BEAM COORDINATION VIA DIFFUSION REDUCED-RANK ADAPTATION OVER ARRAY NETWORKS

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***Abstract*—In this work, we consider a distributed reduced-rank beam coordination problem over array networks. We develop an inherently adaptive combination scheme based on *combination matrix* for beam coordination problem. Two adaptive efficient implementation strategies for diffusion reduced-rank beamforming are proposed. Illustrative simulations validate that the proposed distributed reduced-rank adaptive algorithms could remarkably improve the convergence speed in comparison with the existing techniques under the condition of small samples.**

***Index Terms*—beam coordination, diffusion beamforming, reduced-rank, adaptive combination matrix, distributed strategies**

I. INTRODUCTION

Distributed and collaborative beamforming (DCBF) scheme in wireless sensor networks (WSNs) is receiving newfound interest in recent years [1]. Prior work in the literature has shown that it is benecial to deploy distributed cooperation strategies to coordinate the operation of beamforming arrays [2] or antenna elements [3]–[9] that are scattered over a geographical area and interconnected via some topology. Since early works on distributed beamforming [4], [5] were aimed at performing desirable beam patterns. Later, distributed processings [6], [7] are used to perform optimal beamforming in a noisy environment, while they require a fully connected network or a tree topology and require an ordering of the computations in the nodes. A fully distributed beamforming scheme based on diffusion strategy [10], [11] is proposed in [9], in which each individual array forms a beampattern pointing towards the desired signal through sharing the local beamforming vectors with the neighboring arrays dynamically. Based on [11], [12] describes a distributed approach similar to minimum-variance-distortionless-response (MVDR) beamforming which is suitable for a practical, dynamic environment.

However, under the small-sample conditions, it is challenging in systems with high dimensional weight vectors due to possibly slowed-down convergence speed and poor interference suppression performance. Practically, dimensionality reduction is important for distributed inference problems with large-scale data sets [13]. Several dimensionality reduction methods have been proposed to perform data compression or dimensionality reduction, such as distributed quantized Kalman filtering [14], [15], quantized consensus algorithms [16], distributed principle subspace estimation [17] and Krylov subspace optimization techniques [18]. Nevertheless, the aforementioned algorithms might either be too costly or perform unsatisfactorily when processing a large number of parameters as demonstrated in [13]. Recently, an iterative reduced-rank method known as the joint iterative alternating optimization (JIO) technique has been applied to direction-of-arrival (DOA) estimation [19] and space-time interference suppression [20], etc. More recently, a distributed reduced-rank method based on JIO technique for parameter estimation has been developed, utilizing the combination coefficients to estimate the low-rank weight estimates [21]. However, limited researches have been reported on distributed reduced-rank beamforming applications, and the distributed reduced-rank method [21] could not be directly applied in distributed beam coordination problems.

We herein develop a scheme for distributed beam coordination along with two diffusion reduced-rank adaptive strategies motivated by the joint iterative alternating optimization method [13], [19]–[21]. We derive a combination scheme with *combination matrices* tailored for the beam coordination problem. The combination matrix scheme is inextricably linked with not only the network topology, but also the transformation matrix and low-rank weight estimate. Thus, it is inherently adaptive. The estimation of the transformation matrix at each array utilizes both the local transformation matrix estimate and the local low-rank weight estimate from its neighboring arrays and itself, so does that of the low-rank weight estimate. We validate via simulations that the proposed algorithms could remarkably improve the convergence speed in comparison with the existing techniques under the condition of small samples and they can obtain the comparable perfor-

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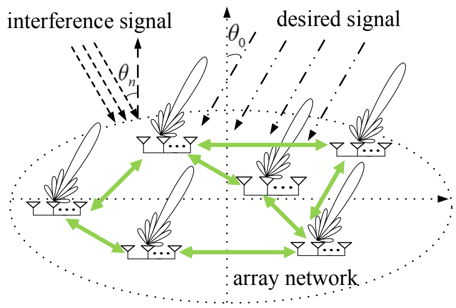

Fig. 1. An illustration of array network.

mance of the full-rank counterparts with the sufficiently large sample size.

