A Geometric Interpretation of Trilateration for RSS-based Localization

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***Abstract*—Trilateration is a popular approach in localization. Many related geometric approaches have been proposed for 2D scenarios. In general, each approach has a standard case in which the main solution is applied, and many specific cases. Each specific case has a particular solution, which makes the algorithm more complex. This paper introduces a novel geometric approach that covers all the cases considered by previous algorithms. It turns out though that this approach is a special case of an existing approach, for which we hence provide a geometric interpretation. Numerical results illustrate the method in RSS-based localization while estimating simultaneously the path loss component.**

Keywords: trilateration, algorithm, localization, RSS-based, path loss estimation

I. Introduction

Nowadays, due to the high demand for determining the position of an object, many positioning systems have been developed, as well as various positioning algorithms have been proposed. Finding a source based on measurements of signals from an array of emitters has been a significant problem [1]. Time of Arrival (TOA), Time Difference of Arrival (TDOA), Angle of Arrival (AOA), Received Signal Strength (RSS) are some approaches commonly used in source localization.

We classify the positioning approaches based on the parameters measured for positioning. In **triangulation**, a mobile station and two base stations form a triangle, where the length and direction of the line segment connecting two base stations are known and the direction of the propagation of a radio-frequency wave can be measured (AOA). In other words, triangulation measures angles, not distances. On the opposite side, **trilateration** measures distances, not angles. In this approach, we calculate the distances from a mobile station and the base stations around and use them to deduce its position. For TOA, the distance is estimated based on the difference between the time instants when the signal was transmitted and when the signal was received. For RSS, we need more parameters to estimate this distance. Beside the ratio between transmitted signal power and received signal power, we also need path loss exponent, a parameter showing the reduction in power density of an electromagnetic wave as it propagates through space. In a 2D model, the received signals from at least 3 base stations are required to estimate position of a mobile station.

RSS-based localization with simultaneous path loss estimation is the subject of several previous research papers [2 - 5].

This paper proposes a new trilateration algorithm that uses 3 base stations of known positions to locate one mobile station, by a geometric approach. It avoids case division, which is the main weakness of some previously proposed algorithms. The original motivation for the proposed approach was to estimate the position of a mobile who possibly has a different height also and hence is not in the plane spanned by the three BS. Compared to the usual trilateration, this scenario corresponds to one possible error case, of the three estimated distances being larger than their true values. In section 2, we summarize some previous algorithms as well as their weaknesses, then propose our own method. Section 3 considers the system model for RSS to allow simultaneous path loss estimation. The results will be demonstrated in section 4. In section 5, we conclude and suggest further research directions.

II. Proposed Geometric Approach

A. Previous works

Many algorithms on trilateration have been proposed so far. In 2D models, the position of a mobile station is considered to be the point whose distance to the each stations is equal to the distance between the mobile station and the corresponding base station. It means that they find the intersection point of the circles whose centers are the base stations (usually 3 base stations). The radius of each circle is the estimated distance from the mobile station to the base station concerned. In the ideal scenario where there is no noise as well as no inaccuracy in measurements, there must be a unique point in which the 3 circles intersect. Undoubtedly, it is the position of the mobile station.

However, such an ideal scenario never exists in reality. The estimated distance is never exactly the correct true distance. Therefore, the 3 circles never intersect in 1 point. [6] summarizes all the possible cases of relative position of 3 circles. 2 circles can intersect each other at 2 common points; **or** touch each other internally or externally (1 common point); **or** lie inside or outside each other (0 common point). Hence, **the total number of intersection points of each pair of 3 circles**

