A Robust LCMP Beamformer with Limited Snapshots

Maaz Mahadi⋆ *Tarig Ballal*⋆⋆ *Mohammad Moinuddin*⋆ *Tareq Y. Al-Naffouri*⋆⋆ *Ubaid Al-Saggaf*⋆

⋆ Department of Electrical and Computer Engineering, King Abdulaziz University, Jeddah, KSA
⋆⋆ CEMSE, King Abdullah University of Science and Technology (KAUST), Thuwal, KSA

***Abstract*—This paper deals with the problem of automatic diagonal loading for the linear constrained minimum power beamformer. To find the beamformer's weights, the linear constrained minimum power problem is reformulated into its generalized sidelobe canceller implementation, which is an unconstrained least-squares problem. To solve this problem, we utilize a bounded perturbation regularization approach where a perturbation matrix with a bounded norm is added to the linear transformation matrix of the least-squares problem in order to enhance the singular-value structure of the matrix. Compared to different diagonal loading methods, the proposed method shows superiority in performance when the number of snapshots is limited.**

***Index Terms*—Robust adaptive beamforming, LCMP, generalized sidelobe canceller, diagonal loading.**

I. Introduction

Beamforming is a spatial filtering technique that allows receiving desired signals impinging on an array from specific directions while suppressing undesired signals impinging from other directions [1].

Linearly Constrained Minimum Power (LCMP) beamforming minimizes the total power of the beamformer output such that a set of linear constraints that control the array beampattern is satisfied [2]. LCMP provides robustness against angle mismatch and perturbation in sensor locations [3]. Practically, the receiver does not have accurate spatial characteristic of a specific scenario. This makes filter designing methods base on assumptions that might not correspond completely to the actual parameters. Several reasons may attribute to this mismatch which include nonstationarity of the environment, multipath, small sample size, steering vector errors, etc. As a result, in the general case, performance of beamforemers deteriorate; hence, Robust Adaptive Beamforming (RAB) techniques that mitigate the effect of such mismatches are required [4]. In the literature, a variety of RAB techniques were proposed.

Interference-plus-Noise Covariance (INC) matrix reconstruction methods aim at reducing the effect of the signal of interest by reconstructing the INC [5], [6]. However, the reconstruction process increases the computational complexity.

An alternative RAB technique is the uncertainty set based technique. Methods that involve this technique estimate the signal of interest's steering vector by specifying a spherical uncertainty constraint on the steering vector [7]. However, the performance of these methods is limited to low Signal to Noise Ratio (SNR). In addition, these methods are computationally inefficient since they require solving second-order cone programming problems [6].

Steering vector projection is another variation of RAB techniques [8], [9]. The steering vector is replaced by its projection on the signal-plus-interference subspace of the sample covariance matrix, which reduces the effect of noise disturbance. Steering vector projection methods perform poorly at low SNRs. Also, they require perfect knowledge of the dimension of the signal-plus-interference subspace.

Diagonal Loading (DL) is a widely used RAB technique in which a constant diagonal matrix is added to the sample covariance matrix. This technique is also known as regularization in the statistical literature [3]. DL's performance depends on the choice of a scalar loading parameter. Choosing the optimal DL automatically is a challenging problem [4]. There is no rigorous way of selecting the parameter since it depends on the noise level or it is based on a norm constraint of the weight vector [10]. A few methods were proposed to tackle the problem of automatically choosing the DL parameter. DL methods are efficient if the exact steering vectors of the signal of interest and interference signals are known or small mismatches are exist.

The General Linear Combination-based (GLC) method [10] estimates the covariance matrix via a shrinkage method. Its estimation is based on the Minimum Mean Square Errors (MMSE) criterion. However, GLC's performance degrades when the number of sensors is relatively large [11]. A recent covariance matrix estimation technique for data sampled from elliptical symmetric distribution is proposed in [12]. It is also based on the estimation of the optimal shrinkage parameters that minimizes the mean squared error. The work in [13] considers computing diagonal loading automatically using a method proposed by Horel, Kennard and Baldwin (HKB) [14], which computes diagonal loading from the regularized least squares formulation. However, its performance degrades when the number of snapshots is relatively large [11].