II. Problem formulation

In this work, we consider a network of K identical antenna arrays, as illustrated in Fig. 1. We consider the uniform linear arrays (ULAs) for simplicity. We assume that one far-field narrowband desired signal $s_{0,i}$ from θ_0 as well as N far-field narrowband interferences $s_{1,i}, s_{2,i}, \cdots, s_{N,i}$ impinge simultaneously on each array k which is composed of M antenna elements. Thus, the $M \times 1$ received signal at array k and time i is given by

$$\boldsymbol{x}_{k,i} = \boldsymbol{a}(\theta_0) s_{0,i} + \sum_{n=1}^{N} \boldsymbol{a}(\theta_n) s_{n,i} + \boldsymbol{\eta}_{k,i}, \tag{1}$$

where $\{\boldsymbol{a}(\theta_n)\}_{n=0}^{N}$ are the $M \times 1$ array steering vectors, $\{\theta_n\}_{n=0}^{N}$ are the corresponding DOAs, and $\boldsymbol{\eta}_{k,i}$ is the zero-mean additive white complex Gaussian noise vector with variance σ_k^2. For the array network in Fig.1, our objective is to find the optimal transformation matrix $\boldsymbol{S}$ and optimal low-rank weight estimate $\bar{\boldsymbol{w}}$ that minimizes the following aggregate global cost across all arrays under the MVDR criteria

$$\min_{\boldsymbol{S}, \bar{\boldsymbol{w}}} \sum_{k=1}^{K} \mathbb{E}\left[|\bar{\boldsymbol{w}}^{\mathrm{H}} \boldsymbol{S}^{\mathrm{H}} \boldsymbol{x}_{k,i}|^2\right], \quad \text{s.t. } \bar{\boldsymbol{w}}^{\mathrm{H}} \boldsymbol{S}^{\mathrm{H}} \boldsymbol{a}(\theta_0) = 1 \tag{2}$$

where $|\cdot|$, $\mathbb{E}[\cdot]$ and $(\cdot)^{\mathrm{H}}$ respectively denote the modulus, mathematical expectation and conjugate transpose, and $\boldsymbol{S}$ transforms $\boldsymbol{x}_{k,i}$ into a vector with a lower dimension $D(D \ll M)$. In what follows, each D-dimensional quantity is denoted with a "bar".

In the diffusion dimensionality reduction scheme, the received vector $\boldsymbol{x}_{k,i}$ at array k is transformed by the transformation matrix $\boldsymbol{S}_{k,i}$ into a lower dimensional vector. The reduced-rank beamformer output of each array k is given by $y_{k,i} = \bar{\boldsymbol{w}}_{k,i}^{\mathrm{H}} \bar{\boldsymbol{x}}_{k,i}$, with the low-rank weight estimate $\bar{\boldsymbol{w}}_{k,i}$. The transformation matrix $\boldsymbol{S}_{k,i}$ and the low-rank weight estimate $\bar{\boldsymbol{w}}_{k,i}$ could be computed at each array k based on the joint iterative alternating optimization strategy [13], [21]. The transformation matrix $\boldsymbol{S}_{k,i}$ and the low-rank weight estimate $\bar{\boldsymbol{w}}_{k,i}$ are shared among the neighboring arrays. The specific derivation can be seen in the next section.

III. Algorithm Development

A. Global and Local Solutions

With the definition of $\boldsymbol{w} \triangleq \boldsymbol{S}\bar{\boldsymbol{w}}$, the constrained global cost (2) can be rewritten as

$$J^{\mathrm{glob}}(\boldsymbol{w}) = \sum_{k=1}^{K} \mathbb{E}[|\boldsymbol{w}^{\mathrm{H}} \boldsymbol{x}_{k,i}|^2] + 2\Re[\lambda(1 - \boldsymbol{w}^{\mathrm{H}} \boldsymbol{a}(\theta_0)], \tag{3}$$

where λ is a scalar Lagrange multiplier, and the operator $\Re[\cdot]$ extracts the real part of the argument. After a series of derivation similar to [10], the local cost function is given by

$$J_k^{\mathrm{loc}}(\boldsymbol{w}) \triangleq \sum_{l \in \mathcal{N}_k} c_{l,k} \mathbb{E}[|\boldsymbol{w}^{\mathrm{H}} \boldsymbol{x}_{k,i}|^2] + 2\Re[\lambda(1 - \boldsymbol{w}^{\mathrm{H}} \boldsymbol{a}(\theta_0))], \tag{4}$$

where $\mathcal{N}_k$ denotes the set of neighboring arrays of array k (including k). The exchange coefficients $\{c_{l,k}\}$ weigh differently the data from the neighbors of array k, and it satisfies

$$c_{l,k} = 0 \text{ if } l \notin \mathcal{N}_k, \quad \boldsymbol{C}\mathbf{1} = \mathbf{1}, \mathbf{1}^{\mathrm{T}}\boldsymbol{C} = \mathbf{1}^{\mathrm{T}}. \tag{5}$$

where $\boldsymbol{C} = [c_{l,k}] \in \mathbb{R}^{K \times K}$, and $\mathbf{1}$ and $(\cdot)^{\mathrm{T}}$ denote the $K \times 1$ vector with unit entries and transpose, respectively.