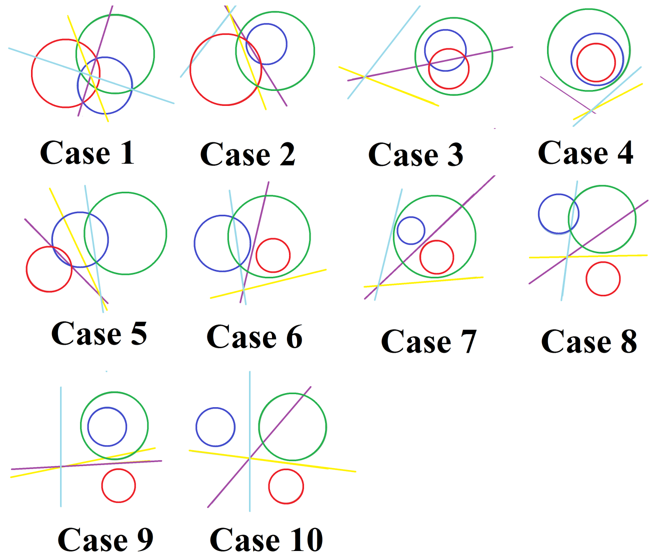

Fig. 1. All possible cases resulting from orientation and relative radii of the 3 circles, as well as their **radical axes**. The yellow line is the radical axis of the red and green circles. The purple line is the radical axis of the red and blue circles. The cyan line is the radical axis of the green and blue circles. The 3 radical axes are always concurrent.

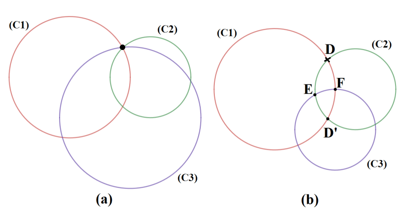

Fig. 2. Intersections of 3 circles in an ideal scenario (a) and in a scenario with erroneous measurements (b).

can be 6 less, or even 0. All ten possible cases are illustrated in Fig. 1.

Frequently, the 3 circles intersect each other at 6 points (case 1). The composition of 3 points that are closest to each other will be selected. [7] proposed a rule to select the correct 3 points needed. This rule selects the 3 points that stay closest to each other. Nonetheless, when the noise is quite considerable, this rule leads to select the wrong composition of 3 points. Fig. 2a shows the ideal scenario when the 3 circles intersect each other at one point. In Fig. 2b, a noticeable noise makes the radius of circle (C3) much smaller. Based on the rule in [7], the selected points are D', E, F because they are closest to each other; instead of the correct composition of points D, E, F. As a result, the estimated position can be sizably erroneous.

The selected 3 points form a triangle. In [7], the **Fermat Point** (the point such that the total distance from the three vertices of the triangle to the point is the minimum possible) of the triangle is taken as the position of the mobile station. The authors of [8] suggested the position to be the **centroid** of the triangle. Nevertheless, those algorithms only work in the standard case where $n = 6$. As for the other cases ($n < 6$), more functions, more analyses need to be performed.

In the paper [9] on localization in a 3D model, the author proposed a method to position in 3D. Similar to 2D models where circles are drawn, in 3D spheres appear. Each sphere has the center at the base station concerned and the radius as the distance between this base station and the mobile station. The intersection points of the 3 spheres are found, they are considered to be the position of the mobile station. Fig. 3 demonstrates a tetrahedron SABC where A, B, C are the base stations and S is the intersection point of the 3 spheres. Although this method could not solve the problem of the cases when the intersection points don't exist, it suggests a novel idea for positioning in 2D. We consider the orthogonal projection point of the apex of the tetrahedron onto the base plane as the position of the mobile station (point H is Fig. 3). We are going to prove that there exists **one and only one** point like this in all the cases, even when there is no intersection point of 3 spheres.

B. The standard case

In this paper, d_{AB} denotes the length of the line segment AB, and (A, d_A) is the circle of center A and radius d_A.

We consider the 3 base stations as the 3 vertices of a triangle. Generally, the 3 spheres intersects each other at *at least* 1 point. This point together with the 3 base stations form a tetrahedron in which it is the apex and the base plane is the plane through 3 base station (Fig. 3). In our work, we consider the orthogonal projection of the apex S on the base plane (ABC) as the estimated position of the mobile station.

We find that orthogonal projection point, as well as the foot of the tetrahedron's altitude through point S.

Defining d_A, d_B and d_C are the measured distance from the mobile station to 3 base stations A, B and C, respectively. We have SA = d_A; SB = d_B and SC = d_C.

Let M be the foot of the altitude through point S of triangle SAB, N be the foot of the altitude through point S of triangle SAC.

In plane (ABC), we draw a line perpendicular to AB through M and a line perpendicular to AC through N. The 2 lines intersect at point H. We have:

SM $\perp$ AB and HM $\perp$ AB $\Rightarrow$ (SHM) $\perp$ AB $\Rightarrow$ SH $\perp$ AB

SN $\perp$ AC and HN $\perp$ AC $\Rightarrow$ (SHN) $\perp$ AC $\Rightarrow$ SH $\perp$ AC

SH $\perp$ AB and SH $\perp$ AC $\Rightarrow$ SH $\perp$ (ABC)

Therefore, H is the foot of the altitude through point S of the tetrahedron SABC. We consider H as the estimated position of the mobile station.