In this paper, we propose a robust LCMP beamformer based on the bounded perturbation regularization approach [15]–[18]. To deal with the constraints in the LCMP beamforming problem, we used the generalized sidlelobe canceller of LCMP that reformulates the problem to an unconstrained least squares problem. The estimated sample covariance matrix which is included in the linear transformation matrix of the LS problem is normally ill-conditioned which makes using regularization approach desirable. The regularization parameter is computed using a procedure that combines a constrained equation with a mean squared error criterion. This allows for automatic adjustment of regularization parameter required by the proposed robust beamformer.

II. Background

A. Signal model and LCMP beamformer

We assume D narrowband far-field signals impinging on an array of N elements $(N > D)$. The n th snapshot of signals received by an array is given by [11]

$$\mathbf{x}(n) = \sum_{i=0}^{D-1} \mathbf{a}_i s_i(n) + \mathbf{v}(n), \tag{1}$$

where $\mathbf{x}(n) \in \mathbb{C}^N$, $\mathbf{a}_i$ denotes the steering vector associated with signal i, the subscript i denotes a set of narrowband signals with $i = 0$ (for the desired signal), $i = 1, 2 \cdots, D-1$ (for interference signals), and $\mathbf{v}(n) \in \mathbb{C}^N$ is a vector of Gaussian noise samples.

The output of a narrowband beamformer is obtained by multiplying the signal $\mathbf{x}(n)$ with a complex weight $\mathbf{w}$ and summing the result to obtain [19]

$$y(n) = \mathbf{w}^H \mathbf{x}(n). \tag{2}$$

For LCMP, these weights are selected to minimize the output power of the beamformer as follows [19]:

$$\mathbf{w}_{\text{opt}} = \underset{\mathbf{w}}{\operatorname{argmin}}\, \mathbf{w}^H \mathbf{R}_{\mathbf{x}} \mathbf{w} \quad \text{s.t.} \quad \mathbf{C}^H \mathbf{w} = \mathbf{g}, \tag{3}$$

where $\mathbf{R}_{\mathbf{x}} = \mathbb{E}[\mathbf{x}(n)\mathbf{x}^H(n)]$ is the data covariance matrix, $\mathbf{C} \in \mathbb{C}^{N \times P}$ is the constraint matrix, and $\mathbf{g}$ is a constraint vector with P elements. The optimum solution is given by

$$\mathbf{w}_{\text{opt}} = \mathbf{R}_{\mathbf{x}}{}^{-1}\mathbf{C}(\mathbf{C}^H \mathbf{R}_{\mathbf{x}}{}^{-1}\mathbf{C})^{-1}\mathbf{g}. \tag{4}$$

Practically, the true covariance matrix is unknown; thus, it is usually replaced by the sample covariance matrix

$$\widehat{\mathbf{R}}_{\mathbf{x}} = \frac{1}{K}\sum_{n=1}^{K} \mathbf{x}(n)\mathbf{x}^H(n), \tag{5}$$

where K is the number of snapshots. The estimated weights of the LCMP beamformer using (5) are given by

$$\mathbf{w}_{\text{lcmp}} = \widehat{\mathbf{R}}_{\mathbf{x}}^{-1}\mathbf{C}(\mathbf{C}^H \widehat{\mathbf{R}}_{\mathbf{x}}^{-1}\mathbf{C})^{-1}\mathbf{g}. \tag{6}$$

B. Generalized sidelobe canceller

The optimization (3) can be formulated differently by decomposing $\mathbf{w}$ into two components: The first one is in the constraint subspace, and the second one is orthogonal to the first [19], i.e.,

$$\mathbf{w} = \mathbf{w}_q - \mathbf{B}\mathbf{w}_a, \tag{7}$$

where $\mathbf{w}_q \in \mathbf{C}^N$ is a fixed *quiescent weight vector* calculated as

$$\mathbf{w}_q = \mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1}\mathbf{g}, \tag{8}$$

and $\mathbf{B} \in \mathbb{C}^{N \times (N-P)}$ is a *blocking matrix* which is orthogonal to $\mathbf{C}$ ($\mathbf{B}^H\mathbf{C} = [\mathbf{0}]_{\boldsymbol{N-P\times P}}$). The blocking matrix is not unique. It is chosen such that $\mathbf{B}^H\mathbf{B} = \mathbf{I}$ which can be calculated as the eigenvectors corresponding to the $N - P$ non-zero eigenvalues of $\mathbf{I} - \mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1}\mathbf{C}^H$.