Thus, minimizing the global cost (3) with respect to $\boldsymbol{w}$ is equivalent to minimizing the following cost function

$$\begin{aligned} J_k^{\mathrm{dist}}(\boldsymbol{w}) = &\sum_{l \in \mathcal{N}_k} c_{l,k} \mathbb{E}[|\boldsymbol{w}^{\mathrm{H}} \boldsymbol{x}_{k,i}|^2] + 2\Re[\lambda(1 - \boldsymbol{w}^{\mathrm{H}} \boldsymbol{a}(\theta_0))] \\ &+ \sum_{l \in \mathcal{N}_k / \{k\}} b_{l,k} \|\boldsymbol{w} - \boldsymbol{w}_l^{\mathrm{loc}}\|^2, \end{aligned} \tag{6}$$

for each array k, where $\|\cdot\|$ denotes the l_2 norm, $\boldsymbol{w}_l^{\mathrm{loc}}$ represents the optimal local weight estimate at lth array, $\{b_{l,k}\}$ is a set of non-negative real coefficients that weigh differently the data from the neighbors. In particular, we are interested in the choice of coefficients such as

$$b_{l,k} = 0 \text{ if } l \notin \mathcal{N}_k, \quad \mathbf{1}^{\mathrm{T}}\boldsymbol{B} = \mathbf{1}^{\mathrm{T}}, \tag{7}$$

where $\boldsymbol{B} = [b_{l,k}] \in \mathbb{R}^{K \times K}$. We therefore have an alternative representation of the global cost (3) in terms of the local estimates $\{\boldsymbol{w}_k^{\mathrm{loc}}\}$ across the network.

Note that weight vectors with large dimension could lead to the slowed-down convergence speed and poor interference suppression performance under the condition of small samples. In order to solve this problem, hereafter, we consider jointly optimizing the transformation matrix $\boldsymbol{S}$ and low-rank weight estimate $\bar{\boldsymbol{w}}$ to implement (6), which is the main motivation for this work.

B. Diffusion Reduced-rank Adaptive Solutions

First, we replace the optimal local weight estimate $\boldsymbol{w}$ in (6) with $\boldsymbol{S}_k\bar{\boldsymbol{w}}_k$. We also replace the optimal local weight estimate $\boldsymbol{w}_l^{\mathrm{loc}}$ with its intermediate estimate that would be available at array l, and is denoted by $\boldsymbol{Q}_l\bar{\boldsymbol{\phi}}_l$. In this way, each array k can proceed to minimize the following modified cost

$$\begin{aligned} J_k^{\mathrm{dist}}(\boldsymbol{S}_k, \bar{\boldsymbol{w}}_k) = &\sum_{l \in \mathcal{N}_k} c_{l,k} \mathbb{E}[|\bar{\boldsymbol{w}}_k^{\mathrm{H}} \boldsymbol{S}_k^{\mathrm{H}} \boldsymbol{x}_{k,i}|^2] \\ &+ 2\Re[\lambda(1 - \bar{\boldsymbol{w}}_k^{\mathrm{H}} \boldsymbol{S}_k^{\mathrm{H}} \boldsymbol{a}(\theta_0))] \\ &+ \sum_{l \in \mathcal{N}_k / \{k\}} b_{l,k} \|\boldsymbol{S}_k\bar{\boldsymbol{w}}_k - \boldsymbol{Q}_l\bar{\boldsymbol{\phi}}_l\|^2. \end{aligned} \tag{8}$$

Note that there are possibly multiple solutions of $\bar{\boldsymbol{w}}_k$ and $\boldsymbol{S}_k$ to (8). Hereafter, we consider the solution to (8) based on the minimum norm criterion.

By instantaneous approximation followed by enforcing the constraint $\bar{\boldsymbol{w}}_{k,i}^{\mathrm{H}}\boldsymbol{S}_{k,i}^{\mathrm{H}}\boldsymbol{a}(\theta_0)=1$ at each array k, we have the following stochastic gradient iterations

$$\boldsymbol{S}_{k,i}=\boldsymbol{S}_{k,i-1}-\mu_{S_k}\sum\nolimits_{l\in\mathcal{N}_k}c_{l,k}y_{k,l,i}^{*}\boldsymbol{P}_i\boldsymbol{x}_{k,i}\bar{\boldsymbol{w}}_{k,i-1}^{\mathrm{H}} + \mu_{S_k}\sum_{l\in\mathcal{N}_k/\{k\}}b_{l,k}\{(\boldsymbol{S}_{k,i-1}\bar{\boldsymbol{w}}_{k,i-1}-\boldsymbol{Q}_l\bar{\boldsymbol{\phi}}_l)\bar{\boldsymbol{w}}_{k,i-1}^{\mathrm{H}}\}, \tag{9}$$