SM is the altitude of the triangle SAB so

$$\mathrm{d}_{SM}^2 = \mathrm{d}_{SA}^2 - \mathrm{d}_{AM}^2 = \mathrm{d}_{SB}^2 - \mathrm{d}_{BM}^2 \quad (1)$$

Thus

$$\mathrm{d}_{AM}^2 - \mathrm{d}_{BM}^2 = \mathrm{d}_{SA}^2 - \mathrm{d}_{SB}^2 = \mathrm{d}_A^2 - \mathrm{d}_B^2 \quad (2)$$

Similarly, triangle SAC, HAB, HAC has altitude SN, HM, HN, respectively.

$$\mathrm{d}_{AN}^2 - \mathrm{d}_{CN}^2 = \mathrm{d}_{SA}^2 - \mathrm{d}_{SC}^2 = \mathrm{d}_A^2 - \mathrm{d}_C^2 \quad (3)$$

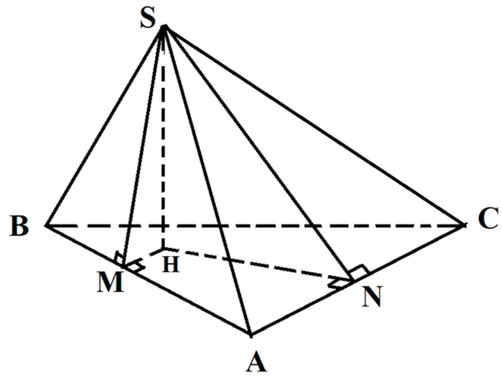

Fig. 3. Tetrahedron SABC formed by 3 base stations A, B, C and the intersection point S of the 3 spheres

$$\mathrm{d}_{AM}^2 - \mathrm{d}_{BM}^2 = \mathrm{d}_{HA}^2 - \mathrm{d}_{HB}^2 \tag{4}$$

$$\mathrm{d}_{AM}^2 - \mathrm{d}_{CN}^2 = \mathrm{d}_{HA}^2 - \mathrm{d}_{HC}^2 \tag{5}$$

From (2)-(5), we deduce that

$$\mathrm{d}_{HA}^2 - \mathrm{d}_{HB}^2 = \mathrm{d}_{SA}^2 - \mathrm{d}_{SB}^2 = \mathrm{d}_{A}^2 - \mathrm{d}_{B}^2 \tag{6}$$

$$\mathrm{d}_{HA}^2 - \mathrm{d}_{HC}^2 = \mathrm{d}_{SA}^2 - \mathrm{d}_{SC}^2 = \mathrm{d}_{A}^2 - \mathrm{d}_{C}^2 \tag{7}$$

Therefore, H stays on the two lines: **radical axis** of circles (A, d_A) and (B, d_B) (*line 1*); **radical axis** of circles (A, d_A), (C, d_C) (*line 2*). As a result, H is the **radical center** of 3 circles (A, d_A), (B, d_B), (C, d_C) [10].

Short explanations: Radical axis of 2 non-concentric circles is the locus of a point having equal power with regard to them. The geometric power of a point with respect to a circle is a real number that reflects the relative distance of the point from the circle. It is *positive* or *0* or *negative* when the point stays *outside* or *on* or *inside* the circle, respectively [10].

C. Suggested algorithm

The interesting thing is that we can always get the point H, even if we cannot form a tetrahedron SABC or even if we cannot form any or all of the 3 triangles SAB, SAC, SBC. Estimating the position of the mobile station becomes calculating the coordinate of the radical center of 3 circles.

1) Name the 3 base stations A, B and C. Estimate the distance d_A, d_B, d_C from the mobile station to each base station, respectively.
2) Determine the 3 circles: (A, d_A), (B, d_B), (C, d_C). Then determine their 3 radical axes, taken in 3 pairs.
3) The 3 radical axes must be concurrent [10]. **The common point of the 3 lines is estimated position of the mobile station (Fig. 1).**

Let x_A, x_B, x_C denote the abscissas of point A, B, C respectively and y_A, y_B, y_C denote the ordinates of point A, B, C respectively.

