estimation based massive MIMO system," *IEEE Transactions on Vehicular Technology*, vol. 67, no. 9, pp. 8549–8560, sep 2018.

[10] E. Hazan, "Introduction to online convex optimization," *Foundations and Trends® in Optimization*, vol. 2, no. 3-4, pp. 157–325, 2016.

[11] M. Zinkevich, "Online convex programming and generalized infinitesimal gradient ascent," in *Proceedings of the 20th international conference on machine learning (icml-03)*, 2003, pp. 928–936.

[12] O. E. Ayach, S. Rajagopal, S. Abu-Surra, Z. Pi, and R. W. Heath, "Spatially sparse precoding in millimeter wave MIMO systems," *IEEE Trans. Wireless Commun.*, vol. 13, no. 3, pp. 1499–1513, 2014.

[13] P. Jain, P. Netrapalli, and S. Sanghavi, "Low-rank matrix completion using alternating minimization," in *Proceedings of the 45th annual ACM symposium on Symposium on theory of computing - STOC 13*. ACM Press, 2013.

[14] Y. Nesterov and A. Nemirovskii, *Interior-point polynomial algorithms in convex programming*. SIAM, 1994, vol. 13.

[15] M. Grant, S. Boyd, and Y. Ye, "CVX: Matlab software for disciplined convex programming," 2008.