$$\bar{\boldsymbol{w}}_{k,i}=\bar{\boldsymbol{w}}_{k,i-1}-\mu_{w_k}\sum\nolimits_{l\in\mathcal{N}_k}c_{l,k}y_{k,l,i}^{*}\bar{\boldsymbol{P}}_{k,i}\boldsymbol{S}_{k,i-1}^{\mathrm{H}}\boldsymbol{x}_{l,i} + \mu_{w_k}\sum_{l\in\mathcal{N}_k/\{k\}}b_{l,k}\{\boldsymbol{S}_{k,i-1}^{\mathrm{H}}(\boldsymbol{S}_{k,i-1}\bar{\boldsymbol{w}}_{k,i-1}-\boldsymbol{Q}_l\bar{\boldsymbol{\phi}}_l)\}. \tag{10}$$

where μ_{S_k} and μ_{w_k} are the positive step sizes, $(\cdot)^*$ denotes the complex conjugate, $y_{k,l,i}=\bar{\boldsymbol{w}}_{k,i-1}^{\mathrm{H}}\boldsymbol{S}_{k,i-1}^{\mathrm{H}}\boldsymbol{x}_{l,i}$, $\boldsymbol{P}_i\triangleq\boldsymbol{I}_M-\frac{\boldsymbol{a}(\theta_0)\boldsymbol{a}^{\mathrm{H}}(\theta_0)}{\|\boldsymbol{a}(\theta_0)\|^2}$ and $\bar{\boldsymbol{P}}_{k,i}\triangleq\boldsymbol{I}_D-\frac{\bar{\boldsymbol{a}}_{k,i}\bar{\boldsymbol{a}}_{k,i}^{\mathrm{H}}}{\|\bar{\boldsymbol{a}}_{k,i}\|^2}$ are the projection matrices, $\bar{\boldsymbol{a}}_{k,i}\triangleq\boldsymbol{S}_{k,i-1}^{\mathrm{H}}\boldsymbol{a}(\theta_0)$, $\boldsymbol{I}_D$ and $\boldsymbol{I}_M$ are identity matrices with dimensions $D\times D$ and $M\times M$, respectively.

It is worth remarking that the iterations of $\boldsymbol{S}_{k,i}$ and $\bar{\boldsymbol{w}}_{k,i}$ in (9) and (10) are intertwined together. Motivated by [13], [19]–[21], we consider herein the joint iterative alternating optimization strategy.

Following [10], [22], we adopt the adapt-then-combine (ATC) diffusion scheme. The iteration (9) can be implemented in two steps by generating the intermediate transformation matrix estimate $\boldsymbol{Q}_{k,i}$ as follows

$$\boldsymbol{Q}_{k,i}=\boldsymbol{S}_{k,i-1}-\mu_{S_k}\sum\nolimits_{l\in\mathcal{N}_k}c_{l,k}y_{k,l,i}^{*}\boldsymbol{P}_i\boldsymbol{x}_{k,i}\bar{\boldsymbol{w}}_{k,i-1}^{\mathrm{H}}, \tag{11}$$

$$\boldsymbol{S}_{k,i}=\boldsymbol{Q}_{k,i}+\mu_{S_k}\sum_{l\in\mathcal{N}_k/\{k\}}b_{l,k}\{(\boldsymbol{S}_{k,i-1}\bar{\boldsymbol{w}}_{k,i-1}-\boldsymbol{Q}_l\bar{\boldsymbol{\phi}}_l)\bar{\boldsymbol{w}}_{k,i-1}^{\mathrm{H}}\}. \tag{12}$$

Similarly, the iteration (10) can be implemented in two steps by generating an intermediate low-rank weight estimate $\bar{\boldsymbol{\phi}}_{k,i}$ as follows

$$\bar{\boldsymbol{\phi}}_{k,i}=\bar{\boldsymbol{w}}_{k,i-1}-\mu_{w_k}\sum\nolimits_{l\in\mathcal{N}_k}c_{l,k}y_{k,l,i}^{*}\bar{\boldsymbol{P}}_{k,i}\boldsymbol{S}_{k,i-1}^{\mathrm{H}}\boldsymbol{x}_{l,i}, \tag{13}$$

$$\bar{\boldsymbol{w}}_{k,i}=\bar{\boldsymbol{\phi}}_{k,i}+\mu_{w_k}\sum_{l\in\mathcal{N}_k/\{k\}}b_{l,k}\{\boldsymbol{S}_{k,i-1}^{\mathrm{H}}(\boldsymbol{S}_{k,i-1}\bar{\boldsymbol{w}}_{k,i-1}-\boldsymbol{Q}_l\bar{\boldsymbol{\phi}}_l)\}. \tag{14}$$