By substituting (7) in (3) and replacing $\mathbf{R}_{\mathbf{x}}$ with $\widehat{\mathbf{R}}_{\mathbf{x}}$, the problem can be reformulated as the following unconstrained LS:

$$\min_{\mathbf{w}_a} (\mathbf{B}\mathbf{w}_a - \mathbf{w}_q)^H \widehat{\mathbf{R}}_{\mathbf{x}} (\mathbf{B}\mathbf{w}_a - \mathbf{w}_q), \tag{9}$$

or

$$\min_{\mathbf{w}_a} \|\mathbf{A}\mathbf{w}_a - \mathbf{b}\|^2, \tag{10}$$

where $\mathbf{A} \triangleq \widehat{\mathbf{R}}_{\mathbf{x}}^{\frac{1}{2}}\mathbf{B} \in \mathbb{C}^{N \times N-P}$ and $\mathbf{b} \triangleq \widehat{\mathbf{R}}_{\mathbf{x}}^{\frac{1}{2}}\mathbf{w}_q$. The above minimization corresponds to the following linear regression model:

$$\mathbf{b} = \mathbf{A}\mathbf{w}_a + \mathbf{z}, \tag{11}$$

where $\mathbf{z} \in \mathbb{C}^N$ is a residual error vector. Since $\widehat{\mathbf{R}}_{\mathbf{x}}^{\frac{1}{2}}$ is normally ill-conditioned, and $\mathbf{b}$ is noisy, the application of regularization to estimate $\mathbf{w}_a$ is desirable. The Regularized Least Squares (RLS) problem is stated as follows:

$$\min_{\mathbf{w}_a} \|\mathbf{A}\mathbf{w}_a - \mathbf{b}\|^2 + \gamma\|\mathbf{w}_a\|^2, \tag{12}$$

which has the closed-form solution

$$\hat{\mathbf{w}}_a = (\mathbf{A}^H\mathbf{A} + \gamma\mathbf{I})^{-1}\mathbf{A}^H\mathbf{b}. \tag{13}$$

Remark: *As can be seen from (13), we consider regularizing* $\mathbf{A}^H\mathbf{A}$*, which is of dimension* $(N-P)\times(N-P)$ *instead of* $\widehat{\mathbf{R}}_{\mathbf{x}}$ *which is of an* $N \times N$ *dimension. Hence, the inversion (13) is valid for fewer snapshots.*

C. The MSE of the RLS estimator

The Mean-Squared Error criterion (MSE) for the RLS is defined as

$$\text{MSE} = \text{tr}\Big[\mathbb{E}\big[(\hat{\mathbf{w}}_a - \mathbf{w}_a)(\hat{\mathbf{w}}_a - \mathbf{w}_a)^H\big]\Big], \tag{14}$$

where tr (.) denotes the matrix trace. The Singular Value Decomposition (SVD) of $\mathbf{A}$ is

$$\mathbf{A} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H, \tag{15}$$

where $\mathbf{U} \in \mathbb{C}^{N \times N}$, $\boldsymbol{\Sigma} = \text{diag}[\sigma_1, \sigma_2, \cdots \sigma_{N-P}, \underbrace{0, \cdots 0}_{P \text{ zeros}}]^T$, with $\sigma_1 > \sigma_2 > \cdots > \sigma_{N-P}$, and $\mathbf{V} \in \mathbb{C}^{N-P \times N}$.

