Near-field source localization of quasi-stationary signals with increased degrees of freedom

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***Abstract*—Near-field source localization has attracted much interest in many applications of radar, sonar, speech and seismology. In near-field, both the direction of arrival (DOA) and range are required for the localization. In order to reduce computational burden, a lot of methods perform 1-D estimation procedures based on symmetric array configurations, but with degrees of freedom about the half of the number of sensors. In this paper, we consider the near-field source localization problem with increased degrees of freedom, by using the property of quasi-stationary signals and Khatri-Rao product. In order to reduce the computational burden, we adopt the dimension-reduced technique by splitting the steering vector in terms of DOA and range. The DOAs are estimated through a one-dimensional search and the ranges are obtained with the estimated DOAs without searching or pairing. Simulations are provided to show the effectiveness of the proposed method.**

***Index Terms*—Near-field, source localization, quasi-stationary signal**

I. Introduction

Source localization plays an important role in array signal processing applications, for example, radar, sonar, speech, and seismology [1]. In the far-field of an array, the wavefronts of signals are planar and the information of DOA is enough for source localization. However, when the sources are close to the array, in the near-field, the wavefronts become spherical and both the DOA and range are required for the localization.

Many methods have addressed the issue of near-field source localization problems. Earlier researches such as the maximum likelihood estimator (MLE) method [2] and the two-dimensional multiple signal classification (2-D MUSIC) method [2], require multi-dimensional (M-D) or 2-D search, which is computational expensive. The weighted linear prediction method [3] makes use of several anti-diagonal sequences in the data covariance matrix of the received signals to localize sources in the near-field. This method has reduced computational cost but requires pairing of parameters. In order to reduce complexity, several one-dimensional (1-D) searching methods are proposed to perform the DOA and range estimation separately. The authors in [4] propose a rank reduced (RARE) method in the estimation of DOA, which makes use of the far-field-like rotational invariance property of the signal subspace. In [5], the main anti-diagonal elements of the data covariance matrix are applied in the estimation of DOA, since they only contain the information of DOA. A dimension-reduced method is proposed in [6] by splitting the steering vector in terms of DOA and range, which results in lower complexity. Nevertheless, these 1-D search methods depend on symmetric array configuration and the maximum resolvable signals are limited to half the number of sensors.

In [7], a Khatri-Rao (KR) subspace method with quasi-stationary signals is proposed to estimate more sources than the number of sensors. Quasi-stationary signal is one class of non-stationary signals whose statistics remain stationary over a certain period of time frame, but change from one frame to another. The property of quasi-stationary signals is also exploited in other underdetermined cases with different array configurations, for example, uniform circular array (UCA) [8], coprime [9] and nested [10] array. However, these researches assume far-field signals. In this paper, we propose to use the feature of quasi-stationary signals and KR product to near-field source localization to increase the degrees of freedom. Since the sources are in near-field, the KR product is jointly applied with the split of DOA and range like in [6] to reduce the computational burden.

The reminder of this paper is organized as follows. Section II gives the signal model. Section III shows the proposed method. Section IV shows its performance with several simulations. The conclusions are drawn in Section V.

II. Signal model

Consider K uncorrelated narrow band sources locating in the near-field of a symmetric uniform linear array (ULA) with $N = 2M + 1$ sensors. The sensors are indexed as $-M, \ldots, 0, \ldots, M$, as shown in Fig. 1.

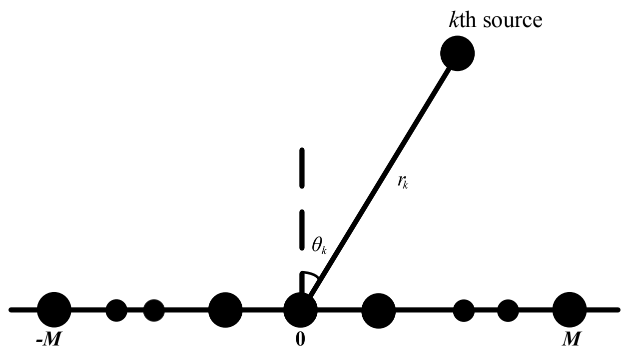

Fig. 1. ULA configuration in near-field source localization.