We further replace $\boldsymbol{Q}_l$ in (12) and (14) by $\boldsymbol{Q}_{l,i}$, replace $\boldsymbol{S}_{k,i-1}$, $\bar{\boldsymbol{\phi}}_l$ and $\bar{\boldsymbol{w}}_{k,i-1}$ by $\boldsymbol{Q}_{k,i}$, $\bar{\boldsymbol{\phi}}_{l,i}$ and $\bar{\boldsymbol{\phi}}_{k,i}$, respectively, and through a series of mathematical derivation, (12) and (14) can be transformed into

$$\boldsymbol{S}_{k,i}=\sum\nolimits_{l\in\mathcal{N}_k}\boldsymbol{Q}_{l,i}\boldsymbol{A}_{l,k,i}, \tag{15}$$

$$\bar{\boldsymbol{w}}_{k,i}=\sum\nolimits_{l\in\mathcal{N}_k}\boldsymbol{D}_{l,k,i}\bar{\boldsymbol{\phi}}_{l,i}, \tag{16}$$

where $\boldsymbol{A}_{l,k,i}$ and $\boldsymbol{D}_{l,k,i}$ are the combination matrices of the transformation matrix and low-rank weight estimate, respectively,

$$\boldsymbol{A}_{l,k,i}\triangleq\begin{cases}\boldsymbol{I}_D+(f_{k,k}-1)\bar{\boldsymbol{\phi}}_{k,i}\bar{\boldsymbol{\phi}}_{k,i}^{\mathrm{H}}, & l=k,\\ f_{l,k}\bar{\boldsymbol{\phi}}_{l,i}\bar{\boldsymbol{\phi}}_{k,i}^{\mathrm{H}}, & l\neq k,\end{cases} \tag{17}$$

TABLE I
THE PROPOSED DRACM-1 AND DRACM-2 ALGORITHMS

Initialization: Start with $\bar{\boldsymbol{w}}_{k,0}^{\mathrm{H}}\boldsymbol{S}_{k,0}^{\mathrm{H}}\boldsymbol{a}(\theta_0)=1$ for each array k. Given non-negative real step-size μ_{S_k} and μ_{w_k} for each array k. For each time instant $i=1,2,\cdots$, repeat:
Adaptation: For each node $k=1,2,\cdots,K$, repeat: $\boldsymbol{Q}_{k,i}=\boldsymbol{S}_{k,i-1}-\mu_{S_k}\sum\nolimits_{l\in\mathcal{N}_k}c_{l,k}y_{k,l,i}^{*}\boldsymbol{P}_i\boldsymbol{x}_{k,i}\bar{\boldsymbol{w}}_{k,i-1}^{\mathrm{H}}$, $\bar{\boldsymbol{\phi}}_{k,i}=\bar{\boldsymbol{w}}_{k,i-1}-\mu_{w_k}\sum\nolimits_{l\in\mathcal{N}_k}c_{l,k}y_{k,l,i}^{*}\bar{\boldsymbol{P}}_{k,i}\boldsymbol{S}_{k,i-1}^{\mathrm{H}}\boldsymbol{x}_{l,i}$.
Combination: For each node $k=1,2,\cdots,K$, repeat: DRACM-1 algorithm: $\boldsymbol{A}_{l,k,i}=\begin{cases}\boldsymbol{I}_D+(f_{k,k}-1)\bar{\boldsymbol{\phi}}_{k,i}\bar{\boldsymbol{\phi}}_{k,i}^{\mathrm{H}}, & l=k,\\ f_{l,k}\bar{\boldsymbol{\phi}}_{l,i}\bar{\boldsymbol{\phi}}_{k,i}^{\mathrm{H}}, & l\neq k,\end{cases}$ $\boldsymbol{S}_{k,i}=\sum\nolimits_{l\in\mathcal{N}_k}\boldsymbol{Q}_{l,i}\boldsymbol{A}_{l,k,i}$, $\boldsymbol{p}_{k,i}=\{\bar{\boldsymbol{\phi}}_{l,i},l\in\mathcal{N}_k\}$, $\boldsymbol{z}_{k,i}=\frac{\boldsymbol{p}_{k,i}^{*}\otimes[\boldsymbol{S}_{k,i}^{\mathrm{H}}\boldsymbol{a}(\theta_0)]}{\|\,\boldsymbol{p}_{k,i}^{\mathrm{T}}\otimes(\boldsymbol{a}^{\mathrm{H}}(\theta_0)\boldsymbol{S}_{k,i})\,\|^2}$, $\boldsymbol{T}_{k,i}=\mathrm{unvec}_{D,Dr_k}\{\boldsymbol{z}_{k,i}\}$, $\bar{\boldsymbol{w}}_{k,i}=\boldsymbol{T}_{k,i}\boldsymbol{p}_{k,i}$.
DRACM-2 algorithm: $\boldsymbol{D}_{l,k,i}=\begin{cases}\boldsymbol{I}_D+(g_{k,k}-1)\boldsymbol{Q}_{k,i}^{\mathrm{H}}\boldsymbol{Q}_{k,i}, & l=k,\\ g_{l,k}\boldsymbol{Q}_{k,i}^{\mathrm{H}}\boldsymbol{Q}_{l,i}, & l\neq k,\end{cases}$ $\bar{\boldsymbol{w}}_{k,i}=\sum\nolimits_{l\in\mathcal{N}_k}\boldsymbol{D}_{l,k,i}\bar{\boldsymbol{\phi}}_{l,i}$, $\boldsymbol{\Xi}_{k,i}=\{\boldsymbol{Q}_{l,i},l\in\mathcal{N}_k\}$, $\boldsymbol{v}_{k,i}=\frac{\bar{\boldsymbol{w}}_{k,i}^{*}\otimes[\boldsymbol{\Xi}_{k,i}^{\mathrm{H}}\boldsymbol{a}(\theta_0)]}{\|\,\bar{\boldsymbol{w}}_{k,i}^{\mathrm{T}}\otimes(\boldsymbol{a}^{\mathrm{H}}(\theta_0)\boldsymbol{\Xi}_{k,i})\,\|^2}$, $\boldsymbol{\Lambda}_{k,i}=\mathrm{unvec}_{Dr_k,D}\{\boldsymbol{v}_{k,i}\}$, $\boldsymbol{S}_{k,i}=\boldsymbol{\Xi}_{k,i}\boldsymbol{\Lambda}_{k,i}$.