Substituting Equation (15) in (11), substituting the result in (13), and substituting the final result in (14) and manipulating, we obtain

$$\begin{aligned}\text{MSE} =& \sigma_z^2 \text{tr}\left[\boldsymbol{\Sigma}^2\left(\boldsymbol{\Sigma}^2 + \gamma\mathbf{I}\right)^{-2}\right] \\ &+ \gamma^2 \text{tr}\left[\left(\boldsymbol{\Sigma}^2 + \gamma\mathbf{I}\right)^{-2}\mathbf{V}^H\mathbf{R}_{\mathbf{w}_a}\mathbf{V}\right],\end{aligned} \tag{16}$$

where σ_z^2 is the noise variance, and $\mathbf{R}_{\mathbf{w}_a} \triangleq \mathbb{E}[\mathbf{w}_a \mathbf{w}_a^H]$. The derivative of (16) with respect to γ can be taken and a critical point can be obtained by solving

$$\begin{aligned}\text{MSE}' = & -2\sigma_z^2 \text{tr}\left[\mathbf{\Sigma}^2\left(\mathbf{\Sigma}^2+\gamma\mathbf{I}\right)^{-3}\right] \\ & + 2\gamma \text{tr}\left[\mathbf{\Sigma}^2\left(\mathbf{\Sigma}^2+\gamma\mathbf{I}\right)^{-3}\mathbf{V}^H\mathbf{R}_{\mathbf{w}_a}\mathbf{V}\right] = 0.\end{aligned} \tag{17}$$

However, a closed-form solution cannot be produced from Equation (17). An approximated formula can be written as follows (Equation (5) in [20]):

$$\begin{aligned}\text{MSE}' \approx & -2\sigma_z^2 \text{tr}\left[\mathbf{\Sigma}^2\left(\mathbf{\Sigma}^2+\gamma\mathbf{I}\right)^{-3}\right] \\ & + 2\gamma\frac{\text{tr}\left(\mathbf{R}_{\mathbf{w}_a}\right)}{N} \text{tr}\left[\mathbf{\Sigma}^2\left(\mathbf{\Sigma}^2+\gamma\mathbf{I}\right)^{-3}\right] = 0.\end{aligned} \tag{18}$$

Solving for γ, we can obtain the approximate minimizer of the MSE in (16). That is

$$\gamma_o \approx \frac{N\sigma_z^2}{\text{tr}(\mathbf{R}_{\mathbf{w}_a})}. \tag{19}$$

However, optimal γ depends on unknown statistics of the signal. The following section describes our method to obtain γ directly from the observed signals.

III. THE BOUNDED PERTURBATION REGULARIZATION APPROACH

The proposed model that is used for estimating a vector quantity $\mathbf{w_a} \in \mathbb{C}^N$ is

$$\mathbf{b} \approx (\mathbf{A}+\Delta)\mathbf{w}_a + \mathbf{z}, \tag{20}$$

where $\Delta \in \mathbb{C}^{N\times N-P}$ is a perturbation matrix. The aim of this perturbation is to modify the singular values of $\mathbf{A}$ in a way that helps improve the estimation of the vector $\mathbf{w}_a$. The perturbation matrix Δ is chosen such that:

$$\|\Delta\|_2 \leq \zeta, \tag{21}$$

where ζ is an unknown constant. The problem formulation for estimating $\mathbf{w}_a$ can be written as a min-max optimization problem

$$\begin{aligned}&\min_{\hat{\mathbf{w}}_\mathbf{a}} \max_{\mathbf{\Delta}} \|\mathbf{b}-(\mathbf{A}+\Delta)\hat{\mathbf{w}}_a\|_2 \\ &\quad \text{subject to} \|\Delta\|_2 \leq \zeta,\end{aligned} \tag{22}$$

that is, we seek an estimate of $\mathbf{w}_a$ that minimizes the maximum residual error over all possible bounded perturbations Δ.

Using Minkowski's inequality [21] and manipulating, the problem can be reformulated as the following equivalent minimization problem [16]:

$$\min_{\hat{\mathbf{w}}_a} ||\mathbf{b}-\mathbf{A}\hat{\mathbf{w}}_a||_2 + \zeta||\hat{\mathbf{w}}_a||_2, \tag{23}$$

which has a solution given by (13) with the following constraint:

$$\gamma = \frac{\zeta\|\mathbf{b}-\mathbf{A}\hat{\mathbf{w}}_a\|_2}{\|\hat{\mathbf{w}}_a\|_2}, \tag{24}$$

where γ is the unknown regularization parameter. Substituting (24) in (13) and manipulating , we obtain the following equation:

$$\begin{aligned}f_1(\gamma) = & \gamma^2\mathbf{b}^H\mathbf{A}(\mathbf{A}^H\mathbf{A}+\gamma\mathbf{I})^{-2}\mathbf{A}^H\mathbf{b} \\ & - \zeta^2\big[\mathbf{b}^H\mathbf{b} - \mathbf{b}^H\mathbf{A}(\mathbf{A}^H\mathbf{A}+\gamma\mathbf{I})^{-1}\mathbf{A}^H\mathbf{b} \\ & - \gamma\mathbf{b}^H\mathbf{A}(\mathbf{A}^H\mathbf{A}+\gamma\mathbf{I})^{-2}\mathbf{A}^H\mathbf{b}\big] = 0.\end{aligned} \tag{25}$$

Applying the SVD (25) and manipulating we obtain

$$f_2(\gamma) = \mathbf{b}^H\mathbf{U}(\mathbf{\Sigma}^2-\zeta^2\mathbf{I})(\mathbf{\Sigma}^2+\gamma\mathbf{I})^{-2}\mathbf{U}^H\mathbf{b} = 0. \tag{26}$$

Equation (26) can be used to find γ. However, it requires knowing the value of ζ. To remove dependency on ζ, we seek an optimal value, ζ_o, that satisfies (26) on average, i.e.,

$$\mathbb{E}\left[\mathbf{b}^H\mathbf{U}\left(\mathbf{\Sigma}^2-{\zeta_o}^2\mathbf{I}\right)\left(\mathbf{\Sigma}^2+\gamma_o\mathbf{I}\right)^{-2}\mathbf{U}^H\mathbf{b}\right] = 0. \tag{27}$$

Solving for ζ_o^2 and manipulating, we obtain

$$\zeta_o^2 = \frac{\text{tr}\left[\mathbf{\Sigma}^2\left(\mathbf{\Sigma}^2+\gamma_o\mathbf{I}\right)^{-2}\mathbf{U}^H\mathbf{R}_\text{b}\mathbf{U}\right]}{\text{tr}\left[\left(\mathbf{\Sigma}^2+\gamma_o\mathbf{I}\right)^{-2}\mathbf{U}^H\mathbf{R}_\text{b}\mathbf{U}\right]}, \tag{28}$$

where $\mathbf{R}_\text{b} = \mathbb{E}[\mathbf{b}\mathbf{b}^H] = \mathbf{U\Sigma V}^H\mathbf{R}_{\mathbf{w}_a}\mathbf{V\Sigma U}^H + \sigma_z^2\mathbf{I}_n$. Substituting for $\mathbf{R}_\text{b}$ in (28) yields

$$\zeta_o^2 = \frac{\sigma_z^2\text{tr}\Big[\mathbf{\Sigma}^2(\mathbf{\Sigma}^2+\gamma_o\mathbf{I})^{-2}\Big] + \text{tr}\Big[\mathbf{\Sigma}^4(\mathbf{\Sigma}^2+\gamma_o\mathbf{I})^{-2}\mathbf{V}^H\mathbf{R}_{w_a}\mathbf{V}\Big]}{\sigma_z^2\text{tr}\Big[\mathbf{\Sigma}^2(\mathbf{\Sigma}^2+\gamma_o\mathbf{I})^{-2}\Big] + \text{tr}\Big[\mathbf{\Sigma}^2(\mathbf{\Sigma}^2+\gamma_o\mathbf{I})^{-2}\mathbf{V}^H\mathbf{R}_{w_a}\mathbf{V}\Big]}. \tag{29}$$