As the 3 radical axes are concurrent, we only need to find the intersection point of any two of them.

The equation of the radical axis of circles (A, d_A) and (B, d_B)

$$(\mathrm{x}_B - \mathrm{x}_A)x + (\mathrm{y}_B - \mathrm{y}_A)y = \mathrm{d}_B^2 - \mathrm{x}_B^2 - \mathrm{y}_B^2 - \mathrm{d}_A^2 + \mathrm{x}_A^2 + \mathrm{y}_A^2 \tag{8}$$

The equation of the radical axis of circles (A, d_A) and (B, d_B)

$$(\mathrm{x}_C - \mathrm{x}_C)x + (\mathrm{y}_C - \mathrm{y}_A)y = \mathrm{d}_C^2 - \mathrm{x}_C^2 - \mathrm{y}_C^2 - \mathrm{d}_A^2 + \mathrm{x}_A^2 + \mathrm{y}_A^2 \tag{9}$$

Let $X = \begin{bmatrix}\mathrm{x}_H & \mathrm{y}_H\end{bmatrix}^T$ be the coordinate vector of the point H. Since H is the intersection point of the 2 radical axes above:

$$\Gamma\,\mathrm{X} = \Psi \tag{10}$$

where

$$\Gamma = \begin{bmatrix}\mathrm{x}_B - \mathrm{x}_A & \mathrm{x}_C - \mathrm{x}_A \\ \mathrm{y}_B - \mathrm{y}_A & \mathrm{y}_C - \mathrm{y}_A\end{bmatrix} \tag{11}$$

$$\Psi = \begin{bmatrix}\mathrm{d}_B^2 \text{ - } \mathrm{x}_B^2 - \mathrm{y}_B^2 \text{ - } \mathrm{d}_A^2 + \mathrm{x}_A^2 + \mathrm{y}_A^2 \\ \mathrm{d}_C^2 \text{ - } \mathrm{x}_C^2 - \mathrm{y}_C^2 \text{ - } \mathrm{d}_A^2 + \mathrm{x}_A^2 + \mathrm{y}_A^2\end{bmatrix} . \tag{12}$$

Hence

$$\mathrm{X} = \Gamma^{-1}\,\Psi . \tag{13}$$

(13) is the equation to compute the coordinate of point H, which is taken as the estimated position of the mobile station.

When the number of base stations N_{BS} is larger than 3, we will obtain N_{BS}(N_{BS} - 1)/2 radical axes. To estimate the radical center of those circles, Ordinary Least Square is applied to get the solution of the overdetermined equation system. It turns out that this proposed method corresponds to method LLS mentioned in [13], where an interesting state of the art appears, including optimally weighted least-squares versions.

III. RSS Model and Attenuation Exponent Estimation

A. RSS Model

RSS is the average power received over a wireless link. Field trials have validated that the disturbance in RSS due to shadowing is log-normal distributed. Accordingly, the log-normal path loss model can be expressed as:

$$P_i = P_0 + 10\alpha\log(d_0) - 10\alpha\log(d_i) + n_i \tag{14}$$

where P_0 is the power received (in dBm) at a reference point at distance d_0, $\log(.)$ stands for $\log_{10}(.)$, d_i is the actual distance from the i-th base station to the mobile station, P_i is the power received (in dBm) at that base station, α is the path loss exponent, n_i is the log normal disturbance. The Path Loss Model (PLM) assumes that this disturbance has a Gaussian distribution with zero mean and a variance of σ^2.

B. Estimation of Path Loss Exponent

To estimate the path loss exponent, beside the N_{BS} base stations given, we also use M_T emitters at known positions. In absence of disturbance:

$$P_{i,j} = P_0 + 10\alpha\log(d_0) - 10\alpha\log(d_{i,j}) + n_{i,j} \qquad (15)$$

where $P_{i,j}$ is the power received at i-th base station, emitted by the j-th emitter, and $d_{i,j}$ is the actual distance between the i-th base station and the j-th emitter and $n_{i,j}$ is the lognormal shadowing. Maximum likelihood (ML) estimation leads to a least-squares cost function. Let us introduce

$$\Gamma_\alpha = \begin{bmatrix} P_{1,1}-P_0 \\ P_{1,2}-P_0 \\ \cdots \\ P_{1,N}-P_0 \\ \cdots \\ P_{M,1}-P_0 \\ P_{M,2}-P_0 \\ \cdots \\ P_{M,N}-P_0 \end{bmatrix}, \ \Psi_\alpha = \begin{bmatrix} 10\log\frac{d_0}{d_{1,1}} \\ 10\log\frac{d_0}{d_{1,2}} \\ \cdots \\ 10\log\frac{d_0}{d_{1,N}} \\ \cdots \\ 10\log\frac{d_0}{d_{M,1}} \\ 10\log\frac{d_0}{d_{M,2}} \\ \cdots \\ 10\log\frac{d_0}{d_{M,N}} \end{bmatrix}, \ n = \begin{bmatrix} n_{1,1} \\ n_{1,2} \\ \cdots \\ n_{1,N} \\ \cdots \\ n_{M,1} \\ n_{M,2} \\ \cdots \\ n_{M,N} \end{bmatrix}. \qquad (16)$$

Then the joint measurements can be written as

$$\Gamma_\alpha = \Psi_\alpha\,\alpha + n\,. \qquad (17)$$

The least-squares or ML estimate is then

$$\hat\alpha = \Psi_\alpha^T\Gamma_\alpha/\Psi_\alpha^T\Psi_\alpha^{-1}\,. \qquad (18)$$

C. ML Position Optimization with Steepest-Descent

The ML estimate (in lognormal shadowing) for the position of the mobile station is

$$(\hat x, \hat y) = \arg\min_{(x,y)} J_{RSS}(x,y) \qquad (19)$$

with

$$J_{RSS}(x,y) = \sum_{i=1}^{N_{BS}}\Big(r_{RSS,i} + 10\hat\alpha\log\sqrt{(x-x_i)^2+(y-y_i)^2}\Big)^2 \qquad (20)$$

where $r_{RSS,i} = P_i - P_0 - 10\,\hat\alpha\,_0$ and $\hat\alpha$ is the (previously) estimated path loss component. (20) can be minimized by the Steepest-Descent iterative procedure Let $X = \begin{bmatrix} x & y\end{bmatrix}^T$

Iterative procedure $X_{k+1} = X_k - \mu\nabla(J_{RSS}(X_k))$

with $\nabla(J(X_k)) = \begin{bmatrix}\frac{\partial J}{\partial x} \frac{\partial J}{\partial y}\end{bmatrix}^T$

where μ is the step-size.

Stopping criterion: $\|X_{k+1} - X_k\|_2 < \epsilon$, where ϵ is a sufficiently small positive constant.

The initiation value X_0 is obtained by the geometric approach provided in section 2, after the following transformation of the RSS data. To reduce the errors in distance estimation, we do not use the RSS data directly. In [11], we learn that if q is a Gaussian variable with mean μ and variance σ^2, then the mean of e^q will be

$$E(e^q) = \exp(\mu + \sigma^2/2)\,. \qquad (21)$$

As for $10^q = e^{(ln10)\,q}$, its mean is

$$E(10^q) = \exp((ln10)\mu + (ln10)^2\sigma^2/2)\,. \qquad (22)$$

As a result, we use the mean of d_i resulting from the RSS with lognormal shadowing as the estimated distance value:

$$\hat{\mathrm{d}}_i = E(\mathrm{d}_i) = \exp\left(\frac{-(ln10)\,r_{RSS,i}}{10\,\hat\alpha} - \frac{(ln10)^2\sigma^2}{200\,\hat\alpha^2}\right) \qquad (23)$$

We use the distance estimates obtained from (23) as the distance data in the geometric approach of section 2 to initialize the ML estimation above.

IV. Simulation Results

A. Environment setup

Our simulation scenario considers a square of size 1000m x 1000m. As for other parameters, P_0 = -45 dBm at d_0 = 10m.

The 3 base stations' coordinators are (200; 200), (800; 200) and (500; 800). 1000 mobiles stations are randomly picked up, their abscissas and ordinates are uniformly distributed from 0 to 1000.

B. Results

To compare the results among the algorithms, we calculate the Average Position Error (AVE), which is defined:

$$AVE \triangleq \sum_{i=1}^{Z}\Big\|\mathrm{X}^{(i)} - \hat{\mathrm{X}}^{(i)}\|_2 \qquad (24)$$

where $X^{(i)}$ is the i-th actual position of the mobile station, $\hat{\mathrm{X}}^{(i)}$ is the i-th estimated position of the mobile station, Z is the number of positions randomly picked up (in this setup, Z = 1000).

Table I illustrates the AVE obtained when we do not re-estimate the path loss component. The value of α varies from 2 to 6, the variance of additive disturbance is 1 and 2.

Table II illustrates the AVE obtained when we re-estimate the path loss component before estimating the position of the mobile station. 5 fixed emitters are placed at (0; 0), (0; 1000), (1000; 0), (1000; 1000) and (500; 500). The estimated value of path loss components are also presented in the table.

	σ = 1	σ = 2
α = 2	79.459	161.481
α = 2.5	57.289	130.180
α = 3	47.187	102.248
α = 3.5	41.074	83.031
α = 4	34.003	75.210
α = 4.5	30.890	63.755
α = 5	27.036	58.434
α = 5.5	25.604	51.040
α = 6	23.480	46.820

TABLE I

Average Position Error in localization without estimation of path loss component

Finally, we provide a comparison of the proposed geometric method (13) ("radii-square-differences" method), with the Fermat point method and the centroid method referred to earlier, in Fig. 4. The proposed method has lower average error and error spread.

	σ = 1		σ = 2	
$\alpha = 2$	$\hat{\alpha} = 2.0055$	76.953	$\hat{\alpha} = 2.0075$	156.388
$\alpha = 2.5$	$\hat{\alpha} = 2.504$	56.524	$\hat{\alpha} = 2.511$	122.647
$\alpha = 3$	$\hat{\alpha} = 3.0026$	47.489	$\hat{\alpha} = 3.0099$	105.778
$\alpha = 3.5$	$\hat{\alpha} = 3.5024$	41.337	$\hat{\alpha} = 3.5148$	88.051
$\alpha = 4$	$\hat{\alpha} = 4.0015$	33.963	$\hat{\alpha} = 4.0166$	74.850
$\alpha = 4.5$	$\hat{\alpha} = 4.5036$	28.722	$\hat{\alpha} = 4.5043$	63.919
$\alpha = 5$	$\hat{\alpha} = 5.0008$	27.663	$\hat{\alpha} = 5.0192$	58.098
$\alpha = 5.5$	$\hat{\alpha} = 5.5045$	25.751	$\hat{\alpha} = 5.5134$	50.735
$\alpha = 6$	$\hat{\alpha} = 6.0018$	22.740	$\hat{\alpha} = 6.0091$	47.981

TABLE II
AVERAGE POSITION ERROR IN LOCALIZATION WITH ESTIMATION OF PATH LOSS COMPONENT

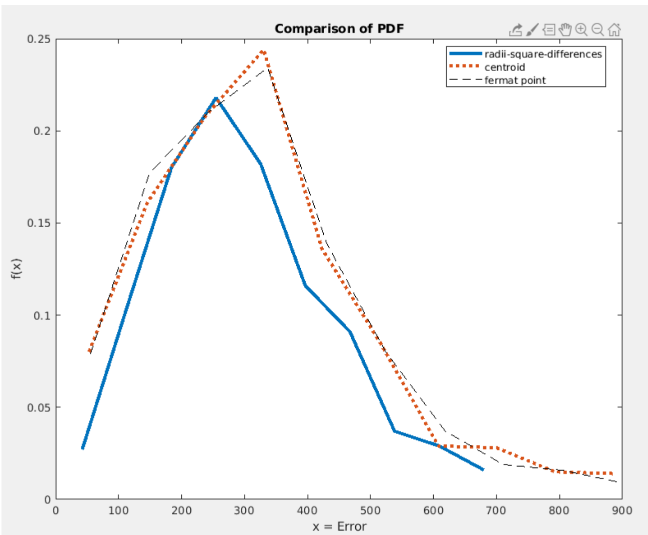

Fig. 4. Position error probability density for the radii-square-differences method, the Fermat point method and the centroid method.

V. CONCLUSION

In the range of trilateration positioning, this paper proposes a new geometric method that can be applied in all measurement error cases. The numerical results shows, compared to the centroid algorithm and the Fermat Point algorithm, that our proposed approach helps to significantly improve the accuracy in localization and reduce the complexity by avoiding case division. Furthermore, experimental results also demonstrate that integrating path loss estimation can make the position estimation more accurate. Nevertheless, only results in 2D simulation are shown. As for 3D models, algorithms are more complicated and are currently being investigated.

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