$$\boldsymbol{D}_{l,k,i}\triangleq\begin{cases}\boldsymbol{I}_D+(g_{k,k}-1)\boldsymbol{Q}_{k,i}^{\mathrm{H}}\boldsymbol{Q}_{k,i}, & l=k,\\ g_{l,k}\boldsymbol{Q}_{k,i}^{\mathrm{H}}\boldsymbol{Q}_{l,i}, & l\neq k,\end{cases} \tag{18}$$

with the real, non-negative combination coefficients $\{f_{l,k},g_{l,k}\}$ satisfying

$$f_{l,k}=g_{l,k}=0 \text{ if } l\notin N_k,\quad \boldsymbol{1}^{\mathrm{T}}\boldsymbol{F}=\boldsymbol{1}^{\mathrm{T}},\quad \boldsymbol{1}^{\mathrm{T}}\boldsymbol{G}=\boldsymbol{1}^{\mathrm{T}}, \tag{19}$$

where $\boldsymbol{F}=[f_{l,k}]\in\mathbb{R}^{K\times K}$ and $\boldsymbol{G}=[g_{l,k}]\in\mathbb{R}^{K\times K}$.

Notice that the iterations of $\boldsymbol{S}_{k,i}$ and $\bar{\boldsymbol{w}}_{k,i}$ are intertwined together. We now derive the iteration of $\bar{\boldsymbol{w}}_{k,i}$ with $\boldsymbol{S}_{k,i}$ fixed. We define

$$\bar{\boldsymbol{w}}_{k,i}=\sum\nolimits_{l\in\mathcal{N}_k}\boldsymbol{D}_{l,k}\bar{\boldsymbol{\phi}}_{l,i}\triangleq\boldsymbol{T}_{k,i}\boldsymbol{p}_{k,i}, \tag{20}$$

where $\boldsymbol{T}_{k,i}\in\mathbb{C}^{D\times(Dr_k)}$ consists of $\{\boldsymbol{D}_{l,k},l\in\mathcal{N}_k\}$ and $\boldsymbol{p}_{k,i}\in\mathbb{C}^{(Dr_k)\times 1}$ consists of $\{\bar{\boldsymbol{\phi}}_{l,i},l\in\mathcal{N}_k\}$, with r_k denoting the cardinality of $\mathcal{N}_k$, i.e., $r_k=|\mathcal{N}_k|$. With the vectorizing of $\boldsymbol{T}_{k,i}$, i.e., $\boldsymbol{z}_{k,i}=\mathrm{vec}\{\boldsymbol{T}_{k,i}\}$, the constraint $\bar{\boldsymbol{w}}_{k,i}^{\mathrm{H}}\boldsymbol{S}_{k,i}^{\mathrm{H}}\boldsymbol{a}(\theta_0)=1$ at each array k (c.f. (3)) can be expressed as