Applying a similar approximation to that used in (18), we reach

$${\zeta_o}^2 \approx \frac{\sigma_z^2\text{tr}\Big[\mathbf{\Sigma}^2(\mathbf{\Sigma}^2+\gamma_o\mathbf{I})^{-2}\Big] + \frac{\text{tr}(\mathbf{R}_{w_a})}{N}\text{tr}\Big[\mathbf{\Sigma}^4(\mathbf{\Sigma}^2+\gamma_o\mathbf{I})^{-2}\Big]}{\sigma_z^2\text{tr}\Big[\mathbf{\Sigma}^2(\mathbf{\Sigma}^2+\gamma_o\mathbf{I})^{-2}\Big] + \frac{\text{tr}(\mathbf{R}_{w_a})}{N}\text{tr}\Big[\mathbf{\Sigma}^2(\mathbf{\Sigma}^2+\gamma_o\mathbf{I})^{-2}\Big]}, \tag{30}$$

which can be written as

$$\zeta_o^2 = \frac{\text{tr}[\mathbf{\Sigma}^2(\mathbf{\Sigma}^2+\gamma_o\mathbf{I})^{-1}]}{\text{tr}[(\mathbf{\Sigma}^2+\gamma_o\mathbf{I})^{-1}]}. \tag{31}$$

Finally, substituting (31) in (26) and replacing γ_o with γ and manipulating yields

$$\begin{aligned}f_3(\gamma) = & \text{tr}\Big[(\mathbf{\Sigma}^2+\gamma\mathbf{I})^{-1}\Big]\text{tr}\Big[(\mathbf{\Sigma}^2+\gamma\mathbf{I})^{-1}\mathbf{d}\mathbf{d}^H\Big] \\ & - N\Big[(\mathbf{\Sigma}^2+\gamma\mathbf{I})^{-2}\mathbf{d}\mathbf{d}^H\Big] = 0,\end{aligned} \tag{32}$$

where $\mathbf{d} \triangleq \mathbf{U}^H\mathbf{b}$. Equation (32) is the BPR equation which can be solved using Newton's or any other suitable method [16].

Finally, we substitute the regularization parameter, γ, obtained from (32) in the loaded version of (6), to obtain the following:

$$\mathbf{w}_\text{DL} = (\widehat{\mathbf{R}}_\mathbf{x}+\gamma\mathbf{I})^{-1}\mathbf{C}(\mathbf{C}^H(\widehat{\mathbf{R}}_\mathbf{x}+\gamma\mathbf{I})^{-1}\mathbf{C})^{-1}\mathbf{g}. \tag{33}$$

IV. Simulation Results

To evaluate performance, the output signal-to-interference-and-noise-ratio (SINR) is considered. Equation (1) can be written as $\mathbf{x}(n) = \mathbf{x}_s(n) + \mathbf{x}_{iv}(n)$, where $\mathbf{x}_s(n) \triangleq \mathbf{a}_0 s_0(n)$ and $\mathbf{x}_{iv}(n) \triangleq \sum_{i=1}^{D-1} \mathbf{a}_i s_i(n) + \mathbf{v}(n)$. The output SINR is defined as follows:

$$\text{SINR} = \frac{\mathbb{E}[|\mathbf{w}^H \mathbf{x}_s(n)|^2]}{\mathbb{E}[|\mathbf{w}^H \mathbf{x}_{iv}(n)|^2]} \quad (34)$$

$$= \frac{\sigma_s^2 |\mathbf{w}^H \mathbf{a}_{0t}|^2}{\mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w}}, \quad (35)$$

where $\mathbf{R}_{i+n} = \mathbb{E}[\mathbf{x}_{iv}(n)\mathbf{x}_{iv}^H(n)]$ is the INC matrix, σ_s^2 is the power of the signal of interest, and $\mathbf{a}_{0t}$ is the actual steering vector of the desired signal.

In all scenarios, we compare the proposed LCMP-BPR with HKB [14], [13], elliptical regularized sample covariance matrix (ELL-RSCM) [12], GLC [10], multichannel wiener filtering based noise reduction with truncated minimum mean square error criterion (MWF-TMMSE) [11], and tridiagonal loading (TRI) [22] methods. Similar to the proposed method, HKB and TRI methods are one parameter diagonal loading methods. The other methods, Ell-RSCP, GLC and MWF-TMMSE, estimate the covariance matrix via a shrinkage method that uses two regularization parameters.

A uniform linear array (ULA) of $N = 10$ elements with $d = 0.5\lambda$ between consecutive elements is used in all simulations, where λ is the wavelength. Uncertainty in the direction of arrival (DOA) of the signal of interest is modeled as a uniform distribution in the range $[-2°, 2°]$. We consider six interference signals $(D = 6)$ impinging on the array.