The source signals are independent quasi-stationary under F time frames and each with a frame length L. The 0th element of the ULA is taken as the reference point. The received signals at the mth sensor under the fth time frame is given by

$$x_m(t) = \sum_{k=1}^{K} s_k(t)e^{j\delta_{mk}} + n_m(t), t \in [(f-1)L+1, fL] \quad (1)$$

where $s_k(t)$ is the kth source signal received at the reference point at time t; $n_m(t)$ is the additive Gaussian noise at the mth sensor with zero mean and variance σ^2 at time t; δ_{mk} is the phase difference of the signals received at the mth sensor and the 0th sensor due to the kth source, which can be written using the second order approximation [4]:

$$\begin{aligned} \delta_{mk} &\approx (-\frac{2\pi d}{\lambda}\sin\theta_k)m + (\frac{\pi d^2}{\lambda r_k}\cos^2\theta_k)m^2 \\ &\approx \omega_k m + \phi_k m^2 \end{aligned} \quad (2)$$

where $\omega_k = -\frac{2\pi d}{\lambda}\sin\theta_k$ and $\phi_k = \frac{\pi d^2}{\lambda r_k}\cos^2\theta_k$; θ_k is the DOA of the kth source, $\theta_k \in [-\frac{\pi}{2}, \frac{\pi}{2}]$; r_k is the range of the kth source, $r_k \in [0.62(D^3/\lambda)^{1/2}, \quad 2D^2/\lambda]$; D is the array aperture; λ is the wavelength; d is the interval between two adjacent sensors and is assumed to be within a quarter-wavelength to avoid phase ambiguity. However, the coupling effect among sensors is not the main focus of this paper, which is not taken into account in the following.

In vector form, (1) can be formulated as

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t), \quad t \in [(f-1)L+1, fL] \quad (3)$$

with

- $\mathbf{x}(t) = [x_{-M}(t), \ldots, x_M(t)]^T$; superscript T stands for transpose operation;
- $\mathbf{s}(t) = [s_1(t), \ldots, s_K(t)]^T$;
- $\mathbf{n}(t) = [n_{-M}(t), \ldots, n_M(t),]^T$;
- $\mathbf{A} = [\mathbf{a}(\theta_1, r_1), \ldots, \mathbf{a}(\theta_K, r_K)]$ is the steering matrix and $\mathbf{a}(\theta_k, r_k) = [e^{-j\omega_k M + j\phi_k M^2}, \ldots, e^{j\omega_k M + j\phi_k M^2}]^T$.

The local covariance matrix under the fth time frame is

$$\mathbf{R}_f = E\{\mathbf{x}(t)\mathbf{x}(t)^H\} = \mathbf{A}\mathbf{S}_f\mathbf{A}^H + \sigma^2\mathbf{I} \quad (4)$$

where $E[.]$ denotes ensemble average; superscript H denotes conjugate transpose operation; $\mathbf{S}_f = E[\mathbf{s}(t)\mathbf{s}^H(t)]$ is the covariance matrix of the source signals under frame f; $\mathbf{I}$ is the $N \times N$ identity matrix.

III. Methodology

In the case of uncorrelated signals, the vectorization of $\mathbf{R}_f$ yields:

$$\text{vec}(\mathbf{R}_f) = (\mathbf{A}^* \odot \mathbf{A})\mathbf{p}_f + \sigma^2\vec{\mathbf{I}} \quad (5)$$

where $\odot$ denotes the Khatri-Rao product; $(\mathbf{A}^* \odot \mathbf{A}) \in \mathbb{C}^{N^2 \times K}$; $\mathbf{p}_f$ is a $K \times 1$ vector which contains the diagonal elements of $\mathbf{S}_f$; $\vec{\mathbf{I}} = \text{vec}(\mathbf{I}) = [\mathbf{e}_1^T, \ldots, \mathbf{e}_N^T]^T$ with $\mathbf{e}_n$ a column vector of all zeros except a 1 at the nth position.

Combining F frames, a new matrix $\mathbf{Y}$ can be written as

$$\begin{aligned} \mathbf{Y} &= [\text{vec}(\mathbf{R}_1), \ldots, \text{vec}(\mathbf{R}_F)] \\ &= (\mathbf{A}^* \odot \mathbf{A})[\mathbf{p}_1, \ldots, \mathbf{p}_F] + \sigma^2\vec{\mathbf{I}}\mathbf{1}_F^T \\ &= (\mathbf{A}^* \odot \mathbf{A})\mathbf{P} + \sigma^2\vec{\mathbf{I}}\mathbf{1}_F^T \end{aligned} \quad (6)$$

where $\mathbf{1}_F$ is an all 1 vector with F elements.

Assume $\begin{bmatrix}\mathbf{P}^T & \mathbf{1}_F\end{bmatrix} \in \mathbb{R}^{F \times (K+1)}$ is of full column rank, which means $F > K + 1$ and the source power distributions over frames are different, the rank of $\mathbf{P}$ is K. On this basis, the unknown noise can be eliminated by using an orthogonal complement projector without damaging the rank condition [7]. Define the projector as $\mathbf{Q}_{\mathbf{1}_F}^{\perp} = \mathbf{I}_F - \frac{1}{F}\mathbf{1}_F\mathbf{1}_F^T$ and the projection can be performed as follows

$$\hat{\mathbf{Y}} = \mathbf{Y}\mathbf{Q}_{\mathbf{1}_F}^{\perp} = (\mathbf{A}^* \odot \mathbf{A})\mathbf{P}\mathbf{Q}_{\mathbf{1}_F}^{\perp}. \quad (7)$$

Given $K < N^2$, the rank of $(\mathbf{A}^* \odot \mathbf{A})$ is K. Therefore, if $K < \min(N^2, F-1)$, the rank of $\hat{\mathbf{Y}}$ would be K at most, which also indicates maximum number of detectable sources by using the proposed method. The singular value decompoistion (SVD) of $\hat{\mathbf{Y}}$ is

$$\hat{\mathbf{Y}} = \begin{bmatrix}\mathbf{U}_s & \mathbf{U}_n\end{bmatrix} \begin{bmatrix}\mathbf{\Phi}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{0}\end{bmatrix} \begin{bmatrix}\mathbf{V}_s^H \\ \mathbf{V}_n^H\end{bmatrix} \quad (8)$$

where $\mathbf{U}_s$ and $\mathbf{V}_s$ denote the left and right singular matrices associated with the K largest non-zero singular values, respectively. $\mathbf{U}_n$ and $\mathbf{V}_n$ denote respectively the left and right singular matrices corresponding to the remaining smaller singular values.

The column vectors of $(\mathbf{A}^* \odot \mathbf{A})$ have the general form $\mathbf{a}^*(\theta, r) \otimes \mathbf{a}(\theta, r)$. Let $\tilde{\mathbf{a}}(\theta, r) = \mathbf{a}^*(\theta, r) \otimes \mathbf{a}(\theta, r)$, we can find the positions of sources by finding the minimum of the MUSIC spectrum as follows:

$$P_{MUSIC}(\theta, r) = \min_{\theta, r} \tilde{\mathbf{a}}^H(\theta, r)\mathbf{U}_n\mathbf{U}_n^H\tilde{\mathbf{a}}(\theta, r). \quad (9)$$

There are two variables θ and r in (9), and we can perform a 2-D search as in [2]. Nevertheless, 2-D search always has a heavy computational burden. The authors in [6] present a reduced dimension method by splitting the steering vector in terms of angle and range:

$$\mathbf{a}(\theta,r)=\underbrace{\begin{bmatrix} e^{j(-M)\omega} & & & & \\ & e^{j(-M+1)\omega} & & & \\ & & \ddots & & \\ & & & 1 & \\ & & \dots & & \\ & e^{j(M-1)\omega} & & & \\ e^{j(M)\omega} & & & & \end{bmatrix}}_{\mathbf{\Gamma}(\omega)}\underbrace{\begin{bmatrix} e^{j(M)^2\phi} \\ e^{j(M-1)^2\phi} \\ \vdots \\ 1 \end{bmatrix}}_{\mathbf{b}(\phi)}. \quad (10)$$

It is obvious that $\mathbf{\Gamma}(\omega)$ only contains the information of DOA while $\mathbf{b}(\phi)$ contains both angle and range. Accordingly, vector $\tilde{\mathbf{a}}(\theta,r)=\mathbf{a}^*(\theta,r)\otimes\mathbf{a}(\theta,r)$ can be rewritten as

$$\begin{aligned}\tilde{\mathbf{a}}(\theta,r)&=(\mathbf{\Gamma}(\omega)\mathbf{b}(\phi))^*\otimes(\mathbf{\Gamma}(\omega)\mathbf{b}(\phi))\\&=\tilde{\mathbf{\Gamma}}(\omega)\tilde{\mathbf{b}}(\phi)\end{aligned} \quad (11)$$

with $\tilde{\mathbf{\Gamma}}(\omega)=\mathbf{\Gamma}^*(\omega)\otimes\mathbf{\Gamma}(\omega)$ and $\tilde{\mathbf{b}}(\phi)=\mathbf{b}^*(\phi)\otimes\mathbf{b}(\phi)$.

Reform the searching spectrum in (9) using (11) as

$$P_{MUSIC}(\theta,r)=\min_{\omega,\phi}\tilde{\mathbf{b}}^H(\phi)\tilde{\mathbf{\Gamma}}^H(\omega)\mathbf{U}_n\mathbf{U}_n^H\tilde{\mathbf{\Gamma}}(\omega)\tilde{\mathbf{b}}(\phi). \quad (12)$$

According to the Rayleigh Quotient Theorem [11], the minimum of the quadratic $P_{MUSIC}(\theta,r)$ in (12) is equal to the smallest eigenvalue $\tau_{min}(\omega)$ of $\tilde{\mathbf{\Gamma}}^H(\omega)\mathbf{U}_n\mathbf{U}_n^H(\omega)\tilde{\mathbf{\Gamma}}(\omega)$, and the associated eigenvector $\tilde{\mathbf{b}}_{min}(\phi)$ can be used to estimate ϕ. In consequence, (12) becomes

$$P_{MUSIC}(\theta,r)=\min_{\omega}\tau_{min}(\omega). \quad (13)$$

In consequence, we simply need a 1-D search for the estimation of DOA, without knowing the range. Given the estimate of angle $\hat{\theta}_k$, we can find the corresponding range of the kth source with its associated eigenvector $\tilde{\mathbf{b}}_{min}(\hat{\phi}_k)$:

$$\tilde{\mathbf{b}}_{min}(\hat{\phi}_k)=\mathbf{b}^*(\hat{\phi}_k)\otimes\mathbf{b}(\hat{\phi}_k)=\begin{bmatrix} 1 \\ e^{j((M-1)^2-M^2)\hat{\phi}_k} \\ \vdots \\ e^{j(-M^2)\hat{\phi}_k} \\ \hline e^{j(M^2-(M-1)^2)\hat{\phi}_k} \\ 1 \\ \vdots \\ e^{j(-(M-1)^2)\hat{\phi}_k} \\ \hline \vdots \\ \hline e^{j(M)^2\hat{\phi}_k} \\ e^{j(M-1)^2\hat{\phi}_k} \\ \vdots \\ 1 \end{bmatrix}. \quad (14)$$

In (14), the unknown variable $\hat{\phi}_k$ only appears in the phase of each element of $\tilde{\mathbf{b}}_{min}(\hat{\phi}_k)$. Therefore, $\angle\tilde{\mathbf{b}}_{min}(\hat{\phi}_k)$ ($\angle$ denotes angle operation) can be viewed as an overdetermined linear system of equations of $\hat{\phi}_k$, which can be solved by using least square approach like in [6]. Each range can be calculated automatically with the estimated DOA, without pairing. The estimates of DOA and range of each source are

$$\hat{\theta}_k=-\text{arcsin}(\frac{\hat{\omega}_k\lambda}{2\pi d}),\quad \hat{r}_k=\frac{\pi d^2}{\lambda\hat{\phi}_k}\cos^2\hat{\theta}_k. \quad (15)$$

It should be noted that the above derivations follow the principles of MUSIC, which can also work with Capon method except that the spectrum in (9) should be reformulated like in [7]

$$P_{Capon}(\theta,r)=\min_{\theta,r}\tilde{\mathbf{a}}^H(\theta,r)(\hat{\mathbf{Y}}\hat{\mathbf{Y}}^H)^{-1}\tilde{\mathbf{a}}(\theta,r). \quad (16)$$

The derivation of (16) and the estimation of parameters by using the Capon method are omitted here since they are obtained in the same way as in [7]. In the following, the proposed methods are respectively called as Proposed method-MUSC and Proposed method-Capon, which can be summarized below:

1) Compute the local covariance matrix $\mathbf{R}_f$ and stack them into matrix $\mathbf{Y}$;
2) Denoise $\mathbf{Y}$ with (7) into $\hat{\mathbf{Y}}$;
3) Perform SVD on $\hat{\mathbf{Y}}$ for the Proposed method-MUSC;
4) Estimate DOA and calculate each range with each obtained DOA.

The computational complexity of the Proposed method-MUSC is $O\{N^2FL+2N^2F^2+N^4(N^2-K)+n_\theta(\frac{N^4}{2}+\frac{7N^6}{16})\}$ while that of Proposed method-Capon is $O\{N^2FL+N^2F^2+N^4F^2+N^6+n_\theta(\frac{N^4}{2}+\frac{7N^6}{16})\}$, respectively. n_θ denotes the searching number.

IV. Simulation

Several numerical simulations are presented to evaluate the performance of the proposed method. Consider a ULA with 9 sensors, that is, $M=4, N=9$. The distance is set as d = $\lambda/5$. In this case, the Fresnel region is [1.25λ, 5.12λ]. The quasi-stationary signals are synthetically generated according to [7] (TABLE II) under Gaussian distribution. The case with 5 sources is considered. The number of signals is larger than half the number of sensors, $K>M$. The locations of these sources are (3.0λ, $-20°$), (5.0λ, $-10°$), (2.0λ, 0°), (4.0λ, 15°), and (3.0λ, 30°), respectively. The frame length in the simulations is fixed as $L=512$ while the number of frames is $F=50$.

In the first simulation, we examine the spectrum of Proposed method-MUSIC when SNR = 15 dB. Fig. 2 and Table I show the estimation of DOA and range, respectively. The peaks in Fig. 2 correspond to the positions of true DOAs. At each estimated DOA, the associated eigenvector is used to calculate the range. According to Table I, the estimates of the ranges are close to the true values of the sources.

In the second simulation, the statistical performance of the proposed methods is evaluated in terms of SNR. Since

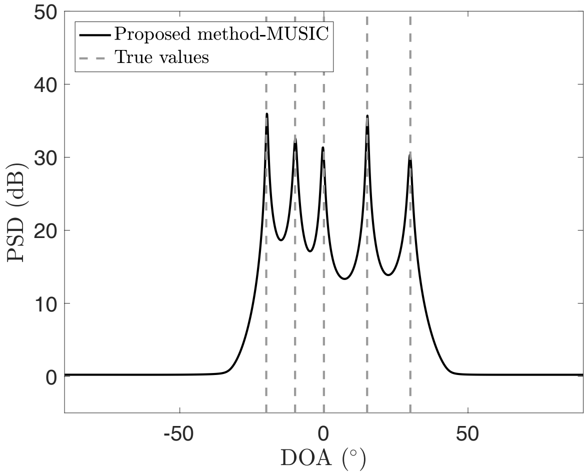

Fig. 2. Spectrum of DOA estimation by using Proposed method-MUSIC with M = 4, K = 5, SNR = 15 dB, L = 512, F = 50.

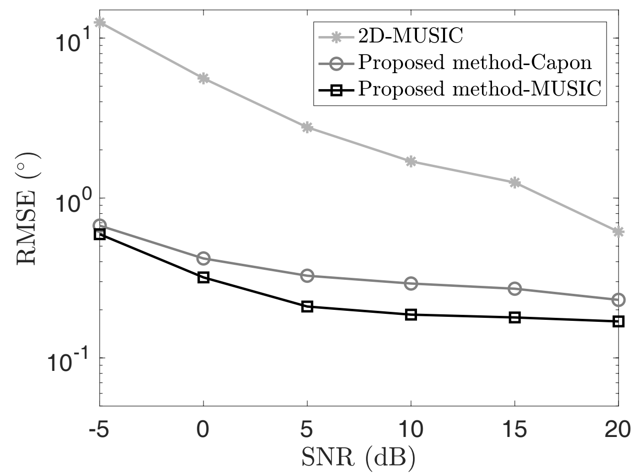

Fig. 3. Statistical performance of the proposed methods and 2D-MUSIC versus SNR, DOA estimation.

TABLE I
RANGE ESTIMATION OF PROPOSED METHOD-MUSIC WITH M = 4, K = 5, SNR = 15 DB, L = 512, F = 50.

True (λ)	3	5	2	4	3
Estimate (λ)	2.82	5.38	2.01	3.72	2.95

the proposed methods have increased freedom, the estimation results of 2D-MUSIC are recorded for comparison. The other settings are the same with the first simulation. The root mean square error (RMSE) is calculated over 200 Monte Carlo trials, which is defined as

$$RMSE = \sqrt{\frac{1}{KU}\sum_{k=1}^{K}\sum_{u=1}^{U}(\hat{\alpha}_{ku} - \alpha_k)^2} \tag{17}$$

where $\hat{\alpha}_{ku}$ is the estimate of α_k at the uth trial; α can either be DOA or range; U is the total number of trials.

Figs. 3 and 4 show the RMSEs of the proposed methods and 2D-MUSIC with respect to SNR. According to Figs. 3 and 4, the RMSE decreases with the increase of SNR, both in the estimation of DOA and range. The RMSEs of 2D-MUSIC are higher than those of the proposed methods while the Proposed method-MUSIC shows the lowest estimation error at each SNR. Besides, the operation time of the Proposed method-MUSIC, Proposed method-Capon and 2D-MUSIC are 0.4383 s, 0.6199s and 2.2895 s respectively, with a computer equipped with a processor unit (CPU) of 2.3 GHz and 8 GB of RAM. As the proposed methods only require 1-D search and the range of each source is estimated automatically, they are more computationally efficient than 2D-MUSIC.

In the third simulation, the performance is evaluated with respect to the number of snapshots. SNR=15 dB. The other settings are the same with the first simulation. Figs. 5 and 6 show the RMSEs for the estimation of DOA and range, respectively. The RMSEs of all the methods decrease as the number of snapshots increases, not only for the estimation of

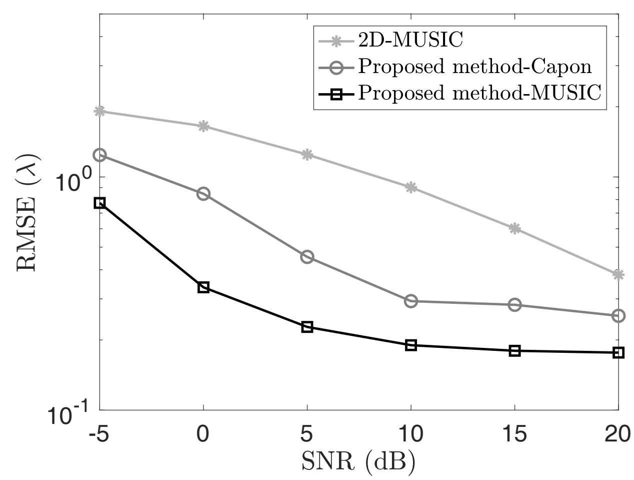

Fig. 4. Statistical performance of the proposed methods and 2D-MUSIC versus SNR, range estimation.

DOA but also for range. Similarly, Proposed method-MUSIC shows the best performance at each number of snapshots, compared with Proposed method-Capon and 2D-MUSIC.

V. CONCLUSION

In this paper, we propose to increase the degrees of freedom of near-field source localization method by exploiting quasi-stationary signal and KR product. Since both DOA and range are required in the localization of near-field sources, we propose to reduce the computational complexity by splitting the steering vector with respect to DOA and range. In consequence, the DOAs can be estimated through a 1-D search according to the Rayleigh Quotient Theorem. While the ranges are automatically calculated with the estimated DOAs and the associated eigenvectors. The proposed method works both with the principles of MUSIC and Capon. Several simulations validate the performance of the proposed method with more

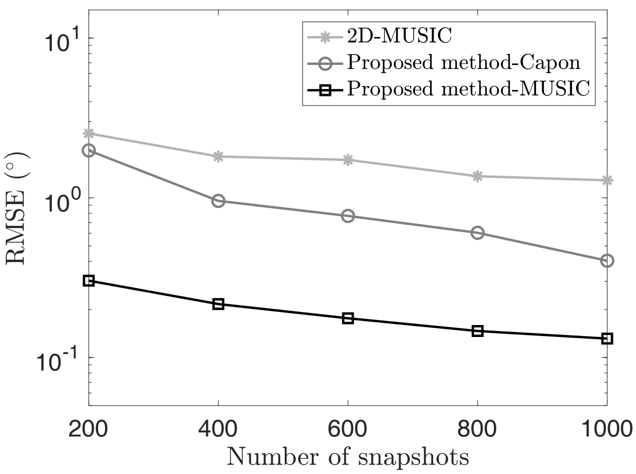

Fig. 5. Statistical performance of the proposed methods and 2D-MUSIC versus number of snapshots, DOA estimation.

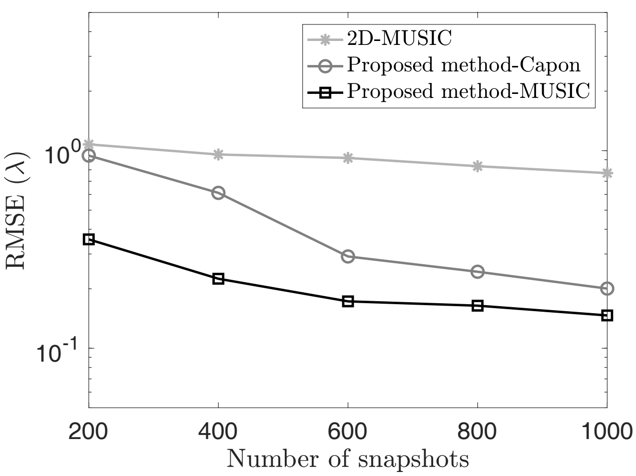

Fig. 6. Statistical performance of the proposed methods and 2D-MUSIC versus number of snapshots, range estimation.

signals than half the number of sensors at different SNRs and different numbers of snapshots. The proposed method with MUSIC principles outperforms that with with Capon and 2D-MUSIC. In the future, we would like to work on correlated or cyclostationary signals.

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