$$[\boldsymbol{p}_{k,i}^{\mathrm{T}}\otimes(\boldsymbol{a}^{\mathrm{H}}(\theta_0)\boldsymbol{S}_{k,i})]\boldsymbol{z}_{k,i}=1. \tag{21}$$

where $\otimes$ denotes the Kronecker product. Notice that there are an infinite number of solutions to the underdetermined equations (21). The minimum norm solution to (21) is given by

$$\boldsymbol{z}_{k,i}=\frac{\boldsymbol{p}_{k,i}^{*}\otimes[\boldsymbol{S}_{k,i}^{\mathrm{H}}\boldsymbol{a}(\theta_0)]}{\|\,\boldsymbol{p}_{k,i}^{\mathrm{T}}\otimes(\boldsymbol{a}^{\mathrm{H}}(\theta_0)\boldsymbol{S}_{k,i})\,\|^2}, \tag{22}$$

$$\boldsymbol{T}_{k,i}=\mathrm{unvec}_{D,Dr_k}\{\boldsymbol{z}_{k,i}\}, \tag{23}$$

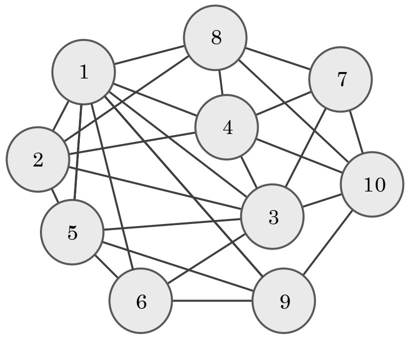

Fig. 2. Network topology

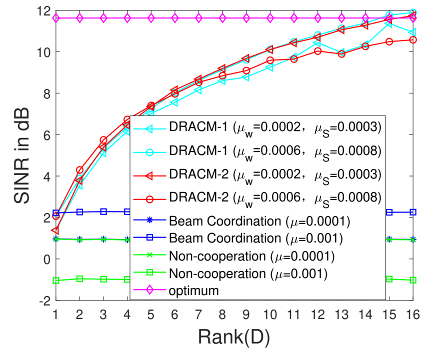

Fig. 3. SINR performance against rank(D)

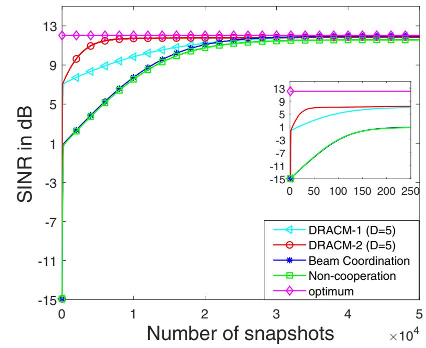

Fig. 4. SINR performance against snapshots

which is unique, where $\mathrm{unvec}_{l,k}$ converts the column vector of lk entries into an $l \times k$ matrix.

In summary, the proposed Distributed Reduced-rank beam coordination with Adaptive Combination Matrix (DRACM-1) algorithm is concluded in Table I.

It is worthy of note that the iterations of $\boldsymbol{S}_{k,i}$ and $\bar{\boldsymbol{w}}_{k,i}$ are intertwined with each other. We could derive the iteration of $\boldsymbol{S}_{k,i}$ with $\bar{\boldsymbol{w}}_{k,i}$ fixed similarly to the iteration of $\bar{\boldsymbol{w}}_{k,i}$ with $\boldsymbol{S}_{k,i}$ fixed. The algorithm is also summarized in Table I.

According to the proposed combination scheme, each array k combines the intermediate transformation matrix estimates from $\mathcal{N}_k$ to obtain the estimate of transformation matrix $\boldsymbol{S}_{k,i}$ via combination matrices $\{\boldsymbol{A}_{l,k,i}, l \in \mathcal{N}_k\}$. These combination matrices are related to not only the network topology, but also the intermediate low-rank weight estimates from $\mathcal{N}_k$. Further, from (17), we notice that the combination matrix $\boldsymbol{A}_{l,k,i}$ of the transformation matrix is obtained by adaptively collecting the intermediate low-rank weight estimates from $\mathcal{N}_k$. From (15), we also note that the transformation matrix $\boldsymbol{S}_{k,i}$ is updated by collecting the intermediate transformation matrix estimates from $\mathcal{N}_k$. Therefore, the estimation of the transformation matrix $\boldsymbol{S}_{k,i}$ at each array k utilizes both the intermediate transformation matrix estimates and intermediate low-rank weight estimates from $\mathcal{N}_k$, so does that of the low-rank weight estimate. Thus, the combination procedure is inherently adaptive. It is also worth nothing that the combination matrix adaptation is inextricably linked with the distributed reduced-rank beam coordination.

IV. Simulation Results

We now compare the proposed DRACM-1 and DRACM-2 algorithms with the full-rank Beam Coordination algorithm [9] under the MVDR criteria and the full-rank non-cooperation counterpart. We consider the topology with $K = 10$ arrays outlined in Fig. 2. The uniform linear array consists of $M = 16$ antenna elements with inter-element spacing half-wavelength. The power of the desired signal and interference signals are 0dB, 10dB, 10dB and 10dB, respectively. The DOAs of these four signals are 20°, -45°, -30° and 60°, respectively. For brevity, we consider utilizing the same step sizes for each array. We use μ_S and μ_w for both proposed algorithms and μ for Beam Coordination and non-cooperation case. The variance of the additive noise at each array k is set to be $\sigma_k^2 = 1$. The averaged results of 100 Monte-Carlo runs are presented herein.

We firstly evaluate the SINR performance of the proposed DRACM-1 and DRACM-2 algorithms with the existing strategies against the rank D using the optimized parameters for all schemes under the small-sample condition with $I = 250$ snapshots, where the snapshot also denotes the time instant. For both two proposed algorithms, the transformation matrices $\boldsymbol{S}_{k,0}$ and low-rank weight estimates $\bar{\boldsymbol{w}}_{k,0}$ are initialized as $\boldsymbol{I}_{M,D} = [\boldsymbol{I}_D \ \boldsymbol{0}_{D,M-D}]^{\mathrm{T}}$ and $[1\ 0 \ldots\ 0]^{\mathrm{T}} \in \mathbb{C}^D$. The full-rank weight vectors for the other two algorithms are initialized as $[1\ 0 \ldots\ 0]^{\mathrm{T}} \in \mathbb{C}^M$, correspondingly. As observed in Fig. 3, the proposed algorithms outperform in convergence speed the existing full-rank Beam Coordination algorithm and non-cooperation counterpart. Moreover, the interference suppression performance of the proposed algorithms is superior to that of the existing full-rank counterparts. Practically, the rank D could be selected to trade-off between the performance of the proposed distributed reduced-rank beam coordination algorithms and the computational complexity.

We also evaluate the steady-state performance of the proposed algorithms with the rank $D = 5$. The step sizes μ_S and μ_w for both proposed algorithms are set to be 0.0003 and 0.0002, respectively, the step sizes for Beam Coordination and non-cooperative case are set to be 0.0001. For both proposed algorithms, the transformation matrices $\boldsymbol{S}_{k,0}$ and low-rank weight estimates $\bar{\boldsymbol{w}}_{k,0}$ are initialized as $\boldsymbol{I}_{M,D} = [\boldsymbol{I}_D\ \boldsymbol{0}_{D,M-D}]^{\mathrm{T}}$ and $[1\ 0 \ldots\ 0]^{\mathrm{T}} \in \mathbb{C}^D$. The full-rank weight vector for the other two algorithms are initialized as $[1\ 0 \ldots\ 0]^{\mathrm{T}} \in \mathbb{C}^M$, correspondingly. As demonstrated in Fig.4, the proposed algorithms can achieve the comparable steady-state performance of the full-rank Beam Coordination algorithm, and both of the proposed algorithms could even outperform the full-rank non-cooperation counterpart in the steady-state performance. Furthermore, the proposed DRACM-2 algorithm is observed to approach the SINR of the optimal counterpart.

V. Conclusions

We propose a fully distributed beam coordination with adaptive combination matrices for array networks, where each array of the network adaptively constructs its own combination matrices by collecting the local transformation matrices and low-rank weight estimates of its neighboring arrays and itself. Simulation results validate that the proposed diffusion reduced-rank algorithms can outperform the existing full-rank algorithms in convergence speed under the small-sample conditions and they can achieve comparable steady-state performance of the existing algorithms with sufficiently large sample size. Furthermore, it is worth nothing that the proposed combination matrix adaptation is inextricably linked with the distributed reduced-rank collaborative beamforming.

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