The signal of interest and interference signals are complex Gaussian data generated randomly with SNR = 5 dB and Interference-to-Noise Ratio (INR = 20 dB). The noise is complex white Gaussian with unit-norm power. Source locations are randomly chosen in every iteration. All SINR curves are obtained by averaging over 2×10^4 independent trials. For this LCMP beamformer, three signals out of the six interference signals are constrained to nulls, i.e., the constraint vector is $\mathbf{g} = [1, 0, 0, 0]^T$ $(P = 4)$.

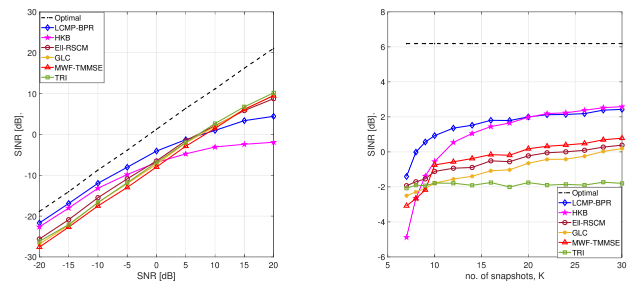
Fig. 1 (a) shows the output SINR versus SNR. As can be seen from the figure, for SNR $\leq$ 10 dB, the proposed LCMP-BPR method achieves the best performance among all the methods. For 10 dB $<$ SNR $\leq$ 20 dB, LCMP-BPR exhibits an inferior performance compared to the other methods except HKB.

Fig. 1 (b) shows the output SINR versus the number of snapshots, K. As can be seen, when $N - P < K < 2N$, LCMP-BPR outperforms all the other techniques. However, for $2N \leq K \leq 3N$, HKB shows a slight advantage over the proposed LCMP-BPR.

It is worth mentioning that the performance of our method is highly sensitive to the effect of the desired signal in the estimated INC. This explains the good performance of our method at low SNRs. To elaborate , we consider estimating the INC matrix from the noise and interference samples generated in the simulation, i.e.,

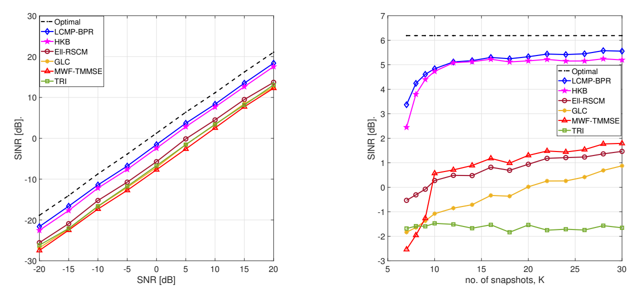
$$\widehat{\mathbf{R}}_{i+n} = \frac{1}{K} \sum_{n=1}^{K} \mathbf{x}_{iv}(n)\mathbf{x}_{iv}^H(n), \quad (36)$$

and use it instead of $\widehat{\mathbf{R}}_{\mathbf{x}}$. Fig. 2 (a) illustrates the output SINR versus SNR performance for INC estimated using (36) . It is clear that our method outperforms all the methods over the entire SNR range. Similarly, Fig. 2 (b) shows the performance of SINR versus the number of snapshots with enhanced estimation of INC. It is evident that both LCMP-BPR and HKB outperform the methods with a good margin. The superiority of BPR-LCMP is more emphasized for $N - P < K \leq 3N$.



(a) SINR versus SNR ($K = 7$) (b) SINR versus K (SNR = 5 dB)

Fig. 1. Output SINR performance.



(a) SINR versus SNR ($K = 7$) (b) SINR versus K (SNR = 5 dB)

Fig. 2. Output SINR performance (better estimation of INC matrix).

V. Conclusion

We propose the LCMP-BPR beamformer based on the bounded perturbation regularization approach. A generalized sidelobe canceller implementation of a linearly constrained minimum power was considered. The constrained LCMP problem was reformulated into an unconstrained least squares problem. Simulation results show that the proposed LCMP-BPR method is effective in scenarios with a ULA of N elements, P constraints, and limited number of snapshot $K \in (N - P, 3N]